Nonparametric classification of time series: Application to the bank share prices in Spanish stock market

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Abstract

A statistical analysis of the series recording the weekly price (per share in euros) of the banks in the Spanish stock market during 2001 and 2002 is performed by using only nonparametric procedures. The analysis consists in estimating, classifying and testing the equality of the trends and then, once the estimated trends have been removed from the model, grouping the residual series by means of a nonparametric cluster method specifically designed to be applied on time series. The used nonparametric methodology can be summarized as follows. The trends are first estimated via local polynomial regression and then compared using several nonparametric test statistics whose asymptotic distributions are approximated by a bootstrap procedure. The residual series are clustered according with a disparity measure between local linear smoothers of the spectra of the series. The dependence of the observations is a crucial feature in the analysis and is kept in mind throughout the work.

Key words: Nonparametric methods, cluster analysis, hypothesis testing, time series.

1 Introduction

Nonparametric curve estimation methods provide a powerful statistical tool for exploring the underlying structure in a data set. These methods are aimed at missing any *ad hoc* parametric assumption about the data. This feature leads to extremely versatile techniques that can be applied on numerous practical situations in many fields. In this work, different nonparametric procedures of kernel type are proposed to analyze the performance of a set of time series that have been simultaneously observed. When several time series are jointly analyzed, besides obtaining appropriate fittings that help us to understand the mechanism generating of the series, an interesting additional question is to investigate how similar the series are. In particular, the assessment of the amount of affinity of the observed series is one of the main concerns of this paper and it is also tackled from a nonparametric point of view.

The study starts by splitting each of series into its deterministic and random components, which are next analyzed separately. First, the trends of the series are estimated via

local polynomial regression by assuming that they are smooth functions. The fitted trends are then classified into similar groups using standard cluster analysis and the equality of trends belonging to each group is checked by means of nonparametric tests that take into account the dependence of the observations. Once the analysis of the trends is over, we focus on the random components. In particular, a specific nonparametric procedure of performing cluster analysis is carried out on the residuals series obtained by subtracting the estimated trends from the original series. Therefore, the final objective is to find similarities and differences among the observed series in order to classify them into a small number of groups. Such a classification might be useful to detect a few representative patterns, forecast future performances, quantify the degrees of affinity, make a survey, ... So, the problem of classifying time series has important practical applications in many fields including physics, economy, medicine, seismology and engineering, among others. Some particular topics studied in literature are the classification of industrial production series (Piccolo (1990)), the comparison of seismological data as the case of distinguishing between earthquake and nuclear explosion waveforms (Kakizawa, Shumway and Taniguchi (1998)), the organization of industrial data according with average wage rates (Galbraith and Jiaqing (1999)) and the classification of children on the basis of similarities in behavior through time (Hirsch and DuBois (1991)).

The set of series analyzed in this work consists of fifteen financial data series[1], each one recording the weekly price (per share in euros) of a specific bank in the Spanish stock market over a time period of two years (2001 and 2002). Note that bank sector is specially interesting to perform our analysis because of its great importance in the stock market (the stock exchange capitalization of this sector represents an important percentage on the total capitalization) and the high heterogeneity of its companies in terms of size, growth expectations, risk, business tendencies, ... The fifteen considered banks are: Andalucía, Atlántico, BBVA, Banesto, Bankinter, BSCH, Castilla, Crédito-Balear, Galicia, Guipuzcoano, Pastor, Popular, Valencia, Vasconia and Zaragozano. These banks make up the whole bank sector in the Spanish stock market and five of them (Banesto, Bankinter, BBVA, BSCH and Popular) belong to IBEX-35 at present, which groups the 35 companies with the highest liquidity and trading volume in the Spanish stock market and its component stocks are reviewed annually. The measurements were recorded in the same time points for all series. In particular, each series is of 103 weekly observations (years 2001 and 2002) and each observation indicates the price per share taken on Thursday. When a Thursday fell on public holiday the observation was taken on Wednesday because previous studies where the price was recorded on Friday led to wrong conclusions due to the so-called weekend effect.

For our comparison purposes the observed series must be in a proper scale, so they have been standardized to have mean zero and unit variance. Graphs of the fifteen standardized series are given in Figure 1.

[1] Source: www.joramon.com

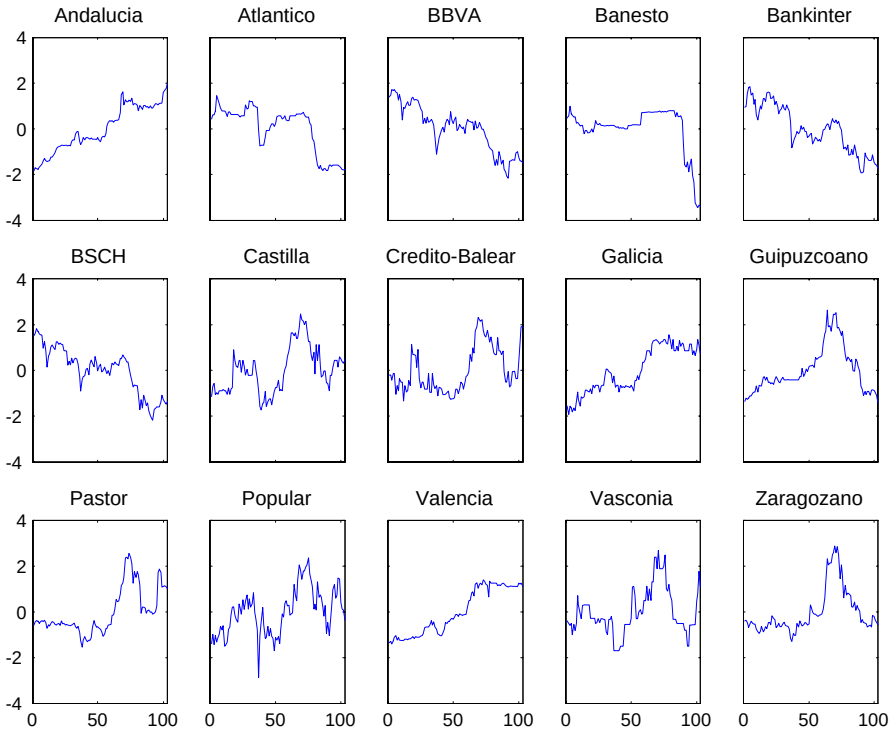


Figure 1: Standardized weekly prices (per share in euros) of banks in the Spanish stock market during the years 2001 and 2002.

Notice that Figure 1 suggests a strong affinity between some series (e.g. BBVA and BSCH exhibit similar decreasing profiles) and clear differences between another ones (e.g. BBVA and Andalucia show opposite trends).

Since all series have been evaluated at the same time points, a fixed regression model can be fitted by considering an equally spaced design on $[0, 1]$, that is

$$Y_{l,t} = m_l(x_t) + \varepsilon_{l,t}, \quad \text{t=1,2, } \ldots \text{,103, } \quad \text{l=1,2,} \ldots \text{,15,} \tag{1}$$

where m_l is the trend component of the series l, $x_t = t/103$ and $\varepsilon_{l,t}$ denotes the random component.

The analysis developed in this work is based on model (1) and consists in estimating, classifying and testing the equality of the deterministic components, m_l, and then, once the estimated trends have been removed from the model, grouping the residual series by means of a cluster method specifically designed to be applied on time series. Indeed, the dependence of the observations is a crucial feature in our analysis and it is kept in mind throughout the work.

The remainder of the article is structured as follows. Section 2 is devoted to estimate and compare the deterministic components. The estimates are obtained via local polynomial regression and are then used to identify clusters and check the hypothesis of equality between

the trends. As the statistic tests are based on nonparametric estimators, their distributions are in general unknown and so asymptotic and bootstrap approximations for these distributions are considered in this section. In Section 3 a cluster algorithm based on a disparity measure between the spectra of the residuals series is employed for classifying the random components in model (1). The results from the cluster analysis are carefully examined and compared with those achieved by using standard cluster algorithms that ignore the dependence structure. Lastly, some conclusions from the study are presented in Section 4.

2 Analysis of the trends

The existence of trends in the observed series looks clear from plots in Figure 1, so that the present section is devoted to estimate, classify and compare the underlying trends, denoted by m_l, $l = 1, \ldots, 15$.

