COMPETING RESEARCH JOINT VENTURES IN PATENT RACES

By

Damiano Bruno SILIPO*

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Abstract

Recent patterns show that most research joint ventures (RJVs) are not industrywide, but take place among a subset of the companies operating in the industry. So, while some members of an industry have formed RJVs, the others have responded by forming competing RJVs.

The paper presents the results of an experimental investigation on the evolution of cooperation in patent races when more than two players are involved in the race, so that competing RJVs may occur. Ideed, the experimental evidence shows that competing coalitions very often emerge, and they spur the incentive to innovate. In addition, the larger the number of the players the lower the probability to collude during the race. Finally, players with an initial advantage in the race often are capable to pre-empt the rivals.

Address:

Dipartimento di Economia e Statistica
Università della Calabria
–87036 Arcavacata di Rende (Cosenza) –ITALY
e-mail: silipo@unical.it

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1. Introduction

In the last decades global competition has changed the way companies engage in R&D. Today, even the largest company could not rely on internally generated and financed research projects to compete in the world market. As a result joint research activities have become an increasing important business strategy among oligopolistic firms. Companies use the joint research efforts as new tools to strengthen their individual competitive positions. Moreover, recent patterns show that research joint ventures (RJVs) are not industrywide, but take place among a subset of the companies operating in the industry. So, as some members of an industry have formed RJVs, the others have responded by forming competing RJVs.

One example of competing RJVs is competition for the development of the high-definition television, to develop the next generation of cable boxes for cable channels and interaction between receivers and broadcasters, for the development of technology enabling television viewers to call up movies of their choice from a central library.[1] Others more recent examples are the race[2] for the prospect of "third-generation" (3G) mobile phones-which will supposedly offer advanced services such as Internet browsing and video-looked, and several RJVs to the sequencing of DNA.[3] Moreover, RJVs are important tools even in competition among nations for the international economic leadership. In 1981 Japan launched the Fifth Generation Computer Systems Project to develop a new-generation computer. In 1984 the European Union established the European Strategic Program for R&D in Information Technologies

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[1] See Kamien and Zang (1993) for the description of these competing RJVs. In addition, Fusfeld and Haklisch (1985) report the main features of the competing RJVs in the eighties. Erdilek (1989) reported the main features of the coalitions in the semiconductor industry.

[2] In this race there are four main competing RJVs involved: Microsoft-Intel; Fujitsu-Siemens; Alcatel-STMicroelectronics; Nokia-Texas Instruments. In addition, Nokia, Motorola, Ericsson, Siemens and Alcatel are important web producers, so the competition is also on the communication platform to affirm as standard in the market.

(ESPRIT), aimed " to provide the European information technology industry with the technological base it needs to become and stay competitive world-wide in the next ten years."[4] Similarly, the stated purpose of The Microelectronics and Computer Technology Corporation (MCC) in the same period (1985) is "to help maintain U.S. technological prominence and international competitiveness in microelectronics and computers".[5] At the moment a competition is taking place between Europe and U.S. to get the leadership in the field of the wireless communications.[6]

Therefore, understanding the nature of the competition when firms undertake RJVs as strategic devices to compete in the R&D and the product markets is an important issue in the analysis of the technical change, the evolution of the markets and the industries.

Notwithstanding the great importance of RJVs for competition among companies as well as nations, little attention as been devoted to understand the circumstances and the mechanism that lead to the formation and breakdown of the RJVs. The first paper to study competing RJVs is due to Kamien and Zang (1993). Although their analysis is constrained by the assumption that the industry is partitioned into symmetric RJVs exogenously given, they have shown that there are circumstances in which competing RJVs provide a greater level of R&D effort than one grand RJV cartel.

By contrast, Yi and Shin (2000) considered endogenous formation of coalitions, and examined the conditions for the existence of stable RJVs coalition structures. They proved that the number and size of coalitions are strictly related to the rules of coalition formation. In particular, the open membership rule leads to a Nash equilibrium RJV structure that is less concentrated than the exclusive membership rule;[7] in addition, the latter provides a higher industry R&D investment than

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[3] "Genetic warfare", The Economist, May 14th, 1998.
[4] "Prospects for the Development of New Policies-Research and Development, Energy and New Technologies", Bulletin of the European Communities Supplement, May 1983, p. 29.
[5] Microelectronics and Computer Technology Corporation, Letter to Carmela S. Haklisch, February 1, 1984.
[6] See, "The tortoise and the hare", The Economist, March 16, 2002.

the former. Bloch (1996) provided more insight on the nature of equilibrium coalition structure, by studying the extensive form of the coalition formation game.[8] In a game in which membership is exclusive and coalitions are formed in sequence, Bloch proved that in equilibrium firms form two asymmetric coalitions, with the dominant coalition comprising three-quarters of the industry members. The first formed coalition admits more members as a way to reduce the size of the rival coalition and to increase the costs of their members.

However, the above quoted papers are not able to deal with the evolution of competition and co-operation along the discovery process. Bloch (1996, p. 118) motivated the above assumptions by the difficulties that arise when a more dynamic framework is adopted in the formation of competing coalitions.

Due to the great difficulties to build up dynamic theoretical models when more than two players are involved in the race, we relied upon the experimental investigation to get an insight on the evolution of the competing joint ventures, by performing an experiment on coalition formation and breakdown as a result of the strategic interaction between players and coalitions along the race.

Specifically, we investigate the behaviour of the players when four or seven players are involved in a race to get a prize. It gets the prize the first player or coalition of players that accumulates a given amount of knowledge. In each period of the race players must decide on the coalition formation and the amount of knowledge to accumulate. Both decisions are made under imperfect information, due to the fact that players decide simultaneously. Coalitions are formed according to the exclusive membership rule. Since coalitions last only one period, the experimental investigation allows us to get an insight on the evolution of cooperation and competition during the course of the race.

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[7] Intuitively, a coalition structure is more concentrated than another is if the former has bigger coalition sizes than the latter.

[8] The extensive form of the game, however, is build up on the assumption that players by accepting the offer to joint a coalition are bound to remain in the coalition, and therefore coalitions cannot compete to attract members.

