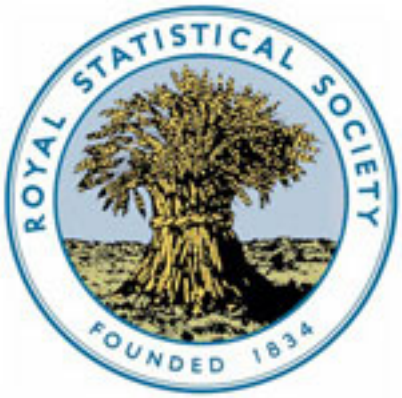




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A Graphic Display for Characterization of Seasonal Pattern Similarities of Time Series

ADI RAVEH
Department of Statistics, The Hebrew University, Jerusalem, and Stanford University, Stanford, CA 94305, USA

A method of graphic displaying of the relative distances among a group of seasonal patterns of time series is provided. This paper presents two ideas: 1. some distance measures of the different series or every two different periods on the same single series, 2. a graphic display technique. Both ideas are not new; nevertheless, its application specifically for finding common seasonal patterns among time series is a new one. One particular Multidimensional Scaling (MDS) algorithm was adopted, that is, the non-metric SSA-I (Smallest Space Analysis) technique. The group of seasonal patterns are transformed into a symmetric matrix by defining indices of similarity. The symmetric matrix is the input for the graphic display techniques. As a result, a set of objects, either the n *various time series or the* n *periods of a single time series is presented in a map, demonstrating the pairwise interrelationships of the set.*

Introduction

In the empirical study of time series we usually try to learn about the pairwise interrelationships of a given set. Often these series are decomposed into their respective trend, seasonal and irregular (random) components. This paper presents a method for comparing time series. The comparison is based on the aspect of seasonality of the proposed series. A series is said to be decomposed into three components: a trend, a seasonal and an irregular (or random) component when each of these components is empirically identified. For example, a multiplicative monthly time series with constant seasonal pattern of length 12 units of time may be described as follows:

$$Y_{aj} = T_{aj} * S_j * I_{aj} \qquad a = 1, \ldots, r; \qquad j = 1, \ldots, 12 \tag{1}$$

where Y_{aj}, T_{aj}, I_{aj} are the original observation, the trend component and the irregular component in the jth month of the ath year, respectively.

S_j is the seasonal component in the jth month. It is convenient to keep the trend component in the scale of original observations by setting the following constraint:

$$\sum_{j=1}^{p} S_j = p, \tag{2}$$

p is the period's length and the model is multiplicative. The vector $S=(S_1, \ldots, S_p)$ is known in the literature under various names such as "Seasonal Pattern", "Seasonality Index", etc., and is expressed usually in percentages.

Seasonal adjustment methods try to estimate either a constant seasonal pattern (see Durbin and Murphy, 1975; Raveh, 1978) or some scheme of varying seasonality (see Burman, 1965; Shiskin *et al.*, 1967). Extension of the above analysis is made by considering the "seasonal similarity" between time series. That is, the similarity of time series seasonal patterns once these are decomposed into their respective components, is investigated. In this paper, given seasonal patterns are treated as vectors for the application of a scaling technique. To do so, the non-metric SSA-I technique of Guttman (1968) is applied to study the "mutual relationships" of multiplicative seasonal components of time series. The "mutual relationships" are defined more specifically by indices of "distance" or "dissimilarity" between every couple of patterns (i.e. between every constant seasonal pattern of two different time series, or between every two seasonal patterns of a single series which has varying seasonality).

Given the three seasonal patterns of Figure 1, it is seen that the patterns

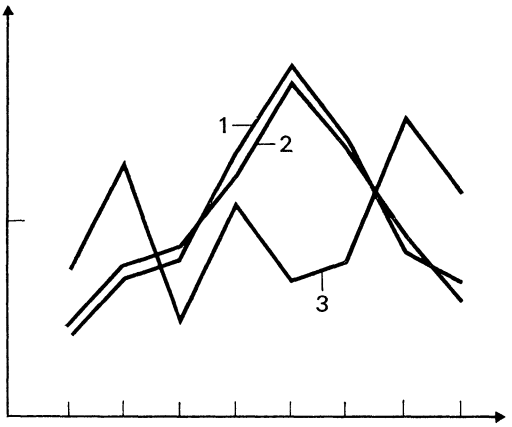


Figure 1

Three seasonal patterns of time series.

of series 1 and 2 are very similar, while that of series 3 is quite different. The proposed method would demonstrate this situation by using a graphic display in which the series 1 and 2 would be located close to each other and series 3 far away.