The smoothing technique chosen in this work has been the weighted local polynomial regression because this estimator presents important advantages over other type kernel methods including: better boundary behavior, adaptation to estimate regression derivatives, easy computation and good minimax properties. See Fan and Gijbels (1996) for a complete study on local polynomial regression.

The local polynomial smoother is obtained by locally fitting a pth degree polynomial to the data by weighted least squares, and as with any smoothing procedure, one has to choose the smoothing parameter which, as it is well-known, controls the amount of local averaging. More specifically, given the sample $\{(x_t, Y_t)\,,\, t = 1, \ldots, n\}$ generated from the regression model $Y_t = m(x_t) + \varepsilon_t$ and assuming that the first $(p+1)$ derivatives of the regression function at point x exist and are continuous, local polynomial fitting permits to estimate the vector $\vec{\beta}(x) = \left(\beta_0(x), \beta_1(x), \ldots, \beta_p(x)\right)^t$, where $j!\beta_j(x) = m^{(j)}(x)$, with $j = 0, 1, \ldots, p$, by minimizing the function

$$\Psi(\vec{\beta}(x)) = \sum_{t=1}^{n} \left(Y_t - \sum_{j=0}^{p} \beta_j(x)(x_t - x)^j \right)^2 \omega_{n,t}, \tag{2}$$

where the weights $\omega_{n,t}$ are given by $\omega_{n,t} = (nh)^{-1} K\left((x_t - x)/h\right)$, with K a kernel function usually chosen to be a unimodal probability density that is symmetric about zero and $h = h_n$ the smoothing parameter so-called bandwidth. The estimator of $\vec{\beta}(x)$, obtained as a solution to the weighted least squares problem given in (2), is called local polynomial kernel estimator or local polynomial smoother.

In matrix notation, the local polynomial kernel estimator takes the form

$$\widehat{\vec{\beta}}_{(n)}(x) = \left(X_{p,(n)}^t W_{(n)} X_{p,(n)} \right)^{-1} X_{p,(n)}^t W_{(n)} \vec{Y}_{(n)} \tag{3}$$

where $\vec{Y}_{(n)} = (Y_1, \ldots, Y_n)^t$, $X_{p,(n)}$ is the $n \times (p+1)$ array whose element (i, j) is $(x_i - x)^j$, for $i = 1, \ldots, n$ and $j = 0, \ldots, p$, and $W_{(n)} = \mathrm{diag}\left(\omega_{n,1}, \ldots, \omega_{n,n}\right)$ is the diagonal array of weights.

Then, the local polynomial estimator of the jth derivative of the regression function, $\hat{m}^{(j)}(x)$, is given by $j!\hat{\beta}_j(x)$, $\hat{\beta}_j(x)$ being the $(j+1)th$ component of the vector in (3), $j = 0, 1, \ldots, p$.

The local polynomial smoother with $p = 1$ (local linear smoother) was used to estimate each single trend m_l from the observed data for each series. The bandwidth h was determined in each case by means of an plug-in algorithm specifically designed to overcome the dependence of the data. The key point of this algorithm is to estimate a parametric correlation model previously assumed for the error process and plug the estimate into the asymptotic expression of the mean integrated squared error of the smoother. A detailed study on the bandwidth selection problem in a context of dependence, including the plug-in algorithm used here, can be seen in Francisco-Fernández and Vilar-Fernández (2001), (2004). The bandwidths obtained from the plug-in algorithm are presented in Table I.

Bank	Andalucía	Atlántico	BBVA	Banesto	Bankinter
h	0.134229	0.087754	0.167225	0.175560	0.175827
Bank	BSCH	Castilla	Crédito-Balear	Galicia	Guipuzcoano
h	0.178553	0.111946	0.080087	0.092995	0.114639
Bank	Pastor	Popular	Valencia	Vasconia	Zaragozano
h	0.115431	0.136151	0.158361	0.112908	0.167420

Table I: Plug-in bandwidths used to estimate the trends.

Attention was also focused on estimating the mean trend of the set of series, let's say m_c. The curve m_c represents the mean weekly prices (per share in euros) of the Spanish bank sector. The mean curve m_c was nonparametrically estimated by smoothing the scatter plot constructed with the points $\left\{\left(x_t, \overline{Y}_{\cdot,t}\right),\ t = 1, \ldots, 103\right\}$, where $\overline{Y}_{\cdot,t} = (1/15)\sum_{l=1}^{15} Y_{l,t}$. The scatter plots of the observed series together with the corresponding local linear estimates for each single trend and for the mean trend are shown in Figure 2.

Plots in Figure 2 suggest that the estimated mean trend cannot be considered as a common pattern since there are marked differences among several estimated individual trends. Therefore, we firstly proceeded to test the hypothesis of equality of trends, that is, the following hypotheses test was considered:

$$H_0 : m_1 = \ldots = m_{15} = m_c \ \text{ versus } \ H_1 : \text{Exists } \ (l, j) \ \text{ such as } m_l \neq m_j,\ l, j \in \{1, \ldots, 15\}. \tag{4}$$

The problem of testing the equality of k regression functions using nonparametric techniques has been broadly studied in recent statistical literature. Some relevant references on this issue are Hall and Hart (1990), King, Hart and Werhly (1991), Kulasekera (1995), Koul and Schick (1997), Dette and Neumeyer (2001) and Neumeyer and Dette (2003), among others. Nevertheless, all of these works assume independence of the observations and this

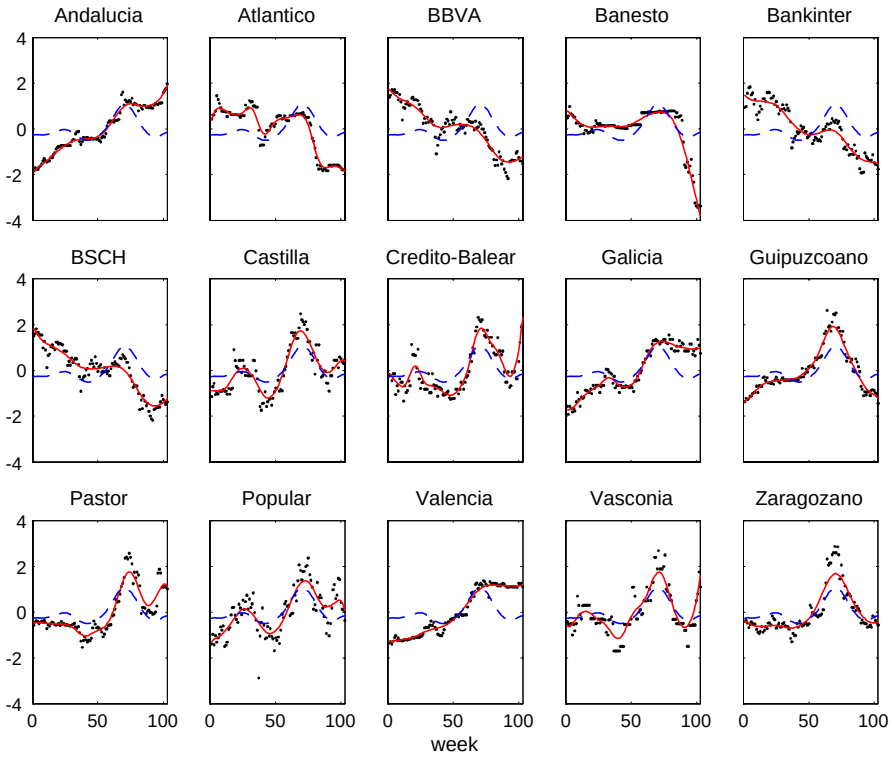


Figure 2: Scatter plots of the observed series together with the local linear fits for each individual trend m_l, $l = 1, \ldots, 15$, (solid curves) and for the mean trend m_c (dashed curve).

assumption is not hold with the data sets in our study. For this reason we have used a test statistic that has been also studied in a dependence context and that can be applied for a number $k > 2$ of curves. Such a test statistic is based on the Crámer-von-Mises type functional distance between kernel type estimators of the unknown regression functions m_l and, in particular, is given by

$$\hat{Q}_n^{(1)} = \sum_{l=2}^{k} \sum_{s=1}^{l-1} \int \left(\hat{m}_{l,h}(x) - \hat{m}_{s,h}(x) \right)^2 \omega(x)\, dx, \tag{5}$$

where $\omega(x)$ is a weight function defined on the support of the design variables, $C = [0, 1]$, and, for each $l = 1, \ldots, k$, $\hat{m}_{l,h}$ is a kernel estimator of m_l obtained with bandwidth h and kernel K.

