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Endogenous prices in mathematical programming models for agricultural policy analysis

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Abstract

This paper proposes a price endogenous model based upon positive mathematical programming methodology for policy and market evaluations. The model is developed preserving the competitive character of farm decisions and considering the aggregate supply response on market prices. The method of aggregation allows one to use the tool for policy evaluation at the sectoral or regional level using individual farm data. The process of simulation adopts the positive mathematical programming calibration property for evaluating farm behaviour dynamics and the estimation of the inverse demand and supply functions for generating endogenous prices relayed on the aggregated individual supply decisions.

Key words: endogenous prices, positive mathematical programming, policy analysis

JEL: C61, Q11

1. Introduction

In recent decades, many policy-makers have encouraged agricultural economists to develop quantitative models able to respond to policy evaluation needs. As a result, mathematical programming (MP) models have assumed a primary role in supporting agricultural policy evaluations. The straightforward interpretation of the MP model outcomes and their capacity to face, often in a poor information context, complex sector prediction problems have determined the MP model's success as tools for interpreting trends in agriculture influenced by agricultural policy mechanisms.

Many researchers have proposed their own MP instruments for evaluating agricultural policies: the work of Buysse et al. (2007) uses a positive mathematical programming (PMP) model to assess the effect of sugar CMO (Common Market Organization) reform on Belgian farmers; while Cortignani and Severini (2008) use an MP model to assess the CAP reform of farm decisions, with a particular emphasis on potential additional crops. It is interesting that all of these analyses have used MP tools to assess the supply side of the problem, that is the effect of scenarios on production levels. Thus, they have avoided providing results related to the effects of policy changes on prices. The simulations considered using such models are generally based on exogenous information about output prices. If one considers the wide range of recent MP models used for policy evaluations that are collected in the volume "Modelling Agricultural Policies: State of the Art and New Challenges" (Arfini, 2005), one may be very surprised to see that most of the MP models presented therein are strictly supply-oriented models and that the output prices are fixed. In recent scientific production, the MP models, even when implemented at the sectoral or territorial level, have failed to take into account the important relationship among supply responses and induced price changes. This relationship seems to be commonly recognised as an econometric issue to resolve using econometric techniques.

Samuelson (1952), building on the seminal paper by Enke (1951) about the famous "electric analogue" specification for linear market functions, explores the ability of linear programming (LP) to evaluate market behaviour using a maximisation problem in which the competitive market conditions are guaranteed. Samuelson's LP model reconstructs the supply and demand functions for products with endogenous responses upon production levels and output price levels. This study spurred a great deal of scientific production, the most significant contribution from which were the fundamental papers by Takayama and Judge (1964a, 1964b), in which multi-product supply and demand functions are internalised into a quadratic programming model representing a given sector. This model provides endogenous price solutions for inputs and outputs, considering the substitution and complementary relationships among activities.

Afterwards, Duloy and Norton (1975) propose a linear transformation of the Takayama and Judge framework that will completely depict the farm production system and incorporate product demand functions into MP models. An LP model built on individual farm data should take into account that in a competitive market context, producers act as price-takers; thus, endogenous prices have to be specified by a sectoral or aggregated objective function able to avoid internal monopolistic behaviour. On this basis, McCarl and Spreen (1980) explain how to appropriately develop price endogenous mathematical programming models for evaluating alternative policy scenarios. The aggregation process is a relevant issue in order to transmit the information about the farm planning allocation to the sector demand function. The aggregation issue as it relates to MP models is a problem that several economists have considered, identifying fundamental criteria that will help to minimise the loss of information during such a process (Day, 1963; Paris and Rausser, 1973; Spreen and Takayama, 1980).

It was not until 2005 that another significant development occurred when Rehman and Yates introduced an advance in the integration of demand functions into MP models. In their work, a large-scale LP model is proposed as an evolution of Martin's stepped LP approximation (Martin, 1972), reaching an equilibrium solution that involves the endogenous estimation of supply and prices but does not consider integrability conditions for demand systems. This latter matter is discussed in Spreen (2006) with reference to the Koopmans-Hitchcock transportation model, whose use is extended to a multiproduct case. Spreen argues that price endogenous models are diffused in empirical practice, even if in the agricultural economics literature after 1990 the MP models generally consider exogenous output prices except when using econometric techniques.

More recently, two other interesting approaches to market behaviour evaluation using MP methodology are presented. The first one was presented by Önal et al. (2009), who further advance the model by McCarl and Spreen, where crop mixed expansion is considered inside policy analysis. The second contribution was made by Arfini et al. (2008), who propose an approach based on the PMP methodology for building a model able to replicate farm behaviour using the estimation of inverse demand and supply functions based on cross-section data. The latter is particularly interesting because it is able to provide agricultural policy information that can inform farm planning responses to agricultural policy modifications and related impacts on output market prices. However, this model does not completely fit the competitive market or the aggregating conditions required for a farm planning model (McCarl and Spreen, 1980) providing an empirical framework far from price-taker behaviour because of individual demand functions inserted inside the individual objective function to maximise.

The purpose of this paper is to present a price endogenous PMP model able to appropriately represent farm planning decisions and simulate market price evolution within a framework of market competition. The first section proposes an extension of the price endogenous MP approach, the second section presents the PMP model with endogenous supply and prices, the third part illustrates how the discussed model can be implemented with a panel of farm data and the fourth section concludes with some main remarks.

2. Mathematical programming with endogenous prices

The producer's objective is to obtain the best economic result with the most limited use of resources. This simple idea originated what Samuelson (1952) called a "new" type of theory, linear programming. Each economic agent may be, thus, interested in obtaining from this theory the best planning organization, among the different choices, given the available input, in order to gain more than they would receive by a self-selection approach. In a competitive framework, according to the theory, this objective should be reached when prices equal marginal costs, where prices constitute a priori information owned, or perceived, by producers.