To encourage the approach developed here, two examples will be considered. The first example presents economic monthly time series of "Tourists Arrival in Israel"; the second (demographic) example demonstrates gradual change in seasonal patterns of the single series "Jewish Marriages" in Israel for the period 1956–68.

Dissimilarity Between Seasonal Patterns

Let us assume n seasonal patterns (vectors of p values) associated to n time series, each with period length of p months, e.g. $p=12$. Each of these patterns is estimated by one of the available decomposition methods (e.g. Shiskin X-11 technique, 1967; Raveh, 1978). Let $S_i=(S_1, \ldots, S_p)$ be a p-vector of seasonality patterns of the ith series, $i=1, \ldots, n$. Each one of the p-coordinates corresponds to a jth unit of time within a whole period. For the multiplicative model, we have by definition (expressed in percentages):

$$\sum_{k=1}^{p} S_{ik}=100.p \qquad (i=1, \ldots, n) \tag{3}$$

Denote by D_{ij} a dissimilarity coefficient between any two patterns, S_i and S_j. This dissimilarity may be defined in several ways. For example, say dissimilarity is defined as the average absolute distance difference between the two seasonal vectors, i.e.

$$D_{ij}=\left(\sum_{k=1}^{p} |S_{ik}-S_{jk}|\right)/p \qquad (i,j=1, 2, \ldots, n) \tag{4}$$

Clearly, (4) defines a symmetric matrix ($D_{ij}=D_{ji}$), and therefore the mutual relationship of seasonal patterns is represented by a $n \times n$ symmetric matrix with zero elements along the diagonal. Other measures having the symmetric property and expressing dissimilarity may be used as well.

Another suggestion for a dissimilarity coefficient between any two patterns S_i and S_j is B_{ij} which is defined as the area between them, e.g. B_{12} is measured by the shaded area in the following Figure 2.

A further approach, that of "Minimum of Possible Dissimilarity", may be to define D'_{ij} in the following way:

$$D'_{ij}=\min_{0 \leqslant l \leqslant p-1}\left\{\left(\sum_{k=1}^{p} |S_{ik}-S_{jk'}|\right)/p\right\} \tag{5}$$

where $k'=(k+l) \bmod p$, that is (for example) for $l=3, p=12$ and $k=1, \ldots,$

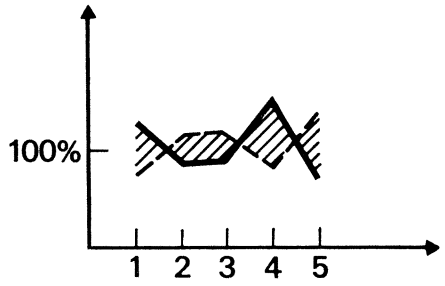


Figure 2

Seasonal patterns: S_1 *is denoted by* ——, S_2 *is denoted by* – – –. *The shaded area is the coefficient of dissimilarity* B_{12}.

12 then $k'=4, 5, \ldots, 12, 1, 2, 3$. The l for which the function obtains its minimum is the desired lag.

In other words, D'_{ij}'s are the minimum absolute mean differences between the two seasonal indices of series i and j where these differences are computed for the 12 (0, ..., 11) various lags. This approach may be adopted to B'_{ij}'s which minimizes the area between the graphs obtained by two seasonal patterns S_i and S_j. The use of this approach discriminates between the following two cases that are graphed in Figure 3. In both graphs (a) and (b) the coefficients of dissimilarity B_{12} ($i=1, j=2$) are equal. By using the approach of "Minimum of Possible Dissimilarity" $B_{12}=0$ for graph (b) with the desired lag $l=1$. $B'_{12}>0$ for graph (a) for $l=0, \ldots, 11$.