Thus, the experimental investigation may be considered a first attempt to study the evaluation of co-operation in patent races when more than two firms are involved in the race.

The main results of the experiments are that, when more than two players are involved in the race, competing coalitions are likely to emerge, and they spur the incentive to innovate. In addition, the larger the number of the players the lower the probability to collude in the race and the higher the probability that leapfrogging will take place.

However, players with an initial advantage in the race often are capable to pre-empt the rivals.

The paper includes five sections. Next section presents the experimental framework, and Section three the main assumptions made in the experiments. Section four gives the main results and Section 5 concludes the paper.

2. The experimental framework

In what follows we adopt the dynamic discovery process of Fudenberg et al. (1983). [9] However, unlike the last authors, we assume N>2 players are involved in a race to obtain a prize worth 60 Euro. In ten races are involved 4 players, and in another ten there are involved 7 players.

We performed these two size experiments since recent theoretical models proved that the equilibrium size of the coalitions shows different pattern below and above five players. For example, Morasch (2000), in a linear Cournot oligopoly model, found that in a game with a number of players $n \leq 5$ only one alliance forms, but with $n > 5$ at least two alliances result. Alliance members behave as a "Stackelberg cartel" relative to the rest of the industry. Similar results are obtained also by Ray and Vohra (1994) and Yi (1997).

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[9] At the outset of the previous paper, we explained why we adopted Fudenberg et al. (1983) framework.

The prize is awarded to the first player or coalition of players to accumulate 50 "units" of knowledge.

If competing players/coalitions accumulate 50 units in the same period, the prize is awarded to the player/coalition with the highest level of knowledge. If they tie, they have equal probability of getting the prize.

The race takes place in discrete time, and the R&D process is deterministic; i.e., there is a deterministic relationship between the effort and the progress made during the race. We assume that each player can accumulate 0, 1 or 2 unit of knowledge in each period, which cost respectively 0, c_1 and c_2, with $c_2 > 2c_1 > 0$. The latter assumption corresponds to a convex-cost knowledge accumulation technology.

In addition, we assume the players know their relative positions, as determined by the amount of knowledge accumulated by the players up to that period.

Each period is characterised by two stages: in the first stage the players make the decision on the coalition formation, and in the second stage they decide the level of effort to undertake for current period.

However, in each stage players make simultaneously their decisions. This latter aspect characterises the game one of imperfect information. So, even though the players know their relative position, the race is characterised by uncertainty on the date of discovery and the likely winner/s of the race.

In each period players decide whether to compete or cooperate with other players.

The nature of the cooperative agreement is the following. Membership is exclusive. Once one coalition has been formed, all the members acquire the same amount of knowledge: this is the maximum amount of the knowledge of the coalition members.

In addition, the progress made by the coalition in each period is the sum of the efforts made by the coalition's members in the same period.

The members decide the effort of the coalition according to the majority rule.

Finally, we assume that subjects belonging to the winning coalition share equally the prize.

The cooperative agreement is binding only for one period, so that the process of coalition formation in the next periods proceeds in a similar way, until one player or coalition will accumulate the amount of knowledge necessary to get the prize.

Let us consider the coalition decision in period t, $t=1,\ldots,T$, with T denoting the data in which one player or coalition accumulate 50 units of knowledge. In period t players know their relative position and simultaneously make a proposal on the coalition they want to make, by hitting the other players' number/s with whom they want to cooperate (if they want to play alone they hit zero). In the same period there form the coalitions among the players that have matching proposals.

As an example, assume there are four players involved in the race, labelled F_i, F_j, F_k, F , and denote by C^λ the coalition proposed by player F_λ, $\lambda = i, j, k,$. Assume in period t player F_i makes the proposal $C^i(t) = \{F_i, F_j, F_k, F \}$, player F_j makes the proposal $C^j(t) = \{F_j, F_i, F \}$, F_k makes the proposal $C^k(t) = \{F_k, F_j, F \}$, and player F makes the proposal $C(t) = \{F_j, F \}$. After checking on the matched proposals, it is straightforward to see that in period t there may form two coalitions: $C_1(t) = \{F_i, F_j\}$ and $C_2(t) = \{F_j, F \}$. Since players may belong to only one coalition, player F_j must choose to what coalition to belong to. We assume that whenever one player must choose among coalitions, it chooses the coalition with the highest level of knowledge. If two or more coalitions have the same level of knowledge, the player chooses the coalition with the largest size, and if coalitions have the same size and the same amount of knowledge, it chooses the coalition that contains more players with whom cooperate in the previous periods, otherwise it chooses randomly between the two coalitions.[10] Without loss of generality, assume player F_j chooses

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[10] So, we assume that whenever one player must choose between coalitions of different size, it chooses the coalition with the highest level of knowledge, independently on the coalition size. Of course, other decision rules may be considered. For example, it may be the case that players choose the coalition that allows getting the highest step-forward, independently on their current positions. However, we believe that the latter aspect does not affect the qualitative results of the experiments.

coalition $C_2(t)$. It follows that in period t there forms the coalition structure $C(t) = \left\{ F_i, F_k, (F_j, F\ ) \right\}$.[11]

Once one coalition has been formed, all the members acquire the same amount of knowledge: this is the maximum amount of the knowledge of the coalition members. The latter characterises the coalition's position in the race.

Recalling the previous example, let denote by $w_\lambda(t)$ knowledge level of player F_λ at time t, $\lambda = i, j, k,\ $. It follows that the level of knowledge of coalition $C_2(t)$ in period t is $w_{C_2}(t) = \max\left\{ w_j(t), w\ (t) \right\}$, and the relative position of the coalitions in period t is $(w_i(t), w_k(t), w_{C_2}(t))$.

The second stage game in period t is the decision on the amount of knowledge to accumulate in current period. So, players make simultaneously a proposal on the level of effort to undertake in period t among the possible options (0,1,2). The level of effort that each coalition member undertakes is the one that gets the highest score among the proposed efforts by the coalition members. If more than one option gets the highest score, the coalition's choice is made randomly.[12] Therefore, the progress made by a coalition in each period is the sum of the (equal) efforts made by the coalition members in the same period.