In the real world, farmers make their decisions upon expected prices, which are determined using their own knowledge about agricultural markets. This knowledge includes past price experience, their own risk attitudes, information about the future environment and information concerning off-market parameters like public rules and agricultural subsidies. Overall farm planning for one large geographical area or for a given large sector leads to modifications in market prices, sometimes inducing a relevant change in expected individual farm revenue. MP models can reproduce the farm system in detail (when information exists) and the farm behaviour to maximise a profit function subjected to several constraints. In such an objective function, prices are exogenous parameters and frequently support the sensitive analysis of market scenario simulations. If the aim is to add information about market responses in terms of modifications to output prices, the model should use an aggregated, rather than an individual, form. In this case, the objective function becomes a variable at endogenous prices (Duloy and Norton, 1975).

Let us start with the usual maximisation producer’s problem in a framework of constant return to scale where the production technology is assumed to be fixed:

$$
\begin{aligned}
\max_{x_j} \quad & \sum_{j=1}^{J} p_j x_j - \sum_{j=1}^{J} c_j x_j \\
& s.t. \\
& \sum_{j=1}^{J} a_{ji} x_j \le b_i \qquad \forall i \\
& x_j \ge 0 \qquad \forall j
\end{aligned} \tag{1}
$$

The objective function represents the farm gross margin to maximise with respect to the output levels x_j for the different farm processes *j* (*j*=1,...,*J*; equal to *k*=1,…,*K*). p_j and c_j represent prices and variable costs for each farm process. The objective function is submitted to a structural constraint according to which the sum of the factors used for the different activities should be less than or equal to the factors available at the farm level *i*, for *i*=1,…, *I*). The parameter a_{ji} provides information about the use of the factor to obtain a unit of process x_j. The Lagrangian transformation of problem (1) is as follows:

$$L = \mathbf{p'x} - \mathbf{c'x} + \mathbf{y'(b - Ax)} \qquad (2)$$

where the vector **y** represents the dual values associated with the limited factor **b**.

The Karush-Kuhn-Tucker (KKT) conditions for obtaining an internal optimal solution through problem (1) are the following:

$$\frac{\partial L}{\partial x_j} = \sum_{i=1}^{I} a_{ji} y_i + c_j - p_j \geq 0 \quad \forall j \qquad (3a)$$

$$\frac{\partial L}{\partial x_j} x_j = 0 \quad x_j \geq 0 \qquad (3b)$$

$$\frac{\partial L}{\partial y_i} = \sum_{j=1}^{J} a_{ji} x_j - b_i \leq 0 \quad \forall i \qquad (4a)$$

$$\frac{\partial L}{\partial y_i} y_i = 0 \quad y_i \geq 0 \qquad (4b)$$

Equations (3) and (4) assure that producer’s problem can achieve an optimal solution. In particular, the relation (3a) is the dual constraint of dual problem of (1) and reveals the competitive equilibrium that is a necessary condition for producer decisional behaviour. Equation (3b) explains how at the optimal level, the primal objective function must be equal to the dual one. Equation (4a) returns the structural constraints in (1), while equation (4b) maintains the same meaning as (3b).

Assume, now, to dispose of information about *N* farms representative of a given sector (i.e., arable crops), so that it is possible to develop and solve *N* LP models like (1). Each farm will produce results in terms of output quantities (**x**) and input marginal value (**y**) in line with conditions (3)-(4).

No individual farm cannot affect the product market price, but the sectoral response determined by farm plan allocations has an evident relationship with price dynamics. This implies that MP models developed at the micro level for a representative sample of farms should produce macro information at the sectoral level.

To correctly aggregate individual farm planning problems, many authors, beginning with Day (1963), have developed sufficient conditions for exact aggregation in MP. These conditions permit one to convert a system of *N* individual producer problems into an aggregated one without loss of information and to guarantee that the sum of the output results obtaining solving N individual problems is equal to the solution for the aggregated outputs. In other terms, the sum of the optimal solutions for farms included in a given sample must be equal to the aggregated model solution:

$$\sum_{n=1}^{N} x_{nj}^{*} = X_{j}^{*} \tag{5}$$

where X_j^* indicates the optimal solution for the aggregated MP model. This condition permits one to affirm that the total gross margin at aggregated level must be equal to the sum of the individual gross margins (Day, 1963). Symmetrically, the total cost of the constrained resources at the aggregated level must be equal to the sum of the individual input total costs descending from the solution of dual problem (1). Then, the dual condition for an exact aggregation is as follows:

$$\sum_{n=1}^{N} b_{ni} y_{ni}^{*} = B_i Y_i^{*} \quad \forall i \tag{6}$$

where B_i and Y_i indicate, respectively, the total available resources *I* for the given group of farms and the marginal costs linked to such resources. The primal (5) and dual (6) conditions contribute to an exact aggregation assuming equality or proportionality among the technical and economic coefficients inside the group of *N* farms. Furthermore, according this formulation, the dimensionality of *N* farms should be the same.

The purpose of an aggregated model is to provide information about the total output decision process at the sectoral level and the related level of prices imposing upon the market after product allocation (McCarl and Spreen, 1980). The assumption of exogenous prices as stated in model (1) is no longer acceptable, so that the sectoral model should consider the linkage between aggregated producer decisions and their effects on market prices. For more insight, consider the following inverse demand function:

$$\mathbf{P}_d = f(\mathbf{X},\boldsymbol{\Gamma}) = \boldsymbol{\alpha} - \mathbf{DX} \tag{7}$$

and the inverse supply function:

$$\mathbf{P}_s = f(\mathbf{X},\boldsymbol{\Theta}) = \boldsymbol{\beta} + \mathbf{QX} \tag{8}$$

where $\mathbf{P}_d$ is the price vector of the outputs, **X** the aggregated endogenous output response and $\boldsymbol{\Gamma}$ the vector of exogenous parameters including the intercept, $\boldsymbol{\alpha}$ (Jx1), and the slope matrix **D**(JxK); $\mathbf{P}_s$ is the price vector of variable factors measured in terms of marginal cost per sectoral output quantity, while $\boldsymbol{\Theta}$ is the vector of exogenous parameters for the intercept, $\boldsymbol{\beta}$ (Jx1), and the slope matrix **Q**(JxK).