Essentially $\hat{Q}_n^{(1)}$ is a consistent estimator of

$$Q = \sum_{l=2}^{k} \sum_{s=1}^{l-1} \int (m_l - m_s)^2 \, \omega,$$

so that the null hypothesis is false if and only if $Q > 0$. The asymptotic behavior of $\hat{Q}_n^{(1)}$ was studied under general dependence conditions in Vilar-Fernández and González-Manteiga (2004). In particular, if the autocovariance function of every error process in

model (1), $\nu_l(\tau) = E(\varepsilon_{l,t}\varepsilon_{l,t+\tau})$, is assumed to satisfy

$$\sum_{\tau=-\infty}^{+\infty} |\tau|\,|\nu_l(\tau)| < \infty, \text{ for } l = 1, \ldots, k,$$

then, Theorem 3 in Vilar-Fernández and González-Manteiga (2004) establishes that, under the null hypothesis of equality of the regression curves,

$$\sqrt{n^2 h}\left(\hat{Q}_n^{(1)} - \frac{k-1}{nh_n} I_\omega C_K \sum_{l=1}^{k} \Gamma_l\right) \longrightarrow N\left(0, \sigma^2_{Q^{(1)}}\right) \text{ in distribution,} \tag{6}$$

where $\Gamma_l = \sum_{\tau=-\infty}^{\infty} \nu_l(\tau)$, for $l = 1, \ldots, k$, $I_\omega = \int \omega$, $C_K = \int K^2$ and

$$\sigma^2_{Q^{(1)}} = 2\left(\int (K * K)^2\right)\left(\int \omega^2\right)\left[(k-1)^2 \sum_{l=1}^{k} \Gamma_l + \sum_{j=1}^{k} \sum_{l=1, l\neq j}^{k} \Gamma_j \Gamma_l\right]$$

with $*$ denoting the convolution operator.

Note that expression (6) involves the unknown parameter values Γ_l so that, in practice, estimates of these quantities, $\hat{\Gamma}_l$, must be obtained from the samples. If suitable estimates $\hat{\Gamma}_l$ are considered then the convergence in (6) holds when Γ_l is replaced by $\hat{\Gamma}_l$, $l = 1, \ldots, k$, (see Theorem 2 in Vilar-Fernández and González-Manteiga (2004)) and so the rejection of the null hypothesis of equality of regression curves would be concluded at a significance level α when

$$\tilde{Q}_n^{(1)} = \sqrt{n^2 h}\, \hat{\sigma}^{-1}_{Q^{(1)}} \left(\hat{Q}_n^{(1)} - \frac{k-1}{nh_n} I_\omega C_K \sum_{l=1}^{k} \hat{\Gamma}_l\right) > z_\alpha, \tag{7}$$

where z_α is such that $\Phi(z_\alpha) = 1 - \alpha$, with $\Phi(\cdot)$ the distribution function of the standard normal.

Criterion (7) was regarded to check the equality of trends of the fifteen financial data series in study. The Nadaraya-Watson estimator (corresponding to make $p = 0$ in (3)) with Gaussian kernel was chosen to estimate m_l and the weight function was taken to be $\omega(x) = 1/0.8$ on $[0.1, 0.9]$ and 0 otherwise. The nonparametric residuals obtained by means of $\hat{\varepsilon}_{l,t} = Y_{l,t} - \hat{m}_{l,h}(x_t)$ were used to estimate Γ_l, $l = 1, \ldots, 15$, and lastly the statistic $\tilde{Q}_n^{(1)}$ was computed for a range of values of the bandwidth h. The achieved results are illustrated in both Figure 3 and Table II. Figure 3 shows the graphs of $\tilde{Q}_n^{(1)} \times 10^{-2}$ and of the p-values associated with each $\tilde{Q}_n^{(1)}$ as a function of h and Table II includes the outcomes for a set of specific values of h.

It is clear from Figure 3 that, as it was expected, the equality of trends is rejected for a wide range of reasonable values of h. Note that as the bandwidth h increases the test tends to accept the null hypothesis of equality. Indeed, this behavior is also expected because the larger the bandwidth, the more smoothing is performed and thus the estimated trends will be very close to the average of the observations. But, on the other hand, the average curve

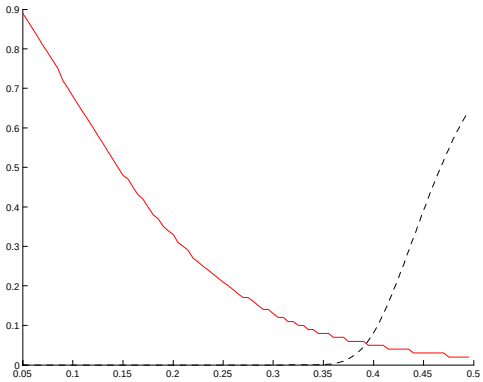


Figure 3: Plots of $\tilde{Q}_n^{(1)} \times 10^{-2}$ (solid curve) and the p-values associated with $\tilde{Q}_n^{(1)}$ (dashed curve) as a function of h.

h	0.100	0.200	0.300	0.350	0.400	0.500
$\tilde{Q}_n^{(1)}$	68.126	32.574	13.091	8.085	5.005	3.135
p-value	0.000	0.000	0.000	0.001	0.081	0.390

Table II: Test of equality of trends for the observed fifteen based on $\tilde{Q}_n^{(1)}$ statistic for specific six values of h.

is the straight line $Y = 0$ for all series because these ones have been previously standardized. Therefore, $\tilde{Q}_n^{(1)}$ in (7) tends to zero as h increases and as a consequence the null hypothesis is accepted and the resulting test presents low power. In summary, the main message from both Figure 3 and Table II is that h has a heavy influence on the behavior of the test so that a suitable selection of the smoothing parameter is crucial to obtain a satisfactory power of the test.

Rejection of the null hypothesis of trends equality motivates a cluster analysis with the fifteen nonparametric estimates of the trends to explore degrees of affinity and group the estimated trends into a few categories. So, the local linear smoothers of the trends previously computed, $\hat{m}_{l,h_l}$, were clustered by agglomerative hierarchical methods using the squared Euclidean distance as disparity measure between two profiles. Thus, the distance between two estimated trends, $d\left(\hat{m}_{l,h_l}, \hat{m}_{s,h_s}\right)$, for $l, s \in \{1, 2, \ldots, 15\}$, is given by

$$d\left(\hat{m}_{l,h_l}, \hat{m}_{s,h_s}\right) = \sum_{t=1}^{103} \left[\hat{m}_{l,h_l}\left(x_t\right) - \hat{m}_{s,h_s}\left(x_t\right)\right]^2.$$

Several agglomerative methods (including Ward's method, centroid method and single, complete and average linkage methods) were used to determine which two groups must be joined at each step of the hierarchical process and, basically, similar cluster solutions were derived in all cases. Figure 4 and Figure 5 show the tree diagrams or dendrograms identifying

the different clusters formed at each step of the clustering process when the average linkage method and Ward's method were respectively used.

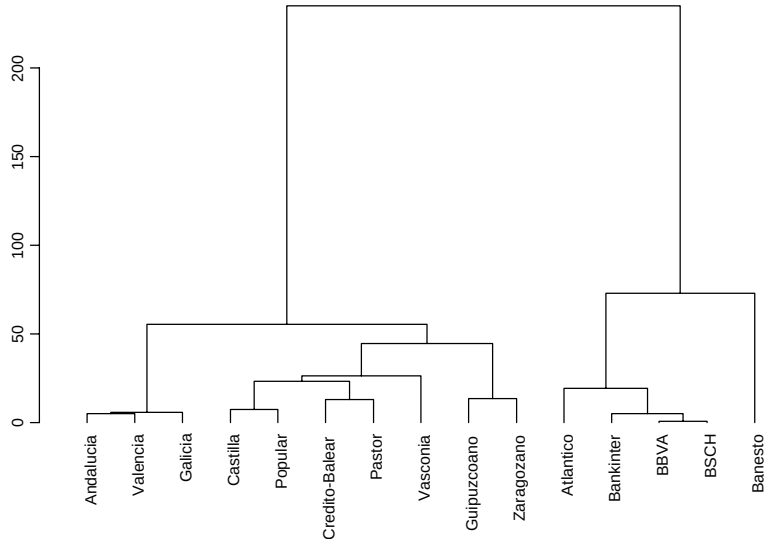


Figure 4: Dendrogram for hierarchical cluster analysis of the local linear estimates of the trends of the fifteen financial data series. Squared Euclidean distance and the average linkage method were used as initial proximity matrix and group proximity measure respectively.