These measures were used as examples of possible indices for characterizing the dissimilarity between the seasonal patterns. In order to

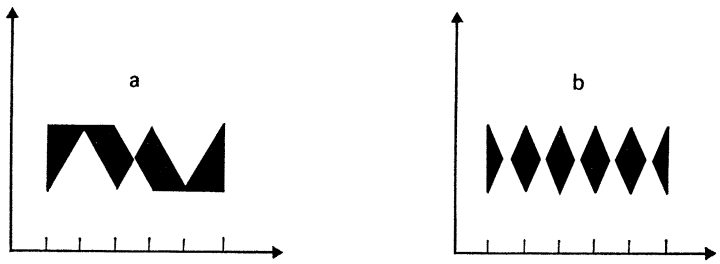


Figure 3

Two seasonal patterns S_1, S_2 *of 5 units and the measure of dissimilarity between them,* B_{12} *based on area*

represent the $n(n-1)/2$ accepted dissimilarity coefficients D_{ij} or B_{ij} simultaneously in a graphic display, the SSA-I technique is used.

The SSA-I Technique: A Brief Review

The family of methods called "Smallest Space Analysis" or briefly, SSA techniques, were developed and constructed by Guttman (1968) (see also Lingoes, 1973). Here the use of the first member of the family SSA-I is demonstrated, since it is particularly suitable for data analysis of symmetric matrices.

SSA-I provides a graphic presentation of pairwise interrelationships of a set of objects (here, n time series). Each series i (here, a seasonal pattern of the i series), $i=1, \ldots, n$, is represented as a point in a space and the SSA-I technique seeks the (Euclidean) space with minimum dimensions that can monotonely reproduce the original dissimilarities D_{ij}. That is, the dimensionality m is sought to be as small as possible for which the Euclidean metric will satisfy the monotonicity condition below:

$$d_{ij} < d_{kl} \quad \text{iff} \quad D_{ij} < D_{kl} \qquad \text{for every } i, j, k, l \tag{6}$$

D_{ij}: The dissimilarity coefficient (input) between series i and j.

d_{ij}: The Euclidean distance (output) in the graphic map between the points i and j.

The matrix of dissimilarities expresses interrelationships between items, while the computer program enables the analysis of these interrelationships in a simple yet comprehensive manner. The main part of the SSA-I output is the *space diagram*, in which each variable is represented by a point. If the dissimilarity coefficient between i and j is larger than the dissimilarity coefficient between k and l, then the Euclidean distance between i and j is larger than that between k and l.

The goodness-of-fit of an output space to the input dissimilarities is assessed by a *coefficient of alienation*. This coefficient is defined by $\theta = \sqrt{1-\mu^2}$ and it varies between 0 and 1 where μ is a coefficient of monotonicity of (D_{ij}, d_{ij}) $(i, j=1, \ldots, n)$; and is defined in (7) below:

$$\mu = \frac{\sum_{i,j=1}^{n} D^*_{ij} . d_{ij}}{\sqrt{\sum_{i,j=1}^{n} D^{*2}_{ij} \sum_{i,j=1}^{n} d^2_{ij}}} \tag{7}$$

The D^*_{ij}'s are chosen to be the rank images of the d_{ij}'s, namely the d_{ij}'s rearranged in the rank order of the D_{ij}'s, allowing for untying of ties. Perfect fit is represented by $\theta=0$, and the worst possible fit is given by $\theta=1$. Inter-

mediate values of the coefficient represent intermediate degrees of goodness-of-fit. How small should the coefficient of alienation be for the fit to be satisfactory? This is a question to which there is no absolute answer (see Guttman, 1977, p. 89). As a rule-of-thumb, a coefficient of alienation of less than 0·15 is considered a good candidate for being "satisfactory"; a more complete answer requires fit to theory. A value of $\theta = 0{\cdot}15$ is equivalent to $\mu = 0{\cdot}99$ which often seems quite high. It is obvious that for a given matrix the coefficient decreases as m – the dimensionality of the space – increases. Mathematically, it is always possible to locate n points in $(n-1)$ dimensional space keeping condition (6) and thus $\theta = 0{\cdot}0$. A configuration of n points in m dimensional space when $m \ll n$ might be helpful for the researcher to characterize the empirical data. In the examples below, two $(m=2)$ and one $(m=1)$ dimensions for the space diagram were sufficient to characterize the data (with θ small enough).