Update of the coalitions' positions completes the decision process in period t.

Co-operative agreements are binding only for one period, so that in each period players must decide whether to propose the same coalition, form a new coalition or play alone.

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[11] In general, a coalition C_i is a subset of the N players, and a coalition structure $C = \left\{ C_1, C_2, ..., C_m \right\}$ is a partition of the players set $F = \left\{ F_1, F_2, ..., F_N \right\}$ such that $C_i \cap C_j = \varnothing$, $\forall C_i, C_j \in C$, $i \neq j$ and $\cup_{i=1}^{N} C_i = F$. This implies that players form non-overlapping coalitions. If all the coalitions contain only one player, we are in the fully competitive case.

[12] This decision rule is made only to simplify the decision process and to avoid the bargaining process on the coalition effort that otherwise would take place in the second stage.

Therefore, the process of coalition formation and breakdown in the next periods proceeds in a similar way, until one player or coalition accumulates 50 units of knowledge.

Notice that when a player joints a coalition there are two countervailing effects. On one hand, it increases the probability of winning the prize, because his progress in the same period is the sum of the efforts made by the coalition members[13], on the other hand, each successful player must share the prize with the other coalition members.

Moreover, an increase in one coalition size determines a negative externality on the other coalitions, by strengthening its competitive position at the expenses of the rivals.

We highlight that the race we are dealing with is characterised by uncertainty and spillover of knowledge. The first feature derives from the simultaneous nature of the firms' decisions, which allows to each firm to monitor the other firms' decisions only one stage later. In addition, although the race is of the winner-takes-all type, it is characterised by spillovers of knowledge, due to the fact that one firm may move from a leading coalition to follower coalitions or one leading firm may form a coalition with followers and spread its knowledge.

Finally, we assume that at both stages players choose the strategy which maximise their expected payoffs, given the relative position of the players in the race. In addition, in each period players may revise their strategies.

For the four-players case above, player F_λ, $\lambda = i, j, k,$, expected payoff at the outset is:

$$E\Pi_{F_\lambda}((w_i(t),...,w\ (t))) = \begin{cases} = \dfrac{V}{n} - \sum_{1}^{T} c(e_{F_\lambda}(w_i(t),...,w\ (t))) \\ = 0 \end{cases}$$

Where V is the value of the prize (in our case equal to $=60$ euros), n is the number of the winning players, and the second term on the right hand side of the previous equation are player F_λ total expected costs necessary to win the race.

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[13] In addition, players that joint the coalition and are behind advantage by acquiring the amount of knowledge of the coalition members that are ahead.

Notice that we assume the losers get nothing but loose nothing in money terms.[14] This assumption mimic a situation in which the participation fees cover all the costs sustained during the race.[15]

In addition, notice that player F_λ expected payoff depends on the coalitions' relative positions and the effort made by each player during the race.

However, predicting firms' behaviour in the framework settled above is a very difficult task, even with a very small number of players.

First of all, most of the commonly used Nash equilibrium refinements cannot apply to the above framework. The notion of Coalition-proof Nash equilibrium (Bernheim et al., 1987) assumes unlimited pre-play communication among players and allows for deviation only by a subset of the original coalition. By contrast, the notion of Strong Nash equilibrium (Aumann, 1959) allows for deviation by every conceivable coalition, but it assumes that the players that not deviated are expected to stay with their equilibrium strategies, which is not our case.

Moreover, since firms make decisions simultaneously, in each period each firm must form believes on the possible decisions the other firms are going to undertake. This corresponds to undertaking believes about the other players' types and strategies; though they may update their believes according to the observed behaviour in previous periods. So, our problem resembles a Bayesian collective-choice problem. However, the solution concepts that are applied to these games seem to be unsatisfactory. In fact they refer the Nash equilibrium concept to the definition of efficient mechanisms rather than efficient outcomes. In addition, several definitions of efficiency are considered (see, Myerson, 1991, Paper 10).

Due to the above-mentioned difficulties, we relied upon the

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[14] However, they have an opportunity cost due to the time they loose to participate to the experiment.

[15] This assumption may lead to the criticism that, by the fact that players had no losses they had an incentive to make always the highest effort. However, we believe that, putting a limit on the number of time they would play without losses it would distortion the players' behaviour.

experimental investigation to get an insight on the evolution of co-operation when firms may form competing RJVs.

To the absence of theoretical investigations, there is also associated very little empirical evidence on the evolution of cooperation in patent races. Vonortas (1997) and Hagerdoon et al. (2000) reported that a significant increase in formation of research partnerships during 1985-'95 in the United States was followed by decreases thereafter. Moreover, Suslow (1992) found that the median life of contractual cartels is only 2.8 years. Interestingly, Nakamura et al. (1996) found that long-lasting joint ventures are those in which partner firms' competitive capabilities have become dissimilar but complementary, whereas the joint ventures in which the parent firms become more alike in their competitive capabilities dissolve earlier.

The last conclusion was supported also by Silipo (2000), which found that stable joint ventures form if firms are in a similar position in the race.[16] However, the duopoly framework they use is not able to deal with the issue of competing research joint venture in patent race.

Therefore, in the lack of theoretical predictions and strong empirical evidence on the evolution of cooperation in patent races, the experimental investigation we present in this paper is a first attempt to insight on the behaviour of the firms in a patent race, when they can undertake joint ventures to compete in the R&D and in the product markets.

3. The design of the experiment

The experiments took place at the University of Calabria (Italy) in November 2002. They involved 110 students, representatives of all the existing Faculties (Economics, Engineering, Political Science, Chemistry, Physics, Mathematics, Geology, Literature and Arts).

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[16] Notice that, Silipo (2000) contains only theoretical and experimental results. Moreover, similar qualitative results are obtained also by Chowdhury and Roy Chowdhury (2001) for joint venture production.