Given functions (7) and (8), it is possible to obtain Samuelson's Net Social Payoff (NSP) in a definite integral form as follows:

$$NSP = \sum_{j=1}^{J}\left\{\int_{0}^{\sum_n x_{nj}}\left(\alpha_j - \sum_k D_{jk}X_k\right)d\left(\sum_n x_{nj}\right)\right\} - \sum_{j=1}^{J}\left\{\int_{0}^{\sum_n x_{nj}}\left(\beta_j + \sum_k Q_{jk}X_k\right)d\left(\sum_n x_{nj}\right)\right\} \tag{9}$$

The economic interpretation of the NSP is that the difference between the total value of the output produced in a sector/region and the total cost of the variable inputs used by producers in the same sector/region is equal to the net margin for that given sector/region. In Takayama and Judge's (1964a) formulation, the NSP is composed of the components in (9) plus the total cost of transportation supported by traders from supply to the demand points. In our proposal, this last added value is substituted for by the total using cost of fixed inputs, like land surface area, so that it represents the producer costs (in terms of constrained resources) of making total production available to markets. Furthermore, it represents the opportunity cost of scarce factors at the sector/regional level. Integral (9) combined with the equation (6) provides a new aggregated result that can be defined as the Net Aggregated Payoff (NAP):

$$NAP = \sum_{j=1}^{J} \alpha_j X_j - \frac{1}{2} \sum_{j=1}^{J} \sum_{k=1}^{K} X_j D_{jk} X_k - \sum_{j=1}^{J} \beta_j X_j - \frac{1}{2} \sum_{j=1}^{J} \sum_{k=1}^{K} X_j Q_{jk} X_k - \sum_{i=1}^{I} B_i Y_i \qquad (10)$$

The NAP can be used as an objective function to maximise into a sectoral/regional model, where N farms produce J activities for a market adopting I fixed inputs. It is important to highlight that each farm acts as an economic agent in a perfect competitive market, so that individual behaviour does not affect the market price. This means that if we develop an aggregated model safeguarding the competition market framework, conditions (3) and (4) should be considered.

According to the previous statements, the aggregated model with endogenous prices can be structured as follows:

$$\max_{X_j, x_{nj}, Y_i, y_{ni}} \sum_{j=1}^{J} \alpha_j X_j - \frac{1}{2} \sum_{j=1}^{J} \sum_{k=1}^{K} X_j D_{jk} X_k - \sum_{j=1}^{J} \beta_j X_j - \frac{1}{2} \sum_{j=1}^{J} \sum_{k=1}^{K} X_j Q_{jk} X_k - \sum_{i=1}^{I} B_i Y_i \qquad (11a)$$

s.t.

$$\sum_{n=1}^{N} x_{nj} = X_j \qquad \forall j \qquad [\mu_j] \qquad (11b)$$

$$\sum_{n=1}^{N} b_{ni} y_{ni} = B_i Y_i \qquad \forall i \qquad [\omega_i] \qquad (11c)$$

$$\sum_{j=1}^{J} A_{nji} x_{nj} \le b_{ni} \qquad \forall n \forall i \qquad [\gamma_{ni}] \qquad (11d)$$

$$\sum_{i=1}^{I} A_{nji} y_{ni} \ge p_{nj} - c_{nj} \qquad \forall n \forall j \qquad [\theta_{nj}] \qquad (11e)$$

$$x_{nj}, y_{ni} \ge 0 \qquad \forall n \forall j \forall i \qquad (11f)$$

The objective function to maximise is the NAP subjected to two aggregating constraints, (11b) and (11c), and the competitive conditions at individual levels, (11d) and (11e); based on (11f), the individual outputs and the marginal value of producer inputs are stated to be non-negative. The symbols in brackets, μ_j, ω_i, γ_{ni} and θ_{nj} are shadow prices associated with the constraints (11b), (11c), (11d) and (11e), respectively.

Let us define the Lagrangian transformation of model (11) as follows:

$$
\begin{aligned}
L \quad = \quad & NAP \\
& +\sum_j \mu_j \left(-X_j + \sum_n x_{nj} \right) \\
& +\sum_i \omega_i \left(B_i Y_i - \sum_n b_{ni} y_{ni} \right) \\
& +\sum_n \sum_i \gamma_{ni} \left(b_{ni} - \sum_j A_{nji} x_{nj} \right) \\
& +\sum_n \sum_j \theta_{nj} \left(-p_{nj} + c_{nj} + \sum_i A_{nji} y_{ni} \right)
\end{aligned} \tag{12}
$$

From (8) it is possible to obtain the KKT conditions:

$$\frac{\partial L}{\partial X_j} = \alpha_j - \sum_k D_{jk} X_k - \beta_j - \sum_k Q_{jk} X_k - \mu_j \le 0 \quad \text{and} \quad \frac{\partial L}{\partial X_j} X_j = 0 \quad \text{for} \quad X_j \ge 0 \tag{13}$$

$$\frac{\partial L}{\partial Y_i} = -B_i + \omega_i B_i \ge 0 \quad \text{and} \quad \frac{\partial L}{\partial Y_i} Y_i = 0 \quad \text{for} \quad Y_i \ge 0 \tag{14}$$

$$\frac{\partial L}{\partial x_{nj}} = \mu_j - \sum_i \gamma_{ni} A_{nji} \le 0 \quad \text{and} \quad \frac{\partial L}{\partial x_{nj}} x_{nj} = 0 \quad \text{for} \quad x_{nj} \ge 0 \tag{15}$$

$$\frac{\partial L}{\partial y_{ni}} = -\omega_i b_{ni} + \sum_j \theta_{nj} A_{nji} \le 0 \quad \text{and} \quad \frac{\partial L}{\partial y_{ni}} y_{ni} = 0 \quad \text{for} \quad y_{ni} \ge 0 \tag{16}$$