An Artificial Example

Let us take the two time series below:

1. $Y_{1t} = \sin \dfrac{2\pi}{p} t \qquad (t = 1, 2, \ldots)$

2. $Y_{2t} = \cos \dfrac{2\pi}{p} t \qquad (t = 1, 2, \ldots).$

These two series might be decomposed into the three components. The trend and the irregular components are equal, $T_t = I_t = 1$. This means that the trend has no slope and there is no randomness. $S_1 = (\sin(2\pi/p), \ldots, \sin 2\pi)$ for given p. For instance, when $p = 4$,

$$S_1 = \left(\sin \frac{\pi}{2}, \sin \pi, \sin \frac{3\pi}{2}, \sin 2\pi\right) = (1, 0, -1, 0)$$

$$S_2 = \left(\cos \frac{\pi}{2}, \cos \pi, \cos \frac{3\pi}{2}, \cos 2\pi\right) = (0, -1, 0, 1)$$

$$D_{12} = \sum_{k=1}^{4} |S_{1k} - S_{2k}|)/4 = 1{\cdot}0$$

The coefficient of "Minimum of Possible Dissimilarity" vanishes ($D_{\hat{l}2} = 0$) when $l = 3$ is the desired lag.

The Method

Given a set of n time series the proposed method is described as a sequence

of steps:

Step 1: Decompose each series into its components using one of the decomposition methods mentioned above in the Introduction. Thus from each series $\{Y_{aj}\}\, a=1,\ldots,r; j=1,\ldots,p$; $S=(S_1,\ldots,S_p)$ is obtained.

Step 2: For each pair of series a coefficient of dissimilarity as mentioned in the section Dissimilarity Between Seasonal Patterns is computed. Thus from n seasonal patterns S_i, $i=1, \ldots, n$, $n(n-1)/2$ coefficients are obtained.

Step 3: The symmetric matrix of coefficients $(n \times n)$ is used as the input to the non-metric technique SSA-I. As a result a graphic display is obtained. This presentation is demonstrated in m-dimension space diagram.

Step 4: Analysing the output of the SSA-I. It might be possible that characterization of the various seasonal patterns would be revealed for m-dimensionality and θ – a coefficient of goodness-of-fit quite low. Thus common seasonal patterns among time series might be found.

These steps are presented in a flowchart in Figure 4.

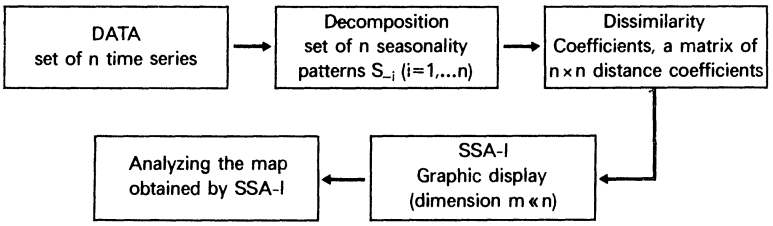


Figure 4

Flowchart of the various steps of the proposed method.

Applications

Below, two examples are considered which demonstrate both the usefulness and applicability of the method introduced in the paper. For convenience, the first is a primarily economic example for which data are available and where the relationship between the seasonality of time series is important. The second, a demographic example, demonstrates a very specific matrix structure: the Simplex. These two real examples are described by reference to the steps in the previous section.

Example 1: Tourists' Arrivals in Israel

In this example $n=16$ series of "Tourists' Arrivals in Israel" during 1974–76 are presented. The series 1–12 are "Tourist arrivals by air from 12 coun-

Table 1

Seasonal indices of the 16 time series on "Tourists Arrival to Israel" (multiplicative model)