We performed ten sessions with four players and ten sessions with seven players. Moreover, in both cases, the players started in half sessions with zero amount of knowledge, and in the other half, one player started with 3 points ahead, one player with 2 points ahead and one player with 1 point ahead. The other players started with 0 points.

Therefore, the last two assumptions were made to test the effects on the coalition formation process of changes in the number of the players and in the initial position (asymmetric/symmetric) in the race.[17]

In both the four and the seven subjects cases, we considered a situation in which subjects are involved in a race to get a prize worth 60 euros. The first subject or coalition of subjects to accumulate 50 units of knowledge gets the prize. If it is a coalition to accumulate first 50 units of knowledge the prize is equally divided among the members of the winning coalition.

However, at the end of each session each winner would get in real money his share of the prize less his costs sustained during the race.[18]

There is no prize whatever if a subject or coalition "looses" the competition. However, we assume that the participation fees cover the costs sustained by the latter.[19]

We assume that each subject can accumulate 0, 1 or 2, units of knowledge per period, which cost respectively $c_0 = 0$ Euro, $c_1 = 0,20$ Euro and $c_2 = 0,50$ Euro. These costs mimic a convex R&D technology.

The progress made by a subject in each period is the effort made by the subject in the same period. Similarly, the progress made by a coalition in each period is the sum of efforts made by their members in the same period. In addition, we assume that the effort of the coalition is decided by the majority rule, or randomly among the coalition members' proposals if no majority arises.

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[17] Notice that, due to our tight budget constraint, we were not able to address other factors affecting the race, such as the value of the price, the amount of knowledge to win the race or the rules of the game (e.g. the open membership versus the exclusive membership game).

[18] Of course, if it were only one winner, he would get the entire prize less total costs.

[19] An additional cost for the players is the time they have lost to participate to the experiment.

By forming a coalition all the players acquired the same amount of knowledge, which is the maximum knowledge owned by the coalition members.

The main features of the experiment are summarised in Table 1.

Table 1		
Features of the experiments		
N. of races	10 with four subjects, among which: 5 with asymmetric initial position and 5 with symmetric initial position	10 with seven subjects, which: 5 with asymmetric initial position and 5 with symmetric initial position
N.of subjects	40	70
Value of the prize	60 euros	60 euros
Cost of the effort in each period (Euro)	$c_0 = 0,\ c_1 = 0,20,$ $c_2 = 0,50$	$c_0 = 0,\ c_1 = 0,20,$ $c_2 = 0,50$
Points to be accumulated to win the prize	60	60

The experiment was made up of twenty sessions: half with the four subjects case, and half with the seven subjects case. In each session was run only one race.

Both in the four players and seven players cases, half of the races started with the same initial position, and in the other half one player started with 2 points ahead, another with one point ahead. In the asymmetric races the initial positions were allocated randomly among the players.

At the beginning of each session there were given the instructions, and there were asked to read them carefully. They are reported in the Appendix. Verbal explanation of the rules of the game followed, and thereafter was asked to fill a questionnaire, checking for their full understanding: additional explanations followed if any answer was incorrect. The subjects played the first time for practice and with no actual money incentives. The second time they played for real money. The "real" experiment started as soon as finished the practice round. Each player had a number, and none known the number attributed to the other players. However, players were located such that there were no ways to communicate between them. Moreover, the number

attributed to each player changed between the practice and the real session. Subjects were allowed to play the game only once.

Figure 1 reports the computer screen as the subjects saw it during the experiments. We consider the four-player case and an initial even position in the race.

Figure 1				
The schedule of the game				
Subject	**1**	**2**	**3**	**4**
It wins 60 euros the subject or coalition of subjects to accumulate first 50 points. In the latter case the prize is divided equally among the winning coalition's members				
Initial position (t=1)	$w_1 = 0$	$w_2 = 0$	$w_3 = 0$	$w_4 = 0$
Propose a coalition by hitting the subject/s number with whom you want to cooperate or hit zero if you do not want to cooperate with other subjects				
Coalitions formed in period 1	1,2		3,4	
Coalitions' positions in period 1	$w = 0$		$w = 0$	
Propose an effort choosing between 0,1 and 2				
Coalitions' positions in period 2	$w = 4$		$w = 4$	
Your sustained costs are:	0,50 euro	0,50 euro	0,50 euro	0,50 euro
Propose a coalition by hitting the subject/s number with whom you want to cooperate or hit zero if you do not want to cooperate with other subjects				
Coalitions formed in period 2	1	2	3,4	
Coalitions' positions in period 2	$w = 4$	$w = 4$	$w = 4$	
Propose an effort choosing between 0,1 and 2				
Coalitions' positions in period 3	$w = 6$	$w = 5$	$w = 6$	
Your sustained costs are:	1 Euro	0,70 Euro	0,70 Euro	0,70 Euro

At the outset the computers showed the subjects' initial positions, as defined by the acquired knowledge.

Players then were asked to make decisions on the coalition formation. As soon as all players have made their membership decision, the program computed and showed on the screen the coalitions formed in the first period and updated their relative positions.

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Afterward, there were asked to each subject to make a decision on the amount of knowledge to accumulate in current period (choosing between 0, 1 and 2).[20]

Therefore, there appeared on the screen the points accumulated by the coalition in period 1 and updated the coalitions' relative position at the beginning of period 2.

Next, the computer showed the costs sustained by each subject during the race. This completed the decision process and the outcome of period one.

In period 2 there were asked to each subject to make again the choice on the membership decision, and thereafter on the level of effort.

The game proceeded in a similar way up to one of the coalitions accumulated 50 points. If more than one coalition accumulated 50 points in the same period, the computer attributed the prize to the coalition with the highest amount of knowledge; otherwise the computer attributed randomly the prize to one coalition with the highest amount of knowledge.

Payments to the winners concluded the session.

4. The experimental evidence

The aim of the experimental investigation is to insight on the following questions: 1) What is the size of the coalitions is going to emerge when more than two players are involved in the race. 2) Do the number of the players in the race affects coalition formation and breakdown; 3) What are the effects of the symmetric/asymmetric initial position on the nature of the race. 4) Is cooperation among players beneficial or detrimental to the speed of innovation?