Dendrograms in Figure 4 and Figure 5 lend support to the five-cluster solution. There is a primary classification where the group of banks {BBVA, BSCH, Bankinter, Atlántico, Banesto} is separated from the rest. Within this first group, $C1 = \{$BBVA, BSCH, Bankinter, Atlántico$\}$ forms a compact cluster, with a decreasing trend and without many oscillations, whereas $C2 = \{$Banesto$\}$ appears isolated until very late in the process because its profile is approximately constant up to the twenty five last weeks where a sharp and noticeable fall is observed. As regards the rest, three different secondary clusters emerge from the pictures: $C3 = \{$Andalucía, Valencia, Galicia$\}$, $C4 = \{$Guipuzcoano, Zaragozano$\}$ and $C5 = \{$Castilla, Popular, Crédito-Balear, Pastor, Vasconia$\}$. In contrast with $C1$, banks included in $C3$ show increasing trends. The trend pattern for group $C5$ is a wiggly curve with the presence of two modes around weeks 25 and 70. Lastly, in $C4$ the profile is similar to that in $C5$ but here the first mode disappears (the trend is flat at the initial period) and the second one is more noticeable. Note that although Popular bank belongs to IBEX-35, its estimated trend does not appear included in $C1$ cluster, where the rest of members of IBEX-35 (BBVA, BSCH, Bankinter and Banesto) are compactly grouped, except for Banesto bank which, in any case, is close to $C1$ cluster.

On the basis of the obtained cluster solution, we proceeded to test the equality of the

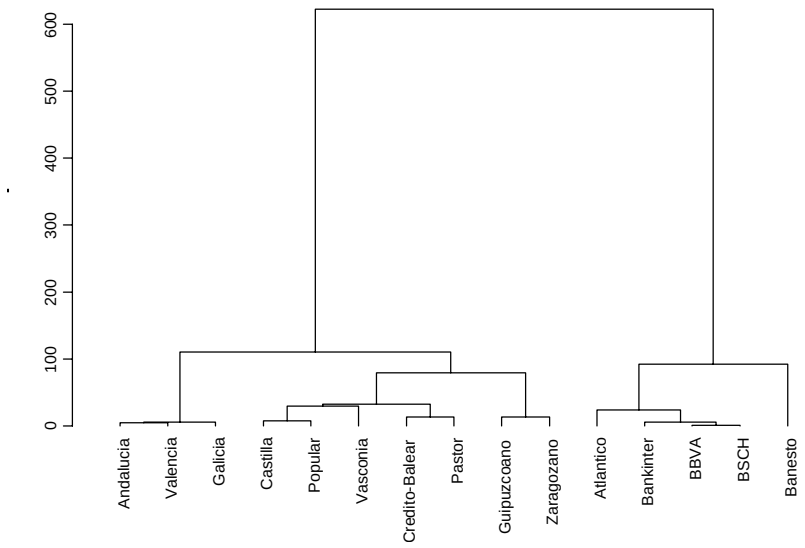


Figure 5: Dendrogram for hierarchical cluster analysis of the local linear estimates of the trends of the fifteen financial data series. Squared Euclidean distance and Ward's method were used as initial proximity matrix and group proximity measure respectively.

trends included within each cluster, so that the test statistic $\tilde{Q}_n^{(1)}$ in (7) was separately computed for each of the four groups of trends: $C1$, $C3$, $C4$ and $C5$. The graphs of the p-values obtained for each group as a function of h are shown in Figure 6. A simple inspection of Figure 6 allows to conclude that the null hypothesis of equality is accepted for the four clusters and for the whole range of bandwidths considered since in all cases the p-value is larger than 0.7.

It is also interesting to check pairwise trends comparisons that is to consider $k = 2$ in (4):

$$H_0 : m_l - m_s = 0 \quad \text{versus} \quad H_1 : m_l - m_s \neq 0, \tag{8}$$

for $l, s \in \{1, \ldots, 15\}$, $l \neq s$.

For checking the equality of $k = 2$ regression curves, statistical literature provides several methods based on nonparametric estimators although again they have been in general studied under independence of the data. Furthermore, even under independence conditions, the sampling distribution of these statistics is difficult to obtain and only asymptotic approximations are available. As these asymptotic approximations frequently depend on unknown population features (see for example $\tilde{Q}_n^{(1)}$ statistic) and present convergence rates rather slow, some authors have proposed resampling procedures to approximate the sampling distributions of these statistics. In fact, some bootstrap algorithms have been developed under dependence conditions.

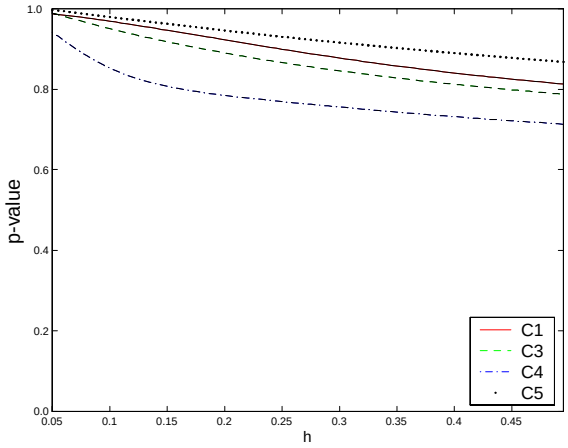


Figure 6: Graphs of the p-values associated with $\tilde{Q}_n^{(1)}$ as a function of h when the equality of the trends included in cluster $C1$ (solid line), $C3$ (dashed line), $C4$ (dot-dashed line) and $C5$ (dotted line) are checked.

We have used up to four different test statistics based on nonparametric methodology to check the hypothesis test (8) with each pair of trends of the five banks included in IBEX-35, that is BBVA, Banesto, Bankinter, BSCH and Popular. Besides the statistic $\tilde{Q}_n^{(1)}$ given in (7), other three statistics denoted by $\tilde{Q}_n^{(2)}$, $\tilde{Q}_n^{(3)}$ and $\tilde{Q}_n^{(4)}$ were used and they are very briefly described below.

Statistic $\tilde{Q}_n^{(2)}$ is defined as the difference between a nonparametric estimator of the variance of the pooled sample and a convex combination of nonparametric variance estimators of the individual samples, that is

$$\tilde{Q}_n^{(2)} = \hat{\sigma}_P^2 - \frac{1}{206}\left(\hat{\sigma}_l^2 + \hat{\sigma}_s^2\right),$$

with

$$\hat{\sigma}_P^2 = \frac{1}{206}\left\{\sum_{t=1}^{103}\left[Y_{l,t} - \hat{m}_{p,g}\left(x_t\right)\right]^2 + \sum_{t=1}^{103}\left[Y_{s,t} - \hat{m}_{p,g}\left(x_t\right)\right]^2\right\}$$

and, for $k = l, s$,

$$\hat{\sigma}_k^2 = \frac{1}{103}\sum_{t=1}^{103}\left[Y_{k,t} - \hat{m}_{k,h_k}\left(x_t\right)\right]^2,$$

where $\hat{m}_{p,g}$ denotes a kernel smoother with bandwidth g of the two combined samples and $\hat{m}_{k,h_k}$ denotes a kernel smoother with bandwidth h_k of each single sample for $k = l, s$. The test based on $\tilde{Q}_n^{(2)}$ was studied under independence conditions by Dette and Neumeyer (2001).

Statistic $\tilde{Q}_n^{(3)}$ is ANOVA type and is given by

$$\tilde{Q}_n^{(3)} = \frac{1}{206}\left\{\sum_{t=1}^{103}\left[\hat{m}_{p,g}\left(x_t\right) - \hat{m}_{l,h_l}\left(x_t\right)\right]^2 + \sum_{t=1}^{103}\left[\hat{m}_{p,g}\left(x_t\right) - \hat{m}_{s,h_s}\left(x_t\right)\right]^2\right\},$$

with $\hat{m}_{p,g}$, $\hat{m}_{l,h_l}$ and $\hat{m}_{s,h_s}$ the kernel smoothers used for constructing $\tilde{Q}_n^{(2)}$. This test was introduced by Young and Bowman (1995) and is motivated by the classical one-way analysis of variance.