Tourists' arrival from	*Jan.*	*Feb.*	*Mar.*	*Apr.*	*May*	*June*	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*
1. USA	98·0	130·3	139·4	110·5	101·0	112·4	145·5^{p}	72·6	62·6	106·8	63·2	57·7^{t}
2. England	57·4	59·9	12·09	176·6^{p}	114·2	84·3	149·2	124·3	101·0	90·4	48·3^{t}	73·4
3. France	42·0	79·0	133·1	85·3	76·8	86·0	225·8	252·2^{p}	63·3	53·2	39·6^{t}	64·6
4. Scandinavia	66·6	97·8	175·0	176·3^{p}	95·4	100·0	81·9	58·7^{t}	64·8	102·9	76·1	104·3
5. Canada	51·8^{t}	108·2	125·7	126·9	141·2	101·7	142·0^{p}	77·9	82·4	109·1	55·0	78·0
6. Germany and Austria	38·6^{t}	68·6	183·1	214·4^{p}	76·9	64·2	97·4	89·4	91·1	136·9	57·1	82·3
7. Switzerland	46·7^{t}	84·0	131·5	208·5^{p}	100·5	54·2	133·4	56·9	98·7	150·1	56·0	79·7
8. Holland	42·6^{t}	70·6	117·0	163·7^{p}	111·2	97·3	175·1	80·9	109·7	114·8	55·3	61·6
9. Italy	48·6^{t}	75·3	119·9	144·4	85·7	72·2	100·5	145·9	167·5^{p}	78·7	56·7	104·6
10. South Africa	171·4^{p}	83·6	85·5	135·2	105·1	93·5	74·1	95·3	89·5	78·9	53·6^{t}	134·3
11. South America	101·3	46·1^{p}	51·7	127·1	93·2	76·4	108·6	94·1	145·4	135·9	74·1	146·0^{p}
12. Oceania	116·5	71·0	100·0	126·1	110·5	91·9	94·0	88·8	81·5	83·3	68·0^{t}	168·3^{p}
13. Total	55·6^{t}	73·1	116·3	134·0	93·1	97·4	153·2^{p}	121·7	90·2	109·0	69·5	87·0
14. By land	53·7^{t}	57·8	117·4	112·4	88·9	118·1	174·2^{p}	127·7	87·2	122·8	68·5	71·2
15. By air	57·2^{t}	79·1	121·2	137·4	95·0	93·9	150·7^{p}	118·2	86·6	105·9	67·0	87·7
16. Other countries	50·0^{t}	64·1	117·6	173·6^{p}	86·7	89·3	160·2	134·9	96·2	78·1	60·6	88·7

Remark: The symbol p indicates the peak of the seasonality, the symbol t indicates the trough of the seasonality.

Table 2

The (symmetric) matrix of distances among the 16 seasonal patterns of the time series as computed from Table 1 using formula (5) with p 12. The values in parentheses are the suitable lags in which the minimum is obtained.

	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
1.	0·0															
2.	24·0	0·0														
	(11)															
3.	37·4	39·4	0·0													
	(11)	(0)														
4.	27·2	30·7	33·3	0·0												
	(0)	(1)	(4)													
5.	17·4	21·2	44·3	26·8	0·0											
	(0)	(0)	(0)	(0)												
6.	40·0	28·8	35·9	25·4	33·2	0·0										
	(11)	(0)	(4)	(0)	(0)											
7.	33·2	23·4	43·4	28·3	24·5	17·3	0·0									
	(0)	(0)	(4)	(0)	(0)	(0)										
8.	26·7	15·0	41·4	27·6	17·7	28·3	21·8	0·0								
	(0)	(0)	(0)	(0)	(0)	(0)..	(0)									
9.	19·2	23·8	30·9	21·1	29·1	27·5	25·6	30·1	0·0							
	(10)	(0)	(11)	(7)	(10)	(7)	(7)	(0)								
10.	23·1	16·5	41·1	25·4	26·3	28·3	27·6	25·5	25·6	0·0						
	(2)	(3)	(7)	(2)	(2)	(3)	(3)	(3)	(8)							
11.	24·1	31·3	44·0	24·8	26·4	36·2	35·8	31·9	22·2	25·6	0·0					
	(6)	(7)	(10)	(6)	(7)	(6)	(6)	(9)	(11)	(4)						
12.	24·4	20·6	42·9	25·4	25·6	27·2	32·7	28·9	24·4	15·4	23·9	0·0				
	(10)	(4)	(3)	(3)	(3)	(3)	(4)	(4)	(4)	(0)	(0)					
13.	21·4	13·9	37·5	22·3	16·2	29·3	26·3	16·0	22·6	18·4	25·4	17·2	0·0			
	(11)	(0)	(0)	(8)	(0)	(0)	(0)	(0)	(0)	(9)	(2)	(9)				
14.	25·2	19·1	35·3	21·2	22·1	34·6	30·3	18·3	30·6	24·2	25·5	21·7	10·5	0·0		
	(0)	(0)	(0)	(8)	(0)	(0)	(9)	(0)	(0)	(6)	(2)	(9)	(0)			
15.	20·0	13·2	36·8	23·6	15·1	29·1	24·7	17·0	22·1	17·6	25·6	17·7	3·1	12·7	0·0	
	(11)	(0)	(0)	(8)	(0)	(0)	(0)	(0)	(0)	(9)	(2)	(9)	(0)	(0)		
16.	24·4	9·7	35·4	26·9	25·3	28·3	26·2	17·3	18·7	13·5	29·5	23·7	11·5	16·9	12·2	
	(11)	(0)	(0)	(8)	(0)	(0)	(0)	(0)	(0)	(9)	(1)	(8)	(0)	(0)	(0)	0·0