Notice that, when more than two players are involved in the race, firms may undertake competing research joint ventures.

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[20] The computer picked up for each coalition the choice with the highest score, or it selected randomly among those with the highest score.

Most of the results are obtained by comparing the experiments in the four-players and the seven-player cases.

First experimental evidence shows that in the more-than-two player case, joint discovery is less likely to take place. The latter occurred only in two out of twenty races, and only in the four-player case.

Looking at the distribution of the size of the winning coalitions (see Figure 2), in the four-player case only 1 race ended with 1 winner, 5 races had 2 winners, and 2 races ended with 3 and 4 winners. In the seven-players case, no race had only 1 winner or all-players winners, 4 races ended with 2 winners, 3 races with 3 winners, 4 races with 2 winners, and, finally, 1 race ended with 5 winners (see Figure 2). On average, the size of the winning coalition was 3 players in the latter case, and it was 2,5 in the races with four players.

We can infer from this results that, the higher the number of the players in the race the bigger the size of the winning coalition.

The result is rather intuitive. The higher the number of players involved in the race, the more players are necessary to have in the winning coalition to offset leapfrogging by competing coalitions.

Figure 2. Distribution of the winning coalition size
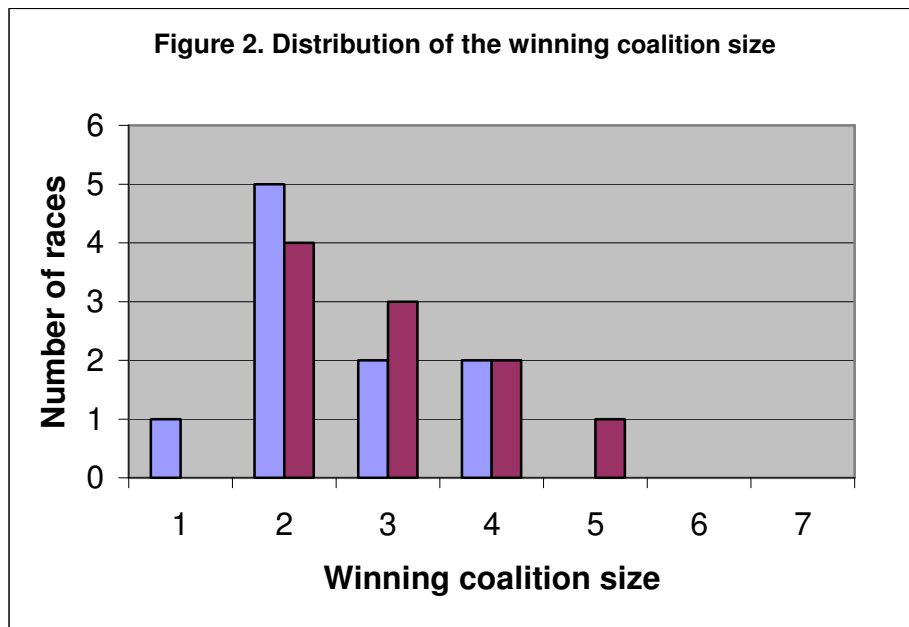
The seven-player case is the darkest.

In addition, the results on the evolution of cooperation shows that competing research joint venture occurred, both in the four-players and the seven-players cases.

Table 2 and 3 show that, most of the times competition took place between two coalitions in the four-player case and between more than two coalitions in the seven-player case.

Therefore, we can conclude that competing research joint venture occurs when more than two firms are involved in the race.

Moreover, the previous results show that, the higher the number of participants in the race the higher the probability that the race is characterised by competition between coalitions.

To insight the factors affecting the probability to win the race, we considered the average size of the coalitions to which belonged winners and losers during the race. The results show that, on average winners belonged during the race to bigger size coalitions than the losers. Only in one race in the seven-player case, the average dimension of the coalitions was higher for the losers than for the winners.

Therefore, we can conclude that, one explanation of the success has been the capabilities of the winners to make bigger-size coalitions during the race.

More insight on the conditions for success is obtained by evaluating the behaviour of the players during the race.

Straightforward investigation of Table 2 and 3 shows that, an additional explanation affecting the probability to succeed is the stability of the coalitions.

In the four-player case all the winners had a greater propensity to cooperate for a longer period than the losers did. This factor seems to be less relevant in the seven-player case. In the latter case, the winners cooperated among themselves for a longer period only in four out of ten races. By contrast, the losers formed more stable coalitions in six out of ten races.

Therefore, the capability to form more stable coalitions is an important factor for success, at least in the four-player case. By contrast, in the seven-player case the most relevant factor to succeed seems to be the bigger size of the coalition.

However, these results do not take account of the fact that, in some races players started in the same position and in others they started in different positions. Thus, we considered the effects of the initial position on the probability of success of the players.

The evidence shows that, winners had an initial advantage in three out of five races. This number increased to four out of five in the seven-player case (see Table 2 and 3). Therefore, at first glance, it seems that the probability of success is positively correlated to an initial advantage in the race.

However, the last conclusion holds only if head-starters maintained their advantage during the race.

With this respect, the evidence is that, players with an initial advantage cooperated among themselves in the first period. In addition, they maintained in the four-player case their advantage all through the race, although in some cases cooperation took place among a subset of them. By contrast, in the seven-player case cooperation among the leaders was less stable, and in two out of five cases leapfrogging occurred in the race.

More precisely, the experimental evidence shows a clear-cut difference in the players' behaviour between the two-size races (see Table 2 and 3). When races involved four players never occurred that the followers leapfrogged the leaders. In addition, the latter did not show a strong incentive to cooperate in order to catch up[21]. So, pre-emption seems to prevail in the four-player case.

A different pattern seems to prevail in the races with seven players. In this case leapfrogging occurred in four out of ten races. In addition, the followers formed competing coalitions in half of the races (Table 3).

The intuition of these results is the following. In the four-player case the size of the following coalition is the same as the size of the leading coalition. By contrast, in the seven-player case, it is easier to leapfrog the leading coalition if the size of the latter does not involve the majority of the players.