If one considers that the total production at the aggregate level is positive, one can state the strict equality of the derivative in (13); this leads us to affirm that $\mathbf{P}_d^* - \mathbf{P}_s^* = \boldsymbol{\mu}^*$, which corresponds to the equivalence between Samuelson's marginal NSP and the marginal values of the aggregated outputs. Thus, μ_j represents the increase in the NAP induced by a unitary increase in sectoral/regional output.

Conditions (14) reveals that for a positive aggregated fixed input value, ω_i is equal to 1, thus indicating that θ_{nj} is the dual depiction of x_{nj} according to equation (11d) at the aggregated level. However, it is not possible to derive the same meaning for γ_{ni} and y_{ni}. γ_{ni} represents Lagrangian multipliers associated with the fixed resources available for producing the *J* activities, and it is strictly relayed on the marginal NSP defined by the KKT derivative (13). This dual value indicates that if one considers an increase of one unit of x_{nj}, the variation at the aggregated level of the NAP is equal to the marginal NSP. γ_{ni} can be interpreted as a social resource value, that is the value of the limited resources (land, water, etc.) assigned at the sectoral/regional level to produce a given level of output. Actually, rearranging (15) through complementary slackness like relation, one obtains the following:

$$\sum_n \mu_j x_{nj} = \sum_n \sum_i \gamma_{ni} A_{nji} x_{nj} \qquad \forall j \tag{17}$$

where the value of NSP per activity is equal to the total social resource cost incurred to produce the aggregated level per activity $\sum_n x_{nj}$.

Model (11) allows exact aggregation with a set of *N* linear programming models, partially contradicting the first theorem of Spreen and Takayama (1980), according to which "given a set of *N* linear programming models on aggregation of linear programming models and an aggregate model, then the aggregate model cannot satisfy exact aggregation […], but may be a semi-exact aggregation model". Indeed, KKT conditions (13) and (15) state the equivalence between sectoral prices and aggregated prices μ_j for all of the farms considered.

The model proposed in this section is rather different from what Samuelson and McCarl et al. propose, but it achieves the same results, with the advantage that it can be used to simulate price scenarios based on different expected prices for farmers. Actually, the set of p_{nj} into equation (11e) can be considered as the expected price inducing sectoral modification in prices based on individual planning decisions.

3. Endogenous prices in PMP

The model developed in the previous section permits one to simultaneously develop solutions for individual and aggregated production plans with a fixed technology and expected prices for the different activities and variable resources. The model outcomes indicate possible allocation scenarios at the farm and sectoral/regional level and the induced price responses at the aggregated level. However, the model (11) can be used for normative purposes because the solutions that such a model provides are prescriptions and not what the real decision reaction should be, with the relative problem of overspecialisation in the most profitable activities. Nevertheless, this type of model may be more linked to the reality if the researcher knows the farm production system in great detail, including the measure of the farmer's risk attitude. In regional models, built with the support of individual information, it is very difficult if not impossible to recover this detailed information at a micro level because the time and costs associated with this endeavour are often too high for such broad information reconstruction. This is why for policy evaluation and micro-based analysis, the classic normative mathematical programming model is substituted for with positive mathematical programming models that allow the use of less individual information, remarkably expanding the number of observed decision units (i.e., farms). The PMP methodology used in agricultural analysis has evidently helped to enrich the support information for agricultural economists and decision-makers.

Skipping the PMP methodology description that can be found in papers such as Howitt (1995), Paris and Howitt (1998) and Heckelei and Wolff (2003), in this section we propose the PMP version of the model (11). The basic solution provided by this new model should replicate the observed situation of each individual farm, and it reacts in term of production and price responses to modifications in market conditions (expected prices) and to changes in the public subsidy system, maintaining a strict connection with the design of the farmers' production plan as captured in the observed situation.

3.1 Inverse demand and supply functions estimation

The estimation of the parameter sets $\mathbf{\Gamma}$ and $\mathbf{\Theta}$ is a prerequisite for implementing model (11). This estimation can be based on exogenous information about prices and output levels as obtained at the farm level in the sample. In particular, the estimation of the parameter set $\mathbf{\Theta}$ is a prerequisite for calibrating the base situation, while knowledge about exogenous prices is not a sufficient condition for achieving a calibrated result. The price of the variable input should be integrated using an added-price component that into the observed farm information is latent but present in the farm decision-making system. This added component is assumed to be a differential cost that includes all those variable costs that the agricultural statistics do not catch: e.g., risk cost (Paris and Howitt, 1998). The added-differential cost is recovered by solving an LP problem in which the observed solution is forced, imposing the so-called calibrating constraint. The problem for *n* farm can be presented below as follows:

$$\begin{aligned} & \max_{\mathbf{x}\geq 0} \quad \mathbf{p'x - c'x} \\ & s.t. \\ & \mathbf{Ax} \leq \mathbf{b} \qquad [\mathbf{y}] \\ & \mathbf{x} \leq \bar{\mathbf{x}} + \varepsilon \quad [\boldsymbol{\lambda}] \end{aligned} \tag{18}$$

As one can see, problem (18) is very similar to the LP problem (1), except for the last constraint, which indicates **x** to be less than or equal to the observed output $\bar{\mathbf{x}}$ plus a very small term ε[1]. The most important term in this problem is the shadow price $\boldsymbol{\lambda}$ associated with the calibrating constraint: it is the differential cost that one must add to the explicit cost **c** and includes all of the economic information taken into account by *n* farmer in deciding to produce $\bar{\mathbf{x}}$.