Last considered statistic, $\tilde{Q}_n^{(4)}$, was proposed by Neumeyer and Dette (2003) and is based on the difference of two marked empirical processes which are constructed from the residuals obtained under the null hypothesis of equality of the two trends. So, under the null hypothesis, the residuals are generated as $\hat{\varepsilon}_{k,t}^0 = Y_{k,t} - \hat{m}_{p,g}(x_t)$, $k = l, s$, and the difference between the corresponding marked empirical processes is given by

$$\hat{R}(r) = \frac{1}{206}\left[\sum_{t=1}^{103} \hat{\varepsilon}_{l,t}^0 \, I(x_t \leq r) - \sum_{t=1}^{103} \hat{\varepsilon}_{s,t}^0 \, I(x_t \leq r)\right],$$

where $r \in [0,1]$ and $I(\cdot)$ denotes the indicator function. Then $\tilde{Q}_n^{(4)}$ is defined as

$$\tilde{Q}_n^{(4)} = \int_0^1 \hat{R}^2(r)\, dr$$

Note that with any of the four considered statistics the criterion is to reject the null hypothesis of equality of a pair of trends for large values of the statistic. The critical values were determined by approximating the sampling distribution of $\tilde{Q}_n^{(k)}$, for $k = 1, 2, 3, 4$, by means of a naive bootstrap which takes into account the dependence of the observations. Basically, the used resampling technique consists on drawing bootstrap resamples of the estimated residuals under the null hypothesis, $\hat{\varepsilon}_{k,t}^0 = Y_{k,t} - \hat{m}_{p,g}(x_t)$, $k = l, s$, but assuming an explicit autoregressive structure for the error process. In this way, the bootstrap algorithm is able to replicate the dependence structure. There also exist alternative resampling techniques to replicate the dependence structure without assuming a specific parametric structure for the error. For more details see Vilar-Fernández *et al.* (2004) where a broad simulation study comparing the performance of different test statistics (including also $\tilde{Q}_n^{(k)}$, $k = 1, 2, 3, 4$) when different bootstrap methods are used to replicate the dependence is presented.

Table III summarizes the p-values obtained for the pairwise comparisons of the five banks included in IBEX-35. These p-values were achieved in all cases on the basis of the bootstrap approximation (columns 3-6), although for $\tilde{Q}_n^{(1)}$ the p-values obtained using its asymptotic distribution (6) are also provided in column 2.

Table III allows to conclude that there are no significant differences between the trends of the weekly prices per share of BBVA, Bankinter and BSCH banks because very high p-values were obtained with all procedures when these banks were pairwise compared. Such a result supports the homogeneity of $C1$ group determined by the cluster analysis previously performed. Since these three banks belong to IBEX-35, any of their trends could be chosen as pattern for bank trends of IBEX-35. On the other hand, rows 4, 9 and 11 in Table III establish that the trend of the weekly prices per share of Popular bank differs significantly from those ones of BBVA, Bankinter and BSCH banks. In fact, this conclusion is derived from all procedures. Therefore it is clear that although Popular bank belongs to IBEX-35, its

Pairwise test	$\tilde{Q}_n^{(1)}$ asymptotic	$\tilde{Q}_n^{(1)}$ bootstrap	$\tilde{Q}_n^{(2)}$ bootstrap	$\tilde{Q}_n^{(3)}$ bootstrap	$\tilde{Q}_n^{(4)}$ bootstrap
BBVA/Banesto	0.103	0.048	0.008	0.024	0.258
BBVA/Bankinter	0.738	0.782	0.332	0.924	0.934
BBVA/Popular	0.000	0.000	0.000	0.000	0.016
BBVA/BSCH	0.854	0.960	0.644	0.994	0.998
Banesto/Bankinter	0.072	0.034	0.004	0.022	0.192
Banesto/Popular	0.436	0.204	0.014	0.132	0.190
Banesto/BSCH	0.208	0.094	0.028	0.078	0.282
Bankinter/Popular	0.000	0.002	0.000	0.000	0.008
Bankinter/BSCH	0.718	0.702	0.434	0.850	0.932
Popular/BSCH	0.000	0.002	0.000	0.000	0.024

Table III: p-values for the pairwise comparisons of the five banks included in IBEX-35 when the test statistics $\tilde{Q}_n^{(k)}$, $k = 1, 2, 3, 4$, are used and their sampling distribution are approximated by bootstrap methodology and also, in the particular case of $\tilde{Q}_n^{(1)}$, by its asymptotic distribution.

behavior has been significantly different from the performance pattern of banks of IBEX-35 and its location in $C5$, the cluster furthest away from $C1$, is completely justified. Concerning Banesto bank, the outcomes of the hypothesis tests do not allow us to obtain definitive conclusions since in general there is not sufficient evidence to reject the null hypothesis or, at least, this evidence is not uniformly found with all used procedures. Pairwise comparisons were also carried out with the rest of banks and the results gave support to the groups determined in the cluster analysis.

3 Cluster analysis of the random components

Next step in our study consisted in removing the estimated trends to get and analyze the estimated noise sequences i.e. the nonparametric residuals $\hat{\varepsilon}_{l,t}$ given by

$$\hat{\varepsilon}_{l,t} = Y_{l,t} - \hat{m}_{l,h_l}(x_t),\ t = 1, 2, \ldots, 103,\ l = 1, 2, \ldots, 15. \tag{9}$$

The fifteen series of residuals are displayed in Figure 7.

The main feature emerging from Figure 7 is that all of the residual series are apparently stationary with zero mean although, indeed, there is no a common pattern to model the dependence structure in the fifteen cases jointly. This fact is supported by Figure 8 where the sample autocorrelation functions of the fifteen residual series are shown.

Using a standard computer statistical package we checked that all series in study can be reasonably well-fitted by means of an autoregressive model or of a moving average model.

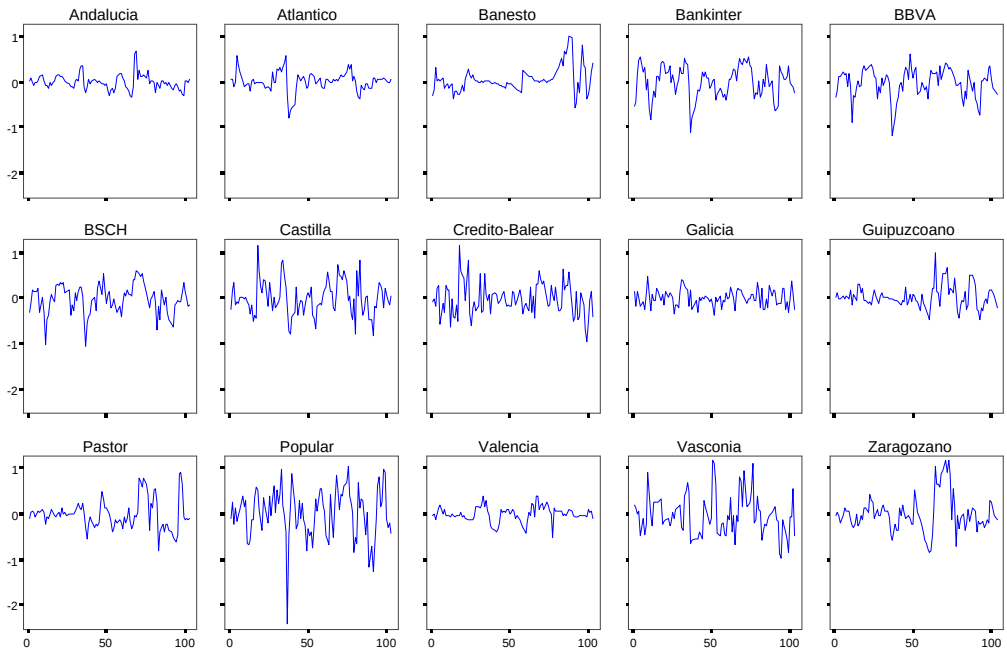


Figure 7: Graphs of the nonparametric residuals sequences $\hat{\varepsilon}_{l,t}$ obtained for each observed series according with (9).

However, in this work, our interest is not focused on fitting a particular dependence model to the residuals series but on classifying them into similar groups. Note that a standard cluster analysis will not provide suitable results unless the used disparity measure has into account the probabilistic characterization of the series.