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[21] In one case, the player starting with the highest point was leapfrogged by the other two head-starters.

However, broadly speaking, followers did show during the race fewer incentives to leapfrog the leader/leaders of the race than we would expect. Indeed, our expectations were that, as soon as a leading coalition formed, followers cooperated among themselves to catch up the leaders, but this was not the case. Instead, the latter showed an incentive to cooperate with the leaders.

Leapfrogging and preemption do not seem to be the only types of behaviour. In one race (race 14) deviation-with-punishment seem to arise. In another race (race 16), players broke down the coalition in the last period of the race, showing a sort of winner-takes-all behaviour. Finally, in two races (races 7 and 10) players have shown the opposite behaviour: they formed the largest coalition in the last period, even when this was not optimal. These players seem to adopt a risk-aversion behaviour (see Table 2).

The important question we address now is whether competition among coalitions spurs the incentive to innovate.

In the previous paper, in a similar framework, we provided experimental evidence that, in the two-player races, the subjects use cooperation to reduce the cost of discovery. In fact, strong evidence proved that, whenever the players cooperated they played almost always 1. The reason is that, in the two-player case the players use cooperation to reduce the cost of innovation and to eliminate competition among them.

This is not the case when the race involves more than two players. In this case, competing research joint ventures spurred the incentive to innovate.

Moreover, the results of the experiments show a different pattern in the behaviour between the four and the seven players cases also in relation to the effort levels (see Table 2 and 3).

As a matter of fact, the experimental evidence shows that, each winner played on average 1,51 per period in the races with four players, and he/she played 1,77 in the races with seven players.

Therefore, we can conclude that, competition between coalitions increases the efforts of the players during the race, and hence it reduces the timing of discovery.

Moreover, in the four-player case the average effort of the winners was higher than those of the followers, apart from one race.

In the seven-player case, the opposite result holds. In fact, the average effort of the players belonging to the loosing coalitions was higher in six races than those of the players belonging to the winning coalition; the opposite was the case only in one race.

The last results confirm the high effort the followers made in the seven-player races, and explain also why leapfrogging occurred in this case.

5. Conclusions

In this paper we addressed the issue of the nature of the race when more than two players are involved in a race to win a prize. We assumed that there is imperfect information among the players, due to the fact that they make simultaneously their decisions: both the membership decision and the effort decision.

Because of the difficulties to get analytical results on the players behaviour when more than two players are involved in a multiperiod and multistage race, we relied upon the experimental evidence.

The latter shows that, the possibility to form competing coalitions does spur the incentive to innovate. In addition, also the number of the players involved in the race does matter. The larger the number of the players the lower the probability to collude in the race. As a matter of fact, players colluded in two races only in the four-player case. Collusion never occurred in the seven-player races. By contrast, in the latter leapfrogging occurred in four out of ten races.

Moreover, the number of players involved in the race seems to be an important explanation whether the pattern is characterised by leapfrogging or pre-emption. In the four-players case leapfrogging never occurred, but the latter was the case in four out ten races in the races with a larger number of players.

However, an initial advantage in the race is an additional important explanation of success.

Therefore, the experimental evidence shows that the pattern of the race is very different from the two-player case when more than two players are involved in the race. Competition among coalitions spurs innovation and in addition does not lead to collusion in the ex post market.

Therefore, the experimental evidence suggests that, it would be desirable to have a large number of firms involved in the race, with no-restriction on the possibility to form coalitions, because it is unlikely that they are going to collude in the R&D and the product markets.

However, more robust results are necessary before to draw policy implications. Within the above framework, first, it is necessary to extend the experiments to a different number of players and to consider how players' behaviour is affected by the value of the prize. Second, it could be desirable to bargaining among the players at both stages, although this would make more difficult the decision process. Third, different rules of the game must be considered (e.g. open membership).

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Appendix
Experiments' instructions

In this experiment you and other three subjects play a game to get a prize worth 60 Euro.[22] It wins the prize the first subject or coalition of subjects to accumulate 50 points. If you are the only subject to accumulate first 50 points YOU GET AT THE END OF THE SESSION IN REAL MONEY 60 EUROS LESS THE COSTS YOU BARED to accumulate 50 points. If you belong to a winning coalition YOU GET AT THE END OF THE SESSION IN REAL MONEY YOUR SHARE OF THE PRIZE (equally divided among the members of the winning coalition) LESS THE COSTS YOU BARED to accumulate 50 points.

IF YOU DO NOT ACCUMULATE 50 POINTS FIRST YOU GET NOTHING.

You can always check on the screen the points accumulated by you and your opponents. In addition, as you and the other subjects make progress, the computer will update the accumulated points.

You can accumulate 0, 1 or 2 points per period, which cost respectively $c_0 = 0$, $c_1 = 0,20$ and $c_2 = 0,50$ Euro.

If you play alone, the points you accumulate in the same period give the progress you make in each period. If you form a coalition with other subjects, the progress you make in each period is the sum of efforts made by coalition's members in the same period. However, in case of success, you have to share the prize with the other coalitions' members.

IN EACH PERIOD you have to make two decisions:

1)   whether to form a coalition with other subjects in the race or going alone;

2)   The level of effort to undertake for current period (whether to play 0, 1, or 2).

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[22] In the handout of the other ten races six substituted three.

All the subjects make both decisions simultaneously. So YOU DO NOT KNOW WHAT THE ARE SUBJECTS ARE DOING BUT ALWAYS YOU CAN CHECK WHAT IS THEIR POSITIONS IN THE RACE.

The following Figure 1 illustrates the decision process, which is a picture of the computer screen as you see it during the experiments.