tries" (or a group of countries). Series 13 is a "Tourist-arrival" total. Series 14 and 15 are "Tourists arriving by land and by air, respectively". Series 16 is "Tourists arriving from other countries" (not included in series 1–12). In step 1, 16 constant seasonality patterns were computed by a non-metric technique using a multiplicative model (see Raveh, 1978, 1980). The seasonal patterns are given in Table 1.

In step 2 the distance between each pair of seasonal patterns is measured by the coefficient of dissimilarity defined in equation (5). The period's length is $p=12$. Thus $(16\times 15)/2=120$ coefficients were obtained and are given in Table 2 . The desired lags, that is, those lags where the $D_{ij}^{\prime}$ obtains its minimum are given in the parentheses, just below the $D_{ij}^{\prime}$ themselves.

In step 3, the matrix of distances (16×16) is used as the input to SSA-I. Applying the technique (SSA-I) for $m=2$ dimensions for the space diagram a graphic display was obtained and is presented in Figure 5. The points in the map indicate the geometric location of the series. Each point represents a time series, or more accurately, its seasonal pattern. Two pairs of points are closer on the map than other pairs of points as their similarity is greater. The coefficient of alienation $\theta=0{\cdot}16$ is small enough to indicate that there

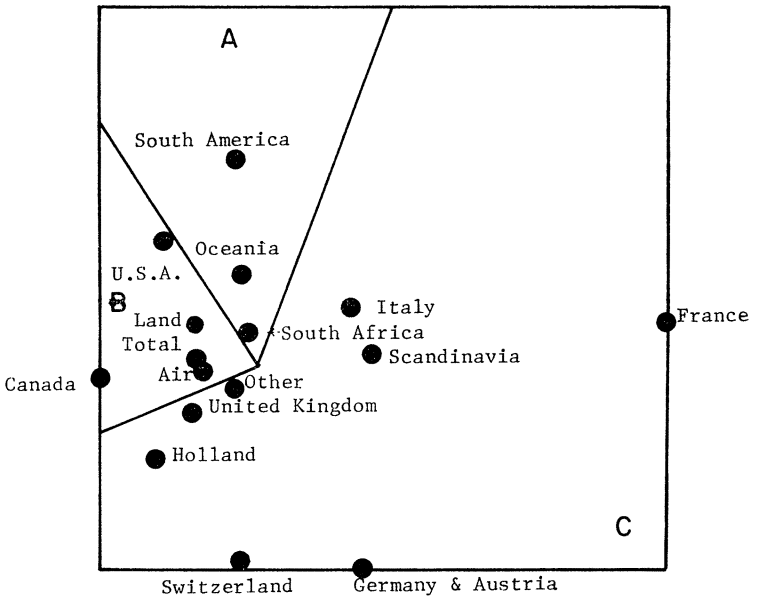


Figure 5

The space diagram in m=*2,* $\theta=0{\cdot}16$.

is a high goodness-of-fit, meaning that reproduction of the original matrix by SSA-I has not distorted the data.