Adopting formulation (8), the total marginal cost for each sample farm that considers both explicit and latent variable costs can be expressed in the following equation:

$$P_{nj}^{s} = f_n(\mathbf{x},\boldsymbol{\Theta}) = c_{nj} + \lambda_{nj}^{*} = \beta_j + Q_{jk}x_{nj}^{*} + u_{nj} \tag{19}$$

where u_{nj} represents the deviation from the total marginal cost, while the aggregated form can be summarised as follows:

$$P_{j}^{s} = f(\mathbf{X},\boldsymbol{\Theta}) = C_j + \Lambda_{j}^{*} = \beta_j + Q_{jk}X_{j}^{*} \tag{20}$$

Terms C_j and Λ_j represent the value of explicit and latent marginal costs in the aggregated specification of model (18). The estimation of parameters β_j beta and Q_{jk} using the known information about total marginal costs permits us to integrate the calibrating constraint in (18), so that if they are inserted into the objective function of a model similar to (18) without calibrating constraints, the optimal solution reached is the same, that is the observed production plan. When we also consider the estimation of the parameters for the inverse demand function, the objective is to derive the parameters according to the following:

$$\begin{bmatrix} \mathbf{P}_d \\ \mathbf{P}_s \end{bmatrix} = \begin{bmatrix} \boldsymbol{\alpha} \\ \boldsymbol{\beta} \end{bmatrix} + \begin{bmatrix} -\mathbf{D} & 0 \\ 0 & \mathbf{Q} \end{bmatrix} \bar{\mathbf{X}} = \begin{bmatrix} \mathbf{P} \\ \mathbf{C} + \boldsymbol{\Lambda} \end{bmatrix} \tag{21}$$

where **P** is the vector of output prices in the aggregated model specification (18) and $\bar{\mathbf{X}}$ is the vector of aggregated observed quantities; parameters $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, **D** and **Q** must be estimated using consistent techniques like OLS or maximum entropy (ME). In the appendix, the ME specification is

[1] This term is introduced in order to avoid the linear dependency among structural and calibrating constraints as suggested by Paris and Howitt (1998).

proposed. The estimation should guarantee the integrability of the inverse demand and supply functions, imposing the symmetry of matrices **D** and **Q**. This result can be obtained using decomposition procedures in order to obtain a symmetric positive semi-definite matrix. This means that matrices **D** and **Q** may contain non-zero parameters in off-diagonal positions, establishing a cross-relationship among different activities.

3.2 Calibrating model

To precisely reproduce the context at play (as indicated in the individual production plans and micro agricultural statistics) and perform the optimisation within a sectoral/regional partial market equilibrium framework with endogenous prices, it is sufficient to substitute equation (11e) into model (11) with the constraint presented below:

$$\sum_{i=1}^{I} A_{nji} y_{ni} \geq p_{nj} - \left\{ \hat{\beta}_j + \hat{Q}_{jk} x_{nj} + \hat{u}_{nj} \right\} \qquad \forall n \forall j \qquad [\theta_{nj}] \qquad (22)$$

It is clear that the only difference with respect to equation (11e) is the different specification of the explicit marginal variable costs, which in this case take the form introduced in equation (19), where the added cost component λ_{nj} enters in as the explicit cost in the new model. The output price p_{nj} remains defined as the exogenous variable, to be interpreted as the expected price for producers, so that it is possible to remain in a price-taker context.

The KKT conditions for the model (11) undergo a change for relation (15), which becomes:

$$\frac{\partial L}{\partial x_{nj}} = -\mu_j + \sum_i \gamma_{ni} A_{nji} + \sum_k \hat{Q}_{jk} \theta_{nj} \geq 0 \qquad \text{and} \qquad \frac{\partial L}{\partial x_{nj}} x_{nj} = 0 \qquad \text{for} \qquad x_{nj} \geq 0 \qquad (23)$$

Rewriting the condition in (23) yields the following:

$$\mu_j \leq \sum_i \gamma_{ni} A_{nji} + \sum_k \hat{Q}_{jk} \theta_{nj} \qquad (24)$$

where the left-hand side of (24) is the marginal value imputed to each activity at the aggregated level, while the right-hand side is the marginal cost of the binding and variable resources necessary for each farm *n* to produce one unit of each aggregated output *j*. Thus, (24) states that the marginal value of the activity at the regional/sector level must be less than or equal to the total marginal cost attributed to fixed and variable factors.

It is straightforward to identify the equivalence between the solutions obtained for the individual farms in the LP model (18) and those obtained in problem (11) modified using equation (22). Actually, $x^*_{nj(18)} = x^*_{nj(11\tilde{m}22)}$ and $y^*_{ni(18)} = y^*_{ni(11\tilde{m}22)}$ where the numbers in parenthesis refer to the solution-related model, and the symbol "$\tilde{m}$" should be read as "modified with".

The PMP model with endogenous prices as presented above simulates competitive behaviour for the group of farms under evaluation, preserving the planning and decision-making behaviour observed in the statistics using individual total marginal cost (19). Because such a model provides responses about quantities at the individual and sectoral/regional level and about prices at the sectoral/regional level, it may become a useful tool for policy and market analysis. Furthermore, the integrability property of the demand and supply functions offers information about the cross-price effect in activities at the aggregated level, so that

$$E^d_{jk} = \frac{dX_j}{dP^d_k}\frac{P^d_k}{X_j} \text{ and } \frac{dX_j}{dP^d_k} = \frac{dX_k}{dP^d_j} \text{ for all } j \neq k \qquad (25a)$$

$$E^s_{jk} = \frac{dX_j}{dP^s_k}\frac{P^s_k}{X_j} \text{ and } \frac{dX_j}{dP^s_k} = \frac{dX_k}{dP^s_j} \text{ for all } j \neq k \qquad (25b)$$

where E^d and E^s are the demand and supply elasticity values, respectively.

