Esercizio 1
Calcolare i seguenti limiti:

(a) $\lim_{x\to 0}\left[\frac{1}{\log(1+x^4)}-\frac{1}{x^4}\right]$; $\left(\frac{1}{2}\right)$

(b) $\lim_{x\to 0^+}\left[1+\log(1+\sqrt{x})\right]^{\frac{1}{x^2}}$; $(+\infty)$

(c) $\lim_{x\to +\infty} x^{\log(1+\frac{1}{x})}$; (1)

(d) $\lim_{x\to +\infty}\frac{\sqrt{1+x^2}}{x+1}$; (1)

(e) $\lim_{x\to +\infty}\left(x+1-\sqrt{1+x^2}\right)$; (1)

(f) $\lim_{x\to +\infty}\left(\sqrt{1+x^2}-\frac{x^2+1}{x+1}\right)$; (1)

(g) $\lim_{x\to +\infty}\left[x-x^2\log\left(1+\frac{1}{x}\right)\right]$; $\left(\frac{1}{2}\right)$

(h) $\lim_{x\to 1}\frac{x-x^x}{1-x+\log(x)}$; (2)

(i) $\lim_{x\to 0}\frac{\mathrm{e}-(1+x)^{1/x}}{x}$; $\left(\frac{\mathrm{e}}{2}\right)$

(j) $\lim_{x\to 0^+}\log(x)\left[\log(1+x)\right]^2$; (0)

(k) $\lim_{x\to +\infty}\left(\frac{\log(x)}{x}\right)^{\frac{1}{x}}$; (1)

(l) $\lim_{x\to 0}\frac{\sqrt{1+x}(1+x)^{\frac{1}{x}}-\mathrm{e}}{x^2}$; $\left(\frac{\mathrm{e}}{12}\right)$

Esercizio 2
Scrivere l'equazione della tangente alla curve $y=f(x)$ nel punto di ascissa x_0:

(a) $y=x^3-2x+3,\ x_0=0$; $(2x+y-3=0)$

(b) $y=\sqrt{x^2+1},\ x_0=\sqrt{3}$; $\left(\sqrt{3}x-2y+1=0\right)$

(c) $y=\mathrm{e}^{\sqrt{x}},\ x_0=1$; $(\mathrm{e}x-2y+\mathrm{e}=0)$

(d) $y=x\left(\log(x)-1\right),\ x_0=\sqrt{\mathrm{e}}$; $(x-2y-2\sqrt{\mathrm{e}}=0)$

(e) $y=x^x,\ x_0=\frac{1}{2}$; $\left(2(1-\log 2)x-2\sqrt{2}y+1+\log 2=0\right)$

Esercizio 3
Applicando le regole di derivazione, dimostare che si ha:

(a) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x(\log(x)-1)\right] = \log(x)$;

(b) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{x-2}{x+2}\right] = \frac{4}{(x+2)^2}$;

(c) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x^\alpha \log(x) - \frac{x^\alpha}{\alpha}\right] = \alpha x^{\alpha-1}\log(x), \quad \alpha \neq 0$;

(d) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1+x^2}{4+x^2}\right] = \frac{6x}{(4+x^2)^2}$;

(e) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{\mathrm{e}^x}{1+x}\right] = \frac{x\mathrm{e}^x}{(1+x)^2}$;

(f) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x\sqrt{x\sqrt{x}}\right] = \frac{7}{4}\sqrt{x\sqrt{x}}$;

(g) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\log_x(a)\right] = \frac{\log(a)}{x(\log(x))^2}$;

(h) $\frac{\mathrm{d}}{\mathrm{d}x}\left[10\sqrt{x} - 20\log(2+\sqrt{x})\right] = \frac{5}{2+\sqrt{x}}$;

(i) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x\sqrt{\frac{x}{2-x}}\right] = \frac{3-x}{2-x}\sqrt{\frac{x}{2-x}}$;

(j) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\log\left(\frac{\sqrt{1+\mathrm{e}^x}-1}{\sqrt{\mathrm{e}^x}}\right)\right] = \frac{1}{2\sqrt{1+\mathrm{e}^x}}$;

(k) $\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{1+\sqrt[4]{x^3}}{1-\sqrt[4]{x^3}}\right] = \frac{3}{2\sqrt[4]{x}\left(1-\sqrt[4]{x^3}\right)^2}$;

(l) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x\sqrt{1+x^2} + \log\left(x+\sqrt{1+x^2}\right)\right] = 2\sqrt{1+x^2}$;

(m) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x^x\right] = x^x(\log(x)+1)$;

(n) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x^{\log(x)}\right] = 2x^{\log(x)-1}\log(x)$;

(o) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x^{\mathrm{e}^x}\right] = x^{\mathrm{e}^x}\mathrm{e}^x\left(\log(x)+\frac{1}{x}\right)$;

(p) $\frac{\mathrm{d}}{\mathrm{d}x}\left[x^{1/x}\right] = x^{1/x-2}\left(1-\log(x)\right)$.

Esercizio 4
Calcolare la derivata terza delle seguenti funzioni:

(a) $y = \frac{\log(x)}{x}$; $\qquad \left(y''' = \frac{11-6\log(x)}{x^4}\right)$

(b) $y = \frac{x}{\log(x)}$; $\qquad \left(y''' = \frac{(\log(x))^2-6}{x^2(\log(x))^4}\right)$

(c) $y = \frac{1+x}{1-x}$; $\qquad \left(y''' = \frac{12}{(x-1)^4}\right)$

(d) $y = \frac{x^2-1}{x^2+1}$; $\qquad \left(y''' = 48\frac{x(x^2-1)}{(x^2+1)^4}\right)$

(e) $y = \frac{x^2}{\mathrm{e}^x}$; $\qquad \left(y''' = \frac{-x^2+6x-6}{\mathrm{e}^x}\right)$