In the map we note the following results: the series 13 and 15, "Tourists arriving" totals and "By air" totals have very similar patterns of seasonality. The series 15 contains 88 per cent of the total tourists (i.e. series 13). The location of these series is approximately in the centre, and in fact represents the "average" weighted seasonality of the other series. "Tourists arriving from France" (3) has quite a different seasonality than that of the other series and is located further from other countries. It is easy to distinguish similar series in the same sectors. The series of "Tourists arriving" from the southern part of the world are in sector A. "Tourists arriving" from North America are located in sector B. In sector C we find "Tourists arriving" from Europe and "other countries".

Example 2: Jewish Marriages – A Demographic Example

In Table 3 (below), moving seasonal patterns of "Jewish Marriages" in Israel in 1956–68 are presented for each year. A seasonal adjustment method for varying seasonality called X-11 is used (multiplicative model) to decompose the time series, see Shiskin *et al.* (1967). The matrix of distances D_{ij} is given in Table 4, all the suitable lags are equal to 0.

Table 3

Seasonal indices of the 13 years of the series "Marriages of Jews" in Israel (1956–68)

Year	*Jan.*	*Feb.*	*Mar.*	*Apr.*	*May*	*June*	*July*	*Aug.*	*Sept.*	*Oct.*	*Nov.*	*Dec.*
1956	90·4	86·4	134·1	88·4	80·3	111·9	81·2	142·6	109·1	92·0	86·1	97·8
1957	90·2	85·8	134·1	88·0	80·0	112·6	80·1	144·9	110·1	90·8	86·6	97·5
1958	90·0	84·4	134·6	86·7	79·8	114·2	77·8	148·8	111·5	88·7	88·2	96·9
1959	88·9	83·0	135·6	85·1	78·9	115·7	74·8	154·4	112·4	87·1	89·8	95·4
1960	88·5	81·8	135·6	83·0	77·8	117·5	71·8	160·5	114·2	85·9	90·4	94·0
1961	88·0	81·1	136·7	81·1	74·5	118·9	70·4	165·9	115·2	86·5	90·4	92·5
1962	88·0	80·5	136·9	79·0	71·5	120·0	71·3	167·8	117·7	86·5	90·6	92·1
1963	87·1	80·3	136·1	77·7	68·4	121·8	73·7	167·3	119·3	86·9	91·0	93·1
1964	86·1	79·8	133·3	76·8	67·5	123·6	76·2	164·4	123·2	86·1	91·3	94·2
1965	85·1	79·1	130·0	77·5	67·0	124·7	78·0	162·1	125·4	85·5	91·8	94·9
1966	84·1	78·2	126·7	78·4	68·4	124·7	79·0	159·6	130·6	84·1	92·8	95·1
1967	83·3	77·8	123·3	79·9	69·9	124·6	78·8	159·0	132·4	83·7	93·0	95·0
1968	83·1	77·7	121·1	80·5	71·0	125·2	78·2	158·8	132·8	83·7	93·0	94·5

In this series there is a gradual change (from year to year) in the varying patterns of seasonality. This implies that the distance between any two patterns is smallest with the neighbouring yearly pattern. Thus, the elements (distances) near the diagonal are the smallest and the elements in the corners (upper right and lower left) are the greatest. Therefore, a unidimensional structure called Simplex is accepted as a representation of the space diagram (see Guttman, 1950, 1954; Anderson, 1959).

Table 4

The (symmetry) matrix of the 13 years as computed from Table 3

	Coefficients matrix												
	1	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*
1.	0·00												
2.	0·72	0·00											
3.	2·14	1·43	0·00										
4.	3·95	3·23	1·81	0·00									
5.	5·67	4·96	3·53	1·72	0·00								
6.	7·14	8·42	5·00	3·19	1·57	0·00							
7.	8·07	7·35	5·92	4·12	2·48	1·08	0·00						
8.	8·42	7·70	6·28	4·47	3·23	2·09	1·20	0·00					
9.	8·62	7·90	6·56	5·15	3·99	3·41	2·75	1·62	0·00				
10.	9·07	8·35	7·04	5·90	4·83	4·66	4·00	2·87	1·37	0·00			
11.	9·55	8·83	7·69	6·55	5·66	5·76	5·10	4·08	2·77	1·48	0·00		
12.	9·85	9·13	7·96	6·82	6·01	6·11	5·60	4·93	3·62	2·35	0·92	0·00	
13.	10·08	9·36	8·08	6·94	6·08	6·18	5·77	5·29	3·97	2·76	1·44	0·54	0·00