In general, cluster analysis of time series has not received much attention in statistical literature in spite of its important practical applications in several fields including indeed economy. A motivating work by Tong and Dabas (1990) shows how various measures of similarity and dissimilarity among time series lead to unexpected and substantially different cluster solutions, so that the authors suggest that the problem of measuring the degree of affinity among several fitted series is a key problem for future research. Some later references where cluster analysis of time series was considered are Piccolo (1990), Maharaj (1996),(2000), Kakizawa, Shumway and Taniguchi (1998) and Vilar-Fernández and Pértega (2004), among others. In most of these works the key point consists in introducing an appropriate distance between two fitted time series models. For example, in Piccolo (1990) a distance between two $ARIMA$ models was defined as the Euclidean distance between the coefficients of their corresponding autoregressive expansions. So, an automatic cluster algorithm was developed by fitting AR models to the observed series and using then the Piccolo's distance as input for

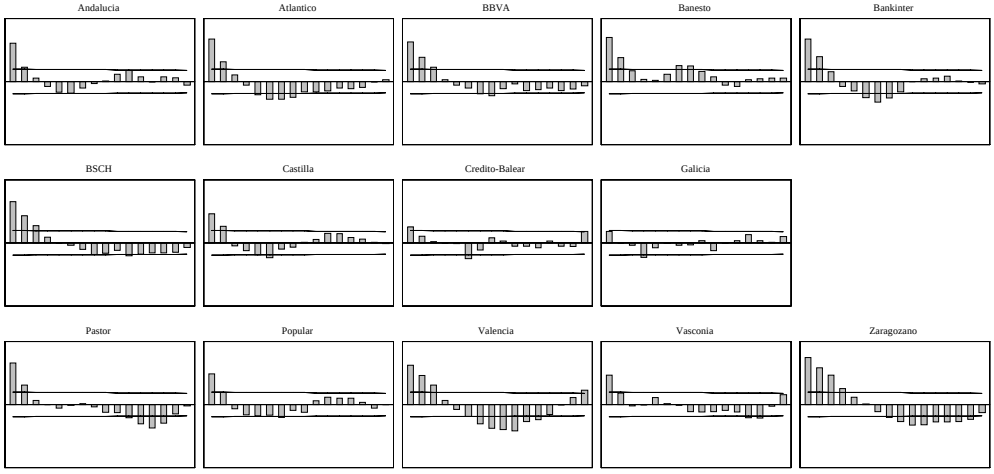


Figure 8: Sample autocorrelation functions of the fifteen residual series $\hat{\varepsilon}_{l,t}$.

the algorithm. Later, also on the basis of the Piccolo's distance, a new test of hypothesis to determine whether two series are generated from significantly different processes was derived by Maharaj (1996). Furthermore, since the p-values associated with the test can be also used to measure the amount of affinity between two series, Maharaj (1996) proposed a new cluster method based on these p-values.

In Kakizawa, Shumway and Taniguchi (1998) and Vilar-Fernández and Pértega (2004) some solutions were reported in the frequency domain. Frequency domain approach is an interesting alternative because the dimensionality problem, usually high in a time series setting, is removed by replacing the covariance matrix representation by its corresponding spectral density representation. In Kakizawa, Shumway and Taniguchi (1998) a general spectral disparity measure between two processes was defined as

$$D_W(f,g) = \frac{1}{4\pi}\int_{-\pi}^{\pi} W\left(f(\lambda)g^{-1}(\lambda)\right)\, d\lambda, \tag{10}$$

where $f(\cdot)$ and $g(\cdot)$ are the spectral densities of each process and $W(\cdot)$ is a divergence function satisfying regularity conditions appropriate to ensure that $D_W(f,g)$ has the quasi-distance property. So, on the basis of measures of the kind (10), the authors construct rules to perform discriminant analysis and cluster analysis with stationary processes. In practice the spectra $f(\cdot)$ and $g(\cdot)$ are usually unknown so that spectral estimates $\hat{f}(\cdot)$ and $\hat{g}(\cdot)$ must be previously computed and this can be done parametrically or nonparametrically. Vilar-Fernández and Pértega (2004) considered the nonparametric via and studied the behavior of discriminant and cluster rules based on $D_W\left(\hat{f},\hat{g}\right)$ when the spectral estimators $\hat{f}$ and $\hat{g}$ are obtained using local polynomial regression techniques. In fact, up to three different approaches to estimate the spectrum by using local polynomial regression techniques (see Fan and Kreutzberger (1998)) were examined and compared.

In this work, the techniques described in Vilar-Fernández and Pértega (2004) were employed to perform cluster analysis with the fifteen residual series in study. The steps of our

cluster procedure are next sketched.

Step 1. The periodograms associated with each residual series are evaluated over the Fourier frequencies. That is, for the lth residual series, $l = 1, \ldots, 15$, the function

$$I_{l,n}(\lambda) = \frac{1}{2\pi n} \left| \sum_{t=1}^{n} \hat{\varepsilon}_{l,t} \exp(-i\lambda t) \right|^2, \ \lambda \in [-\pi, \pi],$$

is evaluated over the frequencies $\lambda_k = 2\pi k/n$, with $k = -N, \ldots, N$, $N = [(n-1)/2]$ and $n = 103$ the series length.

Step 2. The spectral density of the lth error process, $f_l(\lambda)$, for $l = 1, \ldots, 15$, is estimated by using local linear fitting techniques. This is made in three different ways:

(i) Constructing the local linear smoother of the lth periodogram computed in Step 1, where the linear local fit is carried out by least squares as usual. In other words, smoothing directly the data $\{\lambda_k, I_{l,n}(\lambda_k)\}$ by means a standard local linear fit. The resulting estimator will be referred by $\widehat{f}_{l,DLS}$, $l = 1, \ldots, 15$.

(ii) Constructing the local linear smoother of the lth log-periodogram $\{\lambda_k, \log I_{l,n}(\lambda_k)\}$, using least squares again, and then considering the exponential transformation of the obtained log-spectrum smoother. This new estimator will be denoted by $\widehat{f}_{l,LS}$, $l = 1, \ldots, 15$.

(iii) The third estimator, denoted by $\widehat{f}_{l,LK}$, $l = 1, \ldots, 15$, is constructed in a similar form to $\widehat{f}_{l,LS}$ except that the local linear fitting of the lth log-periodogram is performed by using now the maximum local likelihood criterion instead of the least squares one. Note that in this case the log-spectrum smoother must be determined by some numerical procedure (the Newton-Raphson algorithm was used in this work).

The asymptotic properties of these spectral estimators joint with a comparative study among them can be seen in Fan and Kreutzberger (1998) or in Fan and Gjibels (1996). In particular, it is shown that the likelihood-based estimator, $\widehat{f}_{l,LK}$, is recommended from an asymptotic point of view because presents a smaller asymptotic variance than $\widehat{f}_{l,LS}$, without losing in terms of asymptotic bias, and a smaller asymptotic bias at peak regions than $\widehat{f}_{l,DLS}$, maintaining the same asymptotic variance.

Step 3. The distance between each pair of residual series, $\{\hat{\varepsilon}_{l,t}\}$ and $\{\hat{\varepsilon}_{s,t}\}$, $l, s \in \{1, \ldots, 15\}$, is computed by means of a simple modification of the disparity measure $D_W(\cdot,\cdot)$ introduced in (10), which is given by:

$$D_W^{\star}(\hat{f}_l, \hat{f}_s) = D_W(\hat{f}_l, \hat{f}_s) + D_W(\hat{f}_s, \hat{f}_l) = \frac{1}{4\pi} \int_{-\pi}^{\pi} \left\{ W\left(\frac{\hat{f}_l(\lambda)}{\hat{f}_s(\lambda)}\right) + W\left(\frac{\hat{f}_s(\lambda)}{\hat{f}_l(\lambda)}\right) \right\} d\lambda, \quad (11)$$

where $\hat{f}_r$ is any of the three local linear smoothers introduced in Step 2 for estimating the rth spectrum, $r = l, s$, and $W(\cdot)$ is a divergence function that, in this case, was chosen to be

$$W(x) = \log \frac{x+1}{2} - \frac{1}{2} \log x. \quad (12)$$

Note that $D_W(\cdot,\cdot)$ was not directly used to measure the distance between two spectra because equation (12) does not ensure that $D_W(\cdot,\cdot)$ is a symmetrical measure, which is

necessary to perform cluster analysis. The simple modification provided by (11) suffices to obtain a symmetrical measure.