One important property of the model is that the producers' production planning responses take into account the lag between the allocation decision process and market price formation. Indeed, producers generally consider as price signals those that have arisen during the past agrarian year and thus make their decisions based on information that generates production responses, which lead to a final market price that is different from that used in the decision-making process. The competitive market design of the PMP model proposed allows one to take this behaviour into account.

The model can be adapted to specific policy scenarios, including constraints at the sectoral/regional or farm level. The ability of the model to integrate policy restrictions (i.e., product quotas) and/or public incentives (i.e., product and farm subsidies) provides policy-makers with information regarding the likely impact of their decisions with respect to agricultural sectors.

However, the following two weaknesses of this model should be addressed: its lack of statistical properties (Norton and Schiefer, 1980; Paris and Rausser, 1973) and the difficulty of providing representative information at the aggregated level. First of all, it is evident that statistical inference is not applicable to this class of model. Secondly, we frequently lack the data that can prevent the use of very long time series to estimate demand and supply functions. Furthermore, the low quality of agricultural information and the missing values in agricultural databases make the target sample insufficiently representative.

4. Empirical example

The aim of this section is to present a numerical example of the application of the aggregated PMP methodology as detailed above. The examined case considers a group of five farms (N=5) producing three different commodities (J=K=3); for the same of simplicity, the only binding resource is the total agricultural land used by those farms. As indicated in the previous section, the procedure for building the aggregated policy analysis model consists of the following four phases: the estimation of the inverse demand function, the calibration of the observed farm planning situation, the estimation of the inverse aggregated supply function and the calibration of the aggregated model as in problem (11) modified using equation (22).

In this example, the estimation of the inverse demand and supply functions uses only the cross-sectional information derived from the sample of farms. It is important that the estimation of sectoral demand function using a panel of individual farms should be avoided because the resulting curve will reflect individual prices, which in turn generally reflect the contractual relationship between producer and purchaser. This is why two producers providing the same commodity to a market (i.e., soft wheat) could obtain a different price, and it is also why the price might not necessarily be lower for the producer with the largest quantity. In fact, the producer with the biggest quantity is frequently also the most specialised producer and has the highest probability of obtaining the highest-quality product (and, consequently, the best price). Thus, if we estimate the output demand curves using cross-section data, the resulting curves may link the price positively with the quantity level, creating a huge problem in term of economic theory satisfaction.

The parameters of the inverse demand and supply function have been estimated adopting the ME approach[2], and we obtain the following values:

$$\hat{\alpha}_j = \begin{bmatrix} 1.827 & 1.700 & 1.905 \end{bmatrix}$$

$$\hat{D}_{jk} = \begin{bmatrix} 1.535 & -0.239 & -1.503 \\ -0.234 & 2.194 & 0.122 \\ -1.503 & 0.122 & 1.505 \end{bmatrix}$$

$$\hat{\beta}_j = \begin{bmatrix} -0.266 & 1.373 & -0.298 \end{bmatrix}$$

$$\hat{Q}_{jk} = \begin{bmatrix} 0.038 & -0.001 & -0.003 \\ -0.001 & 1.883 & -0.084 \\ -0.003 & -0.084 & 0.038 \end{bmatrix}$$

Matrices $\hat{D}_{jk}$ and $\hat{Q}_{jk}$ are symmetric positive semi-definite as guaranteed by Cholesky's decomposition method adopted in the estimation model. In the inverse supply parameter estimation, the cross-section data has been integrated using the dual information associated with the calibrating constraint derived using model (18). This economic information, which captures the observed production context, is used to obtain the identical farm allocation by solving the endogenous price PMP model.

[2] The probability distribution chosen for the parameter estimation is set up using five probability points that correspond to same number of discretional support values. For more details, see the appendix.

The reconstruction of the inverse demand function allows one to measure the change in the purchased quantity of commodity X_j towards to a change in the price of another commodity P_k when $j \neq k$. This is the measure of demand cross-price elasticity (25a) that one can obtain by estimating equation (7):

$$E_{jk}^{d} = \frac{dX_j}{dP_k^d}\frac{P_k^d}{X_j} = \begin{bmatrix} \frac{dx_1}{dp_1}\frac{p_1}{x_1} & \frac{dx_2}{dp_1}\frac{p_1}{x_2} & \frac{dx_3}{dp_1}\frac{p_1}{x_3} \\ \frac{dx_1}{dp_2}\frac{p_2}{x_1} & \frac{dx_2}{dp_2}\frac{p_2}{x_2} & \frac{dx_3}{dp_2}\frac{p_2}{x_3} \\ \frac{dx_1}{dp_3}\frac{p_3}{x_1} & \frac{dx_2}{dp_3}\frac{p_3}{x_2} & \frac{dx_3}{dp_3}\frac{p_3}{x_3} \end{bmatrix} = \begin{bmatrix} -0.023 & 0.240 & 0.024 \\ 3.062 & -0.552 & -6.238 \\ 0.023 & -0.468 & -0.024 \end{bmatrix}$$

The off-diagonal parameters show the cross-price elasticity for each pair of products where the positive values mean that products j and k are substitutes, while the negatives values express complementary relationships between the products. The values on the diagonal indicate the own-price elasticities for the three commodities.