A Simplex structure matrix (of distances) has the property that its elements incline away from the diagonal [often changes of rows (columns) is needed]. By transforming such a matrix into a map, using SSA-I technique, using one dimension ($m=1$), a (nearly) perfect order structure is obtained like that of Figure 6. The coefficient of goodness-of-fit is $\theta=0{\cdot}09$, indicating a very good fit, i.e. there are only small deviations between the original matrix and a perfect Simplex structure. The centre (arithmetic-average) is represented by the cross.

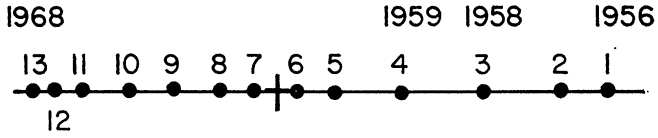


Figure 6

The space diagram in m$=1$ *dimension,* $\theta=0{\cdot}09$.

The matrix of distances given in Table 4 presents another interesting property of a Simplex structure which was revealed by Guttman (see Guttman, 1950, p 323; or Guttman, 1954, pp. 319–24) and is presented in Figure 7 using two ($m=2$) dimensions. This property means that the second component is a U-shaped function of the rank order (i.e. the first component). The bending point is located on the arithmetic-average of the first component (the cross). The coefficient of goodness-of-fit is $\theta=0{\cdot}01$, indicating a very good fit.

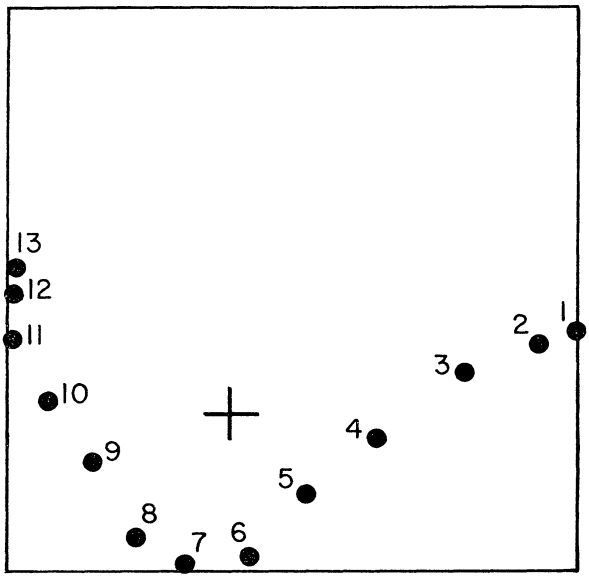


Figure 7

The space diagram in m = *2 dimensions,* $\theta = 0{\cdot}01$.

In this example, the internal order of the points is determined in terms of time. The most different seasonal patterns are those for the years 1956 and 1968. The most similar years in that sense of seasonality are those of 1967 and 1968.

Discussion

A method for the graphic display of the pairwise interrelationships of a set of seasonal patterns of time series is provided. There are many possibilities of empirical study of a given set of time series. The proposed method concentrates on the simultaneous comparison of one aspect of each series, that of the seasonality. The method suggested here is based on a sequence of four steps: decomposing each time series; computing a coefficient of dissimilarity between every pair of seasonal patterns; using a graphic technique for the matrix of distances; analysing and interpreting the map obtained in the previous step.

Each one of the above four steps is not new, nevertheless the application specifically to time series is a new one. The method makes it possible to find common seasonal patterns among time series. Either n various time series

(example 1) or the n periods of a single time series (example 2) could be used for raw material. Other coefficients for dissimilarity among other components, such as trend and irregularities, might be defined as in the second step. Thus the method can be generalized to other aspects of comparison in given sets of time series.

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