Figure 1 The schedule of the game				
1)Subject	1	2	3	4
2) It wins 60 euros the subject or coalition of subjects to accumulate first 50 points. In the latter case the prize is divided equally among the winning coalition's members				
3) Initial position (t=1)	$w_1 = 0$	$w_2 = 0$	$w_3 = 0$	$w_4 = 0$
4)Propose a coalition by hitting the subject/s number with whom you want to cooperate or hit zero if you do not want to cooperate with other subjects				
5) Coalitions formed in period 1				
6) Coalitions' positions in period 1				
7)Propose an effort choosing between 0,1 and 2				
8) Coalitions' positions in period 2				
9)Your sustained costs are:				
10) Propose a coalition by hitting the subject/s number with whom you want to cooperate or hit zero if you do not want to cooperate with other subjects				
11) Coalitions formed in period 2				
12) Coalitions' positions in period 2				
13) Propose an effort choosing between 0,1 and 2				
14) Coalitions' positions in period 3				
15) Your sustained costs are:				

Raw 1 in Figure 1 shows the subjects' numbers that are involved in the race (you can check on the screen which subject you are).

Raw 2 gives the value of the prize and the points to be accumulated to get it.

Raw 3 shows the points that you and your opponents have at the outset of the race (in some races some subjects may have different points at the outset). This defines you and your opponents' initial positions.

In the next raw, you are asked to propose a coalition in period 1. Raw 5 and 6 show respectively the coalitions formed in period 1 and

their relative positions as a result of the simultaneous decisions. A coalition position in period 1 is the maximum level of knowledge of the coalition members.

A COALITION FORMS IN PERIOD 1 IF YOUR PROPOSAL MATCHS THE OTHER PLAYERS' PROPOSALS YOU WISH TO JOINT WITH. For example, if you were player 1 and you propose a coalition with players 2 and 4, the coalition (1,2,4) forms only if the other two players make the same proposal. However, it may happen that only player 2 wishes to make a coalition with you: in the latter case it forms the coalition (1,2).

NOTICE THAT WHEN YOU FORM A COALITION IT INCREASES YOUR PROBABILITY TO WIN THE PRIZE BUT IT DECREASES THE AMOUNT YOU WOULD GET IF YOU WHERE THE ONLY ONE TO WIN THE RACE.

In raw 7 you have to make an effort proposal for period 1. If you are alone your progress in period 1 is the effort you make in the same period. If you belong to a coalition with other subjects, the effort of the coalition is the one proposed by the majority of their members. So, you propose an effort among 0,1 and 2, and the computer automatically attributes to you the effort chosen be the majority of the coalition members. If none effort gets the majority, the computer chooses randomly among the proposed efforts. Notice that the accumulated points by each coalition in each period are the sum of efforts undertaken by their members in the same period. For example, if you are in a coalition with other two subjects, and you propose 1 but the other two subjects propose 2, the computer attributes to each of you the effort 2, and there are 6 points accumulated by your coalition in current period. So, other thinks equals, the greater the coalition size the greater the accumulated points by the coalition members in each period and the lower the share of the prize each member can get if the coalition will win the race.

Raw 8 shows the coalitions' positions at the outset of period 2, and raw 9 shows the costs sustained so far by each player.

In raw 10 you have again to make a proposal on the membership decision in period 2. As before, if you want to propose a coalition you

must hit subject/s number with whom you want to cooperate, or if you want to play alone you must hit zero.

Notice that, your proposal to be realised must match the other players' proposals. So, for example, if you propose the same coalition and the other coalition members do leave in the same period the proposed coalition, you have to play alone in current period.

In the subsequent periods the game proceeds in a similar way as before, until one subject or coalition will accumulate 50 points and will win the prize.

IMPORTANT: please do not try to exit the experiment program even if temporarily. You are not allowed to get up from your seat before the end of the experiment. You are not allowed to speak to any of the participants in the experiments at any time. If you have any query, please call us, and we shall do our best to solve it. Failure to comply with these instructions will imply the termination of the experiment. Your patience is much appreciated.

Besides, notice that each round will end as soon as each participant has made its current decision; therefore, please, do not try to delay the game, and as soon as you make a decision hit the appropriate buttons. However, once you have made a choice you cannot change it.

The computer does not allow to you to go to next step until the current one is finished.

Please, now fill the short enclosed questionnaire. The only purpose of the questionnaire is to make sure that everything is clear before you start the experiment.

TABLE 6

The evolution of cooperation in the four-players case

	*	**	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
RACE 1	1		1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	**1,3**		
		0	4	4	8	10	14	18	22	26	30	34	38	40	44	46	50		
	2		2	2	2	2	2	2	2	2	2,4	2,4	2	2	2	2,4	2,4		
		0	2	4	6	8	10	12	14	16	20	24	26	28	30	34	38		
	3																		
		0																	
	4		4	4	4	4	4	4	4	4			4	4	4				
		0	2	4	6	7	9	11	13	15			25	26	27				

	*	**	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
RACE 2	1		1	1	1,2	1,2	1	1,2	1	1,2	1,2	1,2	1	1,2	1,2	1,2	1,2	**1,2**	
		0	2	3	8	12	14	16	18	22	26	30	32	36	40	44	48	**52**	
	2		2	2			2		2				2						
		0	2	4			13		17				32						
	3		3	3,4	3	3	3,4	3	3	3	3	3,4	3,4	3,4	3,4	3	3,4	3,4	
		0	1	6	7	8	9	11	12	14	16	21	25	29	33	35	39	43	
	4		4		4	4		4	4	4	4					4			
		0	2		7	9		11	13	15	17					35			

	*	**	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
RACE 3	1		1,2	1,2	1	1	1,2	1,2	1,2	1,2	1,2	1	1,2	1,2	1,2	1,2	**1,2**		
		0	4	8	10	12	16	20	24	26	30	32	36	40	44	48	**52**		
	2				2	2						2							
		0			10	12						32							
	3		3,4	3,4	3,4	3	3	3	3	3	3,4	3,4	3,4	3,4	3,4	3,4	3,4		
		0	4	8	12	14	15	17	19	21	26	30	34	38	42	46	50		
	4					4	4	4	4	4									
		0				14	16	18	20	22									

