Esercizio 1

Calcolare i seguenti limiti:

(a) $\lim\limits_{x\to 0}\left[\dfrac{1}{\log(1+x^4)} - \dfrac{1}{x^4}\right]$; $\qquad\left(\dfrac{1}{2}\right)$

(b) $\lim\limits_{x\to 0^+}\left[1 + \log(1+\sqrt{x})\right]^{\frac{1}{x^2}}$; $\qquad(+\infty)$

(c) $\lim\limits_{x\to +\infty} x^{\log\left(1+\frac{1}{x}\right)}$; $\qquad(1)$

(d) $\lim\limits_{x\to +\infty}\dfrac{\sqrt{1+x^2}}{x+1}$; $\qquad(1)$

(e) $\lim\limits_{x\to +\infty}\left(x+1-\sqrt{1+x^2}\right)$; $\qquad(1)$

(f) $\lim\limits_{x\to +\infty}\left(\sqrt{1+x^2} - \dfrac{x^2+1}{x+1}\right)$; $\qquad(1)$

(g) $\lim\limits_{x\to +\infty}\left[x - x^2\log\left(1+\dfrac{1}{x}\right)\right]$; $\qquad\left(\dfrac{1}{2}\right)$

(h) $\lim\limits_{x\to 1}\dfrac{x - x^x}{1 - x + \log(x)}$; $\qquad(2)$

(i) $\lim\limits_{x\to 0}\dfrac{e - (1+x)^{1/x}}{x}$; $\qquad\left(\dfrac{e}{2}\right)$

(j) $\lim\limits_{x\to 0^+}\log(x)\left[\log(1+x)\right]^2$; $\qquad(0)$

(k) $\lim\limits_{x\to +\infty}\left(\dfrac{\log(x)}{x}\right)^{\frac{1}{x}}$; $\qquad(1)$

(l) $\lim\limits_{x\to 0}\dfrac{\sqrt{1+x}(1+x)^{\frac{1}{x}} - e}{x^2}$; $\qquad\left(\dfrac{e}{12}\right)$

Esercizio 2

Scrivere l'equazione della tangente alla curva $y = f(x)$ nel punto di ascissa x_0:

(a) $y = x^3 - 2x + 3$, $x_0 = 0$; $\qquad(2x + y - 3 = 0)$

(b) $y = \sqrt{x^2+1}$, $x_0 = \sqrt{3}$; $\qquad\left(\sqrt{3}x - 2y + 1 = 0\right)$

(c) $y = e^{\sqrt{x}}$, $x_0 = 1$; $\qquad(ex - 2y + e = 0)$

(d) $y = x\left(\log(x) - 1\right)$, $x_0 = \sqrt{e}$; $\qquad\left(x - 2y - 2\sqrt{e} = 0\right)$

(e) $y = x^x$, $x_0 = \frac{1}{2}$; $\qquad\left(2(1 - \log 2)x - 2\sqrt{2}y + 1 + \log 2 = 0\right)$

Esercizio 3

Applicando le regole di derivazione, dimostare che si ha:

(a) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x(\log(x)-1)\right] = \log(x)$;

(b) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x-2}{x+2}\right] = \frac{4}{(x+2)^2}$;

(c) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x^\alpha \log(x) - \frac{x^\alpha}{\alpha}\right] = \alpha x^{\alpha-1}\log(x), \quad \alpha \neq 0$;

(d) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1+x^2}{4+x^2}\right] = \frac{6x}{(4+x^2)^2}$;

(e) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\mathrm{e}^x}{1+x}\right] = \frac{x\mathrm{e}^x}{(1+x)^2}$;

(f) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x\sqrt{x\sqrt{x}}\right] = \frac{7}{4}\sqrt{x\sqrt{x}}$;

(g) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\log_x(a)\right] = -\frac{\log(a)}{x(\log(x))^2}$;

(h) $\frac{\mathrm{d}}{\mathrm{d}x}\left[10\sqrt{x} - 20\log(2+\sqrt{x})\right] = \frac{5}{2+\sqrt{x}}$;

(i) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x\sqrt{\frac{x}{2-x}}\right] = \frac{3-x}{2-x}\sqrt{\frac{x}{2-x}}$;

(j) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\log\left(\frac{\sqrt{1+\mathrm{e}^x}-1}{\sqrt{\mathrm{e}^x}}\right)\right] = \frac{1}{2\sqrt{1+\mathrm{e}^x}}$;

(k) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1+\sqrt[4]{x^3}}{1-\sqrt[4]{x^3}}\right] = \frac{3}{2\sqrt[4]{x}\left(1-\sqrt[4]{x^3}\right)^2}$;

(l) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x\sqrt{1+x^2} + \log\left(x+\sqrt{1+x^2}\right)\right] = 2\sqrt{1+x^2}$;

(m) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x^x\right] = x^x(\log(x)+1)$;

(n) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x^{\log(x)}\right] = 2x^{\log(x)-1}\log(x)$;

(o) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x^{\mathrm{e}^x}\right] = x^{\mathrm{e}^x}\mathrm{e}^x\left(\log(x)+\frac{1}{x}\right)$;

(p) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x^{1/x}\right] = x^{1/x-2}\left(1-\log(x)\right)$.

Esercizio 4

Calcolare la derivata terza delle seguenti funzioni:

(a) $y = \frac{\log(x)}{x}$; $\qquad \left(y''' = \frac{11-6\log(x)}{x^4}\right)$

(b) $y = \frac{x}{\log(x)}$; $\qquad \left(y''' = \frac{(\log(x))^2-6}{x^2(\log(x))^4}\right)$

(c) $y = \frac{1+x}{1-x}$; $\qquad \left(y''' = \frac{12}{(x-1)^4}\right)$

(d) $y = \frac{x^2-1}{x^2+1}$; $\qquad \left(y''' = 48\frac{x(x^2-1)}{(x^2+1)^4}\right)$

(e) $y = \frac{x^2}{\mathrm{e}^x}$; $\qquad \left(y''' = \frac{-x^2+6x-6}{\mathrm{e}^x}\right)$