The PMP model with endogenous prices is designed for policy and market analysis simulations; its equations can take into account the agricultural policy system, so that farm behaviour can be evaluated in the agricultural policy framework on the basis of which producers make their decisions. In this example, the predictive aspect of the model will be tested by simply varying the market price of the first two commodities (1,2) to demonstrate the type of information that can be obtained. The same evaluation can be tested, for example, by modifying the coupled subsidy linked to specific crops in a model with policy constraints. Table 1 shows the results achieved in the initial situation, in which the model exactly reproduces the observed situation and the modifications involved in the three different market scenarios are considered. The information about output and variable input prices P_j^d and P_j^s, the information about the aggregated level of output X_j, the marginal value of the binding resource Y represent the primal solution of the model[3].

Table 1

While Y represents the *a priori* aggregated responses in the binding resource value, $\sum_j \mu_j X_j / B$ represents the *a posteriori* aggregated responses in the binding resource value. This means that the latter is the value of the fixed input as a result of the farmer competitive behaviour. μ_j is the dual solution of the aggregated model given constraint (11b), the marginal value of each unit of sectoral/regional output.

5. Conclusions

Leading up to the 1990s, several authors dealt with the problem of integrating endogenous prices into mathematical programming models to define quantitative tools that would be able to provide useful information on prices in addition to information about output levels. Endogenous price mathematical programming models have since been recognised as a powerful instrument for supporting policy-maker decisions and evaluation. Despite involving econometric techniques that require a huge amount of information that is frequently not available in agricultural statistics, this class of mathematical programming models can be easily used in the agricultural context because of the flexibility involved in reproducing policy constraints in such a sector.

The inclusion of prices in mathematical programming models must be based on the competitive behaviour underlying farmer decisions. This is why the model must guarantee the price-taker condition during the optimisation process involving input distribution to farm activities. As a result, prices are fixed at individual levels, while at the regional or sectoral level, prices are endogenous.

[3] The developments of the model and its resolution have been carried out using the GAMS software and the solver CONOPT.

The mathematical programming models using endogenous prices involve two levels of analysis: the micro level, which reflects the allocation choices of each farm, and the macro level, which is the response in terms of sectoral or regional prices generated by the aggregated response of all the farms considered.

To evaluate farmer reactions with respect to policy and/or market impulses, the mathematical programming model proposed in this paper must be able to describe farm behaviour along with the relative constraints on planning decisions. The detailed reproduction of the farm system adopted by each farm considered in a regional or sectoral analysis is completely unrealistic given the high costs of information collection and systematisation. One methodology that permits us to overcome this issue and estimate, in an economic manner, technological and risk attitude constraints is positive mathematical programming. This paper attempts to introduce endogenous prices into a positive mathematical programming model in which the economic information about basic allocation choices is used to predict farm behaviour in the simulation phase.

The positive mathematical programming model using endogenous prices reproduces farm behaviour in a competitive context and provides output and input prices at an aggregated level. The net aggregated pay-off objective function should not be considered as indicating the difference between producer and social surplus, but instead should it be seen as reflecting the difference between the value of agricultural production assigned by the market and the total cost sustained by producers as they seek to make their total production available in a given sector or in a given region. The objective function value can be interpreted as the part of the output value that remains inside the first agricultural chain. The PMP model using endogenous prices is clearly a partial equilibrium model.

Appendix A

Inverse demand function parameters estimation by adopting maximum entropy method

$$\max_{p(\bullet)} \quad Hd(p) = -\sum_{j=1}^{J}\sum_{p=1}^{P} pd_{jp} \log pd_{jp} - \sum_{j=1}^{J}\sum_{j'=1}^{J'}\sum_{p=1}^{P} Pl_{jj'p} \log Pl_{jj'p}$$
$$-\sum_{j=1}^{J}\sum_{j'=1}^{J'}\sum_{p=1}^{P} Ph_{jj'p} \log Ph_{jj'p} - \sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{p=1}^{P} pe_{njp} \log pe_{njp}$$

s.t.

$$pr_j^{\circ} = \sum_{p=1}^{P} zd_{jp} pd_{jp} - \sum_{k=1}^{K}\left\{\sum_{j'=1}^{J'} R_{jj'} R_{kj'}\right\}\sum_{n=1}^{N}\bar{x}_{nk} \quad ,\forall j$$

$$pr_{nj} = \sum_{p=1}^{P} ze_{njp} pe_{njp} + \sum_{p=1}^{P} zd_{jp} pd_{jp} - \sum_{k=1}^{K}\left\{\sum_{j'=1}^{J'} R_{jj'} R_{kj'}\right\}\sum_{j'=1}^{J'}\bar{x}_{nk} \quad ,\forall n \forall j$$

$$R_{jj'} = \left(\sum_{k=1}^{K}\sum_{p=1}^{P} Zl_{jkp} Pl_{jkp}\right)\left(\sum_{k=1}^{K}\sum_{p=1}^{P} Zd_{jkp} Pd_{jkp}\right)^{1/2} \quad ,\forall j \forall j'$$

$$1 = \sum_{p=1}^{P} pd_{jp} \quad ,\forall j \quad ; \quad 1 = \sum_{p=1}^{P} Pl_{jj'p} \quad ,\forall j \neq j'$$
$$1 = \sum_{p=1}^{P} Pd_{jj'p} \quad ,\forall j = j' \quad ; \quad 1 = \sum_{p=1}^{P} pe_{njp} \quad ,\forall n \forall j$$

where:

$Hd(p)$: entropy function value;

pd_{jp} : probability variables for the set of probabilities *p* (for *p*=1,2,…,5) linked to activity *j* (*j*=1,2,…,*J*) concerning the intercept;

pe_{njp} : probability variables for the set of probabilities *p* linked to activity *j* and farm *n* (*n*=1,2,…,*N*) concerning the deviation terms;

zd_{jp} : support values for the set of probabilities *p* linked to activity *j* concerning the intercept;

ze_{njp} : support values for the set of probabilities *p* linked to activity *j* concerning farm deviations;

$Pl_{jj'p}$: probability variables for the off-diagonal parameters;

$Ph_{jj'p}$: probability variables for the diagonal parameters;

Zl_{jkp} : support values for the probabilities Pl_{jkp};

Zd_{jkp} : support values for the probabilities Ph_{jkp};

R_{jk} : Cholesky's decomposition matrix; $R_{jk} = \left[L_{jk} D_{jk} \right]^{1/2}$;

pr_j° : price for each activity *j* recovered at regional or sectoral level;

pr_{nj} : observed price for each activity *j* collocated on the market by farm *n*;

$\overline{x}_{nk}$: observed output level for each activity *k* in each farm *n*.