Step 4. Using the dissimilarity matrix between the residual series based on $D_W^\star$ both the Ward's and the average linkage methods were used to produce the agglomerative hierarchical cluster analysis.

The resulting dendrogram when the cluster analysis was performed using $\widehat{f}_{l,DLS}$ as nonparametric spectral estimator and the average linkage method as criterion to evaluate the proximity between groups in an intermediate step is shown in Figure 9. The dendrogram displayed in Figure 10 was obtained when the cluster analysis was carried out with the same features except that $\widehat{f}_{l,LK}$ was used instead of $\widehat{f}_{l,DLS}$.

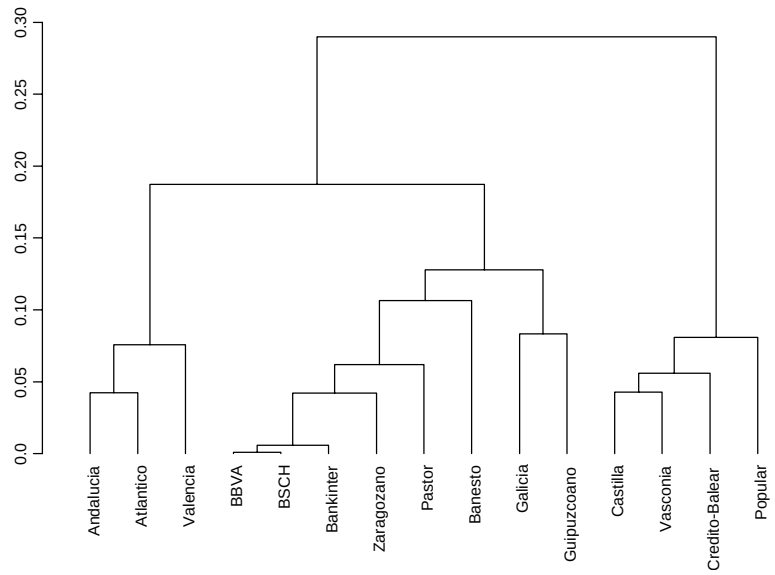


Figure 9: Dendrogram for hierarchical cluster analysis of the fifteen residual series when $\widehat{f}_{l,DLS}$, $D_W^\star$ and the average linkage method are used as nonparametric spectral estimator, series dissimilarity measure and groups proximity measure respectively.

Dendrograms in Figure 9 and Figure 10 show very similar nested sequences of cluster solutions. In both cases a primary classification leads to a three-cluster solution where the residual series would be grouped in the following groups: $S1 = \{$Vasconia, Castilla, Crédito-Balear, Popular$\}$, $S2 = \{$Valencia, Andalucía, Atlántico$\}$ and $S3 = \{$Zaragozano, Pastor, Bankinter, BBVA, BSCH, Galicia, Guipuzcoano$\}$, with Banesto included in $S2$ or in $S3$ depending that the used smoother is $\widehat{f}_{l,LK}$ or $\widehat{f}_{l,DLS}$ respectively. Also a five-cluster solution with the two new groups $S4 = \{$Banesto$\}$ and $S5 = \{$Galicia, Guipuzcoano$\}$ appears to be reasonable.

Some conclusions can be fairly derived from any of the obtained dendrograms:

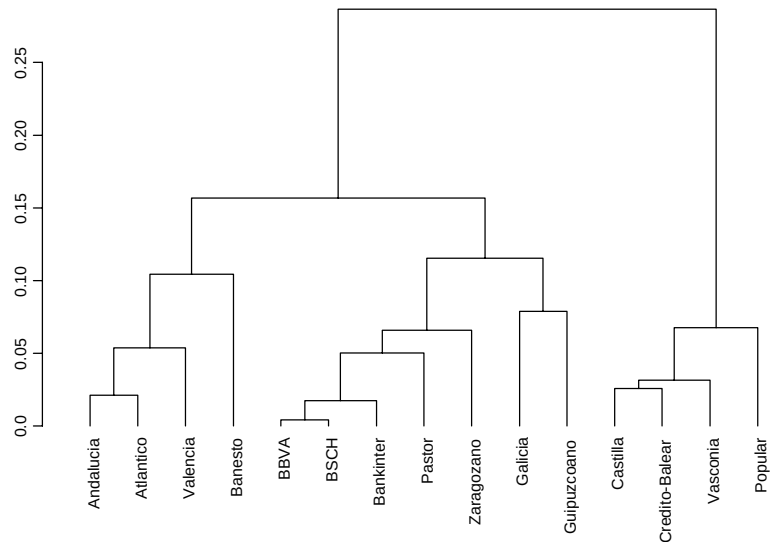


Figure 10: Dendrogram for hierarchical cluster analysis of the fifteen residual series when $\widehat{f}_{l,LK}$, $D_W^\star$ and the average linkage method are used as nonparametric spectral estimator, series dissimilarity measure and groups proximity measure respectively.

(i) Once the trends have been removed, three models (or five if the five-cluster solution is adopted) should suffice to describe the dependence structures of the fifteen residuals series. So, the behavior of future values as function of past values will be very similar for those banks confined in a same group and somewhat different otherwise and, indeed, this kind of information can be very valuable in a stock market context.

(ii) Whether parametric structures are required to model the dependence and, for example, the modelling is restricted to an autoregressive model or a moving average model, then the residual series in all resulting groups can be reasonably well-fitted by $AR(1)$ processes, $\hat{\varepsilon}_{l,t} = \phi\hat{\varepsilon}_{l,t-1} + z_{l,t}$, with $|\phi| < 1$, $z_{l,t}$ iid $N\left(0, \sigma^2\right)$ and the values of ϕ and σ^2 depending on the particular cluster where the lth series is confined.

So, if the five-cluster solution is considered, then series in group $S1$ present the smallest value for $\phi = \phi_{S1} \approx 0.4$ and the largest standard deviation $\sigma = \sigma_{S1} = 0.33$. Crédito-Balear bank, with a smaller estimated autoregressive coefficient, around 0.26, and Popular bank, with a estimated variance slightly higher, 0.4, are the profiles farthest from the centroid of $S1$. This proximity design is well illustrated in Figure 9, whereas is a bit obscured in dendrogram in Figure 10 where Vasconia bank is included in $S1$ later than Crédito-Balear. Series in $S2$ form a compact group where $\phi = \phi_{S2} \approx 0.6$ and $\sigma = \sigma_{S2} = 0.13$, whereas for series in $S3$, $\phi = \phi_{S3}$ is shifted between 0.64 and 0.74 and $\sigma = \sigma_{S3} = 0.24$ in all cases. The estimated values for Banesto bank were $\phi = \phi_{S4} \approx 0.74$ and $\sigma = \sigma_{S4} = 0.23$ so that

$S3$ would be its closest cluster when the $ARMA$ representation is considered as proximity criterion. This fact appears again better illustrated in Figure 9 than in Figure 10. Lastly, in $S5$ are located the residual series of Guipuzcoano and Galicia banks although both profiles are joined very late in the process and they can be regarded as isolated points similarly to Banesto.

In summary, our cluster procedure leads to group the residual series quite according with their fitted $ARMA$ models. This was expected since our cluster algorithm is based on measuring the discrepancy between spectra or, equivalently, between covariance structures. However, it must be noted that our cluster algorithm does not require any previous assumption on the probabilistic structure of the series because is based on nonparametric estimators of the spectral densities. So, from a theoretical point of view, the only requirements on the stochastic processes of our cluster algorithm are limited to general regularity conditions under which the consistency of the spectral smoothers is ensured (see Fan and Gijbels (1996)).

(iii) A comparison between the cluster analysis with the residuals series and the cluster analysis carried out with the trends in Section 2 (see again Figure 4 or Figure 5) allows us to conclude that there are similarities between the classification of the residuals and the classification of the trends but there are essential differences as well. For example, both the trends and the residuals series of Bankinter, BBVA and BSCH are joined together in the earliest stages of the corresponding agglomerative processes. Therefore, it is clear that these three banks behave in the same way in terms of their deterministic performance but also in terms of their stochastic performance. In the opposite side, it is observed that, e.g., the residuals series of Atlántico bank was located in $S2$ group together with those of Valencia and Andalucía banks, although the trends of these two banks are increasing in contrast to the decreasing trend of Atlántico bank.