Appendix B

Inverse supply function parameters estimation by adopting maximum entropy method

$$\max_{p(\bullet)} \quad Hc(p) = \quad -\sum_{j=1}^{J}\sum_{p=1}^{P} p\alpha_{jp}\log p\alpha_{jp} - \sum_{j=1}^{J}\sum_{j'=1}^{J'}\sum_{p=1}^{P} P\varphi_{jj'p}\log P\varphi_{jj'p}$$
$$-\sum_{j=1}^{J}\sum_{j'=1}^{J'}\sum_{p=1}^{P} Pw_{jj'p}\log Pw_{jj'p} - \sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{p=1}^{P} pu_{njp}\log pu_{njp}$$

s.t.

$$\lambda_j^{\bigcirc} + c_j^{\bigcirc} = \sum_{p=1}^{P} p\alpha_{jp} z\alpha_{jp} + \sum_{j'=1}^{J'}\left\{\sum_{k=1}^{K}\left(T_{jk}T_{kj'}\right)\right\}\sum_{n=1}^{N}\overline{x}_{nk} \quad ,\forall j$$

$$\lambda_{nj} + c_{nj} = \sum_{p=1}^{P} p\alpha_{jp} z\alpha_{jp} + \sum_{j'=1}^{J'}\left\{\sum_{k=1}^{K}\left(T_{jk}T_{kj'}\right)\right\}\overline{x}_{nk} + \sum_{p=1}^{P} pu_{njp}\, zu_{njp} \quad ,\forall n\forall j,\ \text{for } \overline{x}_{nk} > 0$$

$$\lambda_{nj} + c_{nj} \le \sum_{p=1}^{P} p\alpha_{jp} z\alpha_{jp} + \sum_{j'=1}^{J'}\left\{\sum_{k=1}^{K}\left(T_{jk}T_{kj'}\right)\right\}\overline{x}_{nk} + \sum_{p=1}^{P} pu_{njp}\, zu_{njp} \quad ,\forall n\forall j,\ \text{for } \overline{x}_{nk} = 0$$

$$T_{jj'} = \sum_{j'=1}^{J}\left\{\sum_{p=1}^{P}\left(p\varphi_{jj'p} z\varphi_{jj'p}\right)\sum_{p=1}^{P}\left(pw_{jj'p} zw_{jj'p}\right)^{1/2}\right\}$$

$$\sum\nolimits_{p=1}^{P} p\alpha_{(\cdot)} = 1\ ;\ \sum\nolimits_{p=1}^{P} p\varphi_{(\cdot)} = 1\ ;\ \sum\nolimits_{p=1}^{P} pw_{(\cdot)} = 1\ ;\ \sum\nolimits_{p=1}^{P} pu_{(\cdot)} = 1$$

where:

$Hc(p)$: entropy function value;

$p\alpha_{jp}$: probability variables for the set of probabilities *p* (for *p*=1,2,…,5) linked to activity *j* (*j*=1,2,…,*J*) concerning the intercept;

$P\varphi_{jj'p}$: probability variables for the off-diagonal parameters;

$Pw_{jj'p}$: probability variables for the diagonal parameters;

pu_{njp} : probability variables for the set of probabilities *p* linked to activity *j* and farm *n (n*=1,2,…,*N)* concerning the deviation terms;

$z\alpha_{jp}$: support values for the set of probabilities *p* linked to activity *j* concerning the intercept;

zu_{njp} : support values for the set of probabilities *p* linked to activity *j* concerning farm deviations;

$z\varphi_{jj'p}$: support values for the probabilities $P\varphi_{jj'p}$;

$zw_{jj'p}$: support values for the probabilities $Pw_{jj'p}$;

T_{jk} : Cholesky's decomposition matrix; $T_{jk} = \left[L_{jk} D_{jk} \right]^{1/2}$;

$\lambda_j^{\bigcirc} + c_j^{\bigcirc}$: total marginal cost for activity *j* related to the entire sample of farm;

$\lambda_{nj} + c_{nj}$: total marginal cost for activity *j* related to each farm *n* included inside the sample;

$\overline{x}_{nk}$: observed output level for each activity *k* in each farm *n*.

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Tables

Table 1. Solutions of the PMP model with endogenous prices

Outcomes	Initial value	Market Scenarios		
		S1	S2	S3
P_1^d	130.1	91.094	108.632	91.099
P_2^d	215.3	232.788	239.188	232.625
P_3^d	135.6	172.315	155.608	172.157
P_1^s	105.5	107.554	105.927	107.756
P_2^s	153.8	144.246	135.838	144.462
P_3^s	89.1	90.533	89.870	90.708
X_1	374.31	379.95	375.42	380.52
X_2	17.77	17.41	16.81	17.44
X_3	376.06	379.28	375.92	379.86
Y	243.149	329.291	239.482	327.089
$\sum_j \mu_j X_j / B$	247.721	234.586	244.924	233.102

Description of scenarios:

S1: commodity 1 price increased by 10% with respect to its initial value;

S2: commodity 2 price decreased by 10% with respect to its initial value;

S3: commodity 1 price increased by 10% and commodity 2 price decreased by 10% with respect their initial value