(iv) As it was previously pointed out, a cluster algorithm to classify time series must keep in mind the existence of a probabilistic structure underlying to the series because in other case no appropriate cluster solutions are obtained. To highlight this fact, we performed a standard cluster algorithm with our set of residuals series using the squared Euclidean distance and the average linkage method. The derived results were essentially different and, in particular, some series with different fitted autoregressive structures were located together in the same group. For example, Galicia and Andalucía banks are surprisingly joined very early, at the third stage of the process, immediately after Bankinter, BBVA and BSCH which are kept together. Note that in our cluster procedure Galicia and Andalucía banks belong to groups placed faraway each other and they would be only joined at the end of the process.

(v) The particular choice of the spectral smoother does not play a key role since that very similar results were obtained with any of the three used local linear estimates $\widehat{f}_{i,DLS}$, $\widehat{f}_{i,LS}$ and $\widehat{f}_{i,LK}$. In fact the main differences concerned to the location of the banks farthest from any common pattern (e.g. Banesto), at the latest stages of the cluster procedure, so no substantial differences in the cluster solution were reported. In this point it must be noted

that the theoretical advantages of the likelihood-based estimator, $\widehat{f}_{i,LK}$, are obtained in an asymptotic sense, so such advantages can be negligible with finite samples.

4 Concluding remarks

The general philosophy of this paper has been to show as nonparametric methodology provides a wide range of procedures to perform a whole statistical analysis of a set of observed time series, including estimation, testing of hypotheses and cluster analysis. Besides being free of rigid parametric constraints, nonparametric techniques can be perfectly adjusted to deal with dependent observations as it is shown throughout this work.

The set of examined series consisted of the weekly price (per share in euros) of the banks in the Spanish stock market during the years 2001 and 2002. One of the main objectives was to classify the banks into a few categories, so series located in the same category present a similar performance pattern. Therefore, the problem of fitting a particular model to each of the series was not the main concern in this paper, but finding similarities and differences among the observed series. To achieve this goal it is necessary to analyze separately the trends and the stochastic components of the series.

Local linear smoothers of the trends allowed us to obtain a general view of the set of deterministic components (see Figure 2) suggesting, in particular, that there likely exist different performance patterns among the trends. A recently developed nonparametric test statistic led to reject the hypothesis of equality of the trends giving so statistical support to this intuition. A standard cluster analysis with the fitted trends suggested a five-cluster solution where the resulting groups were formed by $C1 = \{$BBVA, BSCH, Bankinter, Atlántico$\}$, $C2 = \{$Banesto$\}$, $C3 = \{$Andalucía, Valencia, Galicia$\}$, $C4 = \{$Guipuzcoano, Zaragozano$\}$ and $C5 = \{$Castilla, Popular, Crédito-Balear, Pastor, Vasconia$\}$. Furthermore, a description of the performance pattern for the trends of each group was made and it is worth pointing out that this kind of description was feasible on the basis of the nonparametric characterization. Lastly, pairwise trends comparisons were checked within each cluster using up to four different test statistics and, in all cases, the results allowed us to corroborate the cluster solution and confirm our classification of the deterministic components. The four used test statistics were again based on nonparametric techniques and their sampling distributions were approximated by a bootstrap algorithm constructed to take into account the underlying dependence structure.

Once the deterministic components of the banks series were classified and characterized, our attention was concentrated on their random components. The problem was now to cluster stationary time series with zero mean, so similarities and differences must be determined in terms of the amount of affinity of their covariance structures. Therefore, the chosen cluster procedure must consider distances involving covariance matrices that, in a time series setting, will have in general a high dimensionality. To overcome the dimensionality problem,

the residual series were clustered in the frequency domain, that is, the disparity between covariance structures was measured in terms of disparity between the corresponding spectral representations. This was done obtaining first local linear estimates of the spectra and computing then the distance between them by means of a spectral disparity measure whose asymptotic theoretical properties have been studied by the authors in a previous work.

The results of the cluster analysis with the residual series identified also five groups: $S1 = \{$Vasconia, Castilla, Crédito-Balear, Popular$\}$, $S2 = \{$Valencia, Andalucía, Atlántico$\}$, $S3 = \{$Zaragozano, Pastor, BBVA, BSCH, Bankinter$\}$, $S4 = \{$Banesto$\}$ and $S5 = \{$Galicia, Guipuzcoano$\}$. Such a classification appeared to be quite consistent with the corresponding fitted $ARMA$ models, so our cluster procedure was able to group the series according with their dependence structures. This is a relevant fact since any previous assumption on the dependence structure was made, in contrast to other methods in literature specifically designed to deal with time series but assuming particular structures on the underlying dependence models. In summary, it seems that, besides presenting good asymptotic properties, our cluster procedure also behave reasonably well with finite sample and can be successfully applied to group sequences of real data.

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References

1. Dette, H. and Neumeyer, N. (2001). Nonparametric analysis of covariance. *Ann. Stat.*, 29, 5, 1361–1400.

2. Fan, J. and Gijbels, I. (1996). *Local Polynomial Modelling and Its Applications*. Chapman and Hall. London.

3. Fan, J. and Kreutzberger, E. (1998). Automatic local smoothing for spectral density estimation. *Scand. J. Stat.*, 25, 359–369.

4. Francisco-Fernández, M. and Vilar-Fernández, J.M. (2001). Local polynomial regression estimation with correlated errors. *Commun. Statist. Theory Meth.*, 30, 7, 1271–1293.

5. Francisco-Fernández, M.and Vilar-Fernández, J.M. (2004). Weighted local nonparametric regression with dependent errors: study of real private residential fixed investment in the USA. *Statistical Inference for Stochastic Processes*, 7, 1, 69–93.

6. Galbraith, J.K. and Jiaqing, L. (1999). Cluster and discriminant analysis on Time Series as a research tool. *UTIP Working Paper Number 6*, Austin: Lyndon B. Johnson School of Public Affairs. The University of Texas at Austin.

7. Hall, P. and Hart, J.D. (1990). Bootstrap test for difference between means in nonparametric regression. *J. Am. Stat. Assoc.*, 85, 412, 1039–1049.

8. Hirsch, B. and DuBois, D. (1991). Self-Esteem in early adolescence: The identification and prediction of contrasting longitudinal trajectories. *Journal of Youth and Adolescence*, 20, 1, 53–72.

9. Kakizawa, Y., Shumway, R.H. and Taniguchi, M. (1998). Discrimination and clustering for multivariate time series. *J. Am. Stat. Assoc.*, 93, 328–340.

10. King, E.C., Hart, J.D. and Wehrly, T.E. (1991). Testing the equality of two regression curves using linear smoothers. *Stat. Probab. Lett.*, 12, 3, 239–247.

11. Koul, H. L. and Schick, A. (1997). Testing for the equality of two nonparametric regression curves. *J. Stat. Plan. Infer.*, 65, 293–314.

12. Kulasekera, K.B. (1995). Comparison of regression curves using quasi-residuals. *J. Am. Stat. Assoc.*, 431, 90, 1085–1093.

13. Maharaj, E.A. (1996). A significance test for classifying $ARMA$ models. *J. Statist. Comput. Simul.*, 54, 305–331.

14. Maharaj, E.A. (2000). Clusters of time series. *J. Classification*, 17, 297–314.

15. Neumeyer, N. and Dette, H. (2003). Nonparametric comparison of regression curves: an empirical process approach. *Ann. Stat.*, 31, 3, 880–920.

16. Piccolo, D. (1990). A distance measure for classifying $ARIMA$ models. *J. Time Series Anal.*, 11, 2, 153–164.

17. Tong, H. and Dabas, P. (1990). Cluster of time series models: an example. *J. Appl. Stat.*, 17, 187–198.

18. Vilar Fernández, J.A. and Pértega, S. (2004). Discriminant and cluster analysis for Gaussian stationary processes: Local linear fitting approach. To appear in *Nonparametr. Stat.*

19. Vilar Fernández, J.M. and González Manteiga, W. (2004). Nonparametric comparison of curves with dependent errors. To appear in *Statistics.*

20. Vilar Fernández, J.M., Vilar Fernández J.A. and González Manteiga, W. (2004). Bootstrap tests for nonparametric comparison of regression curves with dependent errors. Preprinted submitted to *Comput. Stat. Data Anal.*

21. Young, S.G. and Bowman, A.W. (1995). Nonparametric analysis of covariance. *Biometrics*, 51, 920–931.