	*	**	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
RACE 4	1		1,2	1,2	1,2	1,2	1,2	1,2	1,2	1,2,4	1,2,4	1	1,3	1,2,3	**1,2,3**				
		3	5	7	11	15	19	21	25	31	37	39	43	49	**52**				
	2											2,4	2						
		1										41	43						
	3		3	3	3	3	3	3	3	3	3	3							
		0	2	4	6	8	10	12	13	15	17	19							
	4		4	4	4	4	4	4	4				4	4	4				
		2	4	6	8	10	12	14	16				43	45	47				

	*	**	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
RACE 5	1		1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	1,3	**1,3**			
		2	7	11	13	17	21	23	27	29	33	37	41	45	47	**51**			
	2		2	2	2	2,4	2,4	2,4	2	2	2,4	2	2	2	2	2			
		0	0	2	3	11	11	15	16	17	19	20	22	24	26	28			
	3																		
		3																	
	4		4	4	4				4	4		4	4	4	4	4			
		1	3	5	7				17	18		20	22	24	26	28			

* Players   ** Initial position

TABLE 6 (continued)

RACE 6

*	**	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1		1	1	1	1	1	1	1	1	1	1	1	1	1				
	3	4	5	5	5	6	8	10	12	13	15	16	18	20				
2		2,3,4	2,3,4	2,3,4	2,3,4	2,3,4	2,3,4	2,3,4	2,3,4	2,3,4	2,3,4	2,3,4	2,4	2				
	2	5	8	11	14	17	20	23	29	35	41	44	48	50				
3													3	3				
	1												46	48				
4														4				
	0													50				

RACE 7

*	**	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1		1	1,2,4	1,2,4	1,2	1,2	1	1,2,4	1,2,4	1	1,2,3,4							
	2	4	11	14	18	22	22	32	38	38	50							
2		2					2,4			2,4								
	1	3					26			42								
3		3	3	3	3	3	3	3	3	3								
	0	0	0	0	0	0	0	0	0	0								
4		4			4	4												
	3	5			15	17												

RACE 8

*	**	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1		1,3	1,3	1,3	1	1,3	1,3	1,3	1,2	1,2	1,2,3	1,2,3	1,2	1,2,3				
	0	4	8	12	14	18	22	26	30	32	38	44	48	54				
2		2,4	2,4	2,4	2,4	2,4	2,4	2,4										
	0	4	6	10	14	18	22	26										
3					3				3	3			3,4					
	0				14				28	30			48					
4									4	4	4	4		4				
	0								28	30	32	34		50				

RACE 9

*	**	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1		1,3,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4
	0	3	7	11	13	15	19	21	25	27	29	33	37	41	45	47	49	51
2		2	2	2	2	2	2,3	2,3	2	2	2	2,3	2,3	2	2,3	2	2	2
	0	2	4	6	8	10	14	16	18	20	22	26	26	28	30	32	34	36
3			3	3	3	3			3	3	3			3		3	3	3
	0		4	5	5	6			17	18	19			27		31	32	34
4																		
	0																	

RACE 10

*	**	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1		1,3	1	1	1	1	1	1	1	1,2,3,4	1,2,3,4							
	3	5	5	7	9	11	13	15	15	47	55							
2		2,4	2,3,4	2,3,4	2,3,4	2,3,4	2,3,4	2,3,4	2,3,4									
	2	6	12	18	21	27	30	33	39									
3																		
	1																	
4																		
	0																	

TABLE 7
The evolution of cooperation in the seven-players case

RACE 1

*	**	1	2	3	4	5	6	7	8	9	10	11	12	13
1		1,2	1,3,7	1,2,3	1,2,3	1,2,3	1,2	1,2	1,3,7	1,3,7	1,2,3	1,2,3	1	
	0	4	10	16	22	28	30	32	35	38	44	47	49	
2			2						2	2,5			2,3	
	0		6						34	38			51	
3		3					3	3,7						
	0	2					30	34						
4		4,5	4,5	4,5	4,5	4,5	4,5	4,5	4,5	4	4,6	4	4,5	
	0	4	8	12	14	18	22	26	30	32	36	38	42	
5											5	5		
	0										40	42		
6		6	6	6	6,7	6,7	6,7	6	6	6		6	6	
	0	2	4	6	14	14	16	18	20	21		38	40	
7											7	7	7	
	0										40	42	44	

RACE 2

*	**	1	2	3	4	5	6	7	8	9	10	11	12	13
1		1,2,3	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3	1,2,3					
	0	6	12	18	24	30	36	42	48					
2														
	0													
3														
	0													
4		4	4,5,6	4,5,6,7	4,5,6,7	4,5,6,7	4,5,6,7	4,5,6,7	4,5,6					
	0	1	6	14	22	30	38	46	52					
5		5,6,7												
	0	3												
6														
	0													
7			7						7					
	0		9						48					

RACE 3

*	**	1	2	3	4	5	6	7	8	9	10	11	12	13
1		1,4	1,4	1,3,4	1,3,4	1,3,4	1,3,4	1,3,4	1,3,4	1,4	1,4			
	0	4	8	14	20	26	32	38	44	48	50			
2		2	2,5	2,5	2,5	2,5	2,5	2,5,6	2,5,6	2,5,6	2,5,6			
	0	2	6	8	9	13	15	22	28	34	40			
3		3,6	3,6							3	3			
	0	2	6							46	48			
4														
	0													
5		5												
	0	0												
6				6	6	6,7	6,7							
	0			8	10	14	16							
7		7	7	7	7			7	7	7	7			
	0	2	3	5	6			18	20	22	24			

RACE 4

*	**	1	2	3	4	5	6	7	8	9	10	11	12	13
1		1	1	1	1	1	1	1	1,3	1	1	1	1	1
	0	2	4	6	8	10	12	14	18	20	22	24	26	28
2		2,6	2,6	2	2	2	2,6	2,5,7	2,7	2	2,4,7	2,4,7	2,4,7	2,4,7
	0	2	6	8	10	11	15	21	25	29	32	38	44	50
3		3,5	3,5	3,5	3	3	3	3		3,5	3	3	3	3
	0	0	2	6	8	10	12	14		27	29	31	33	35
4		4,7	4,7	4,7	4,7	4,7	4,7	4	4	4,7				
	0	4	8	12	16	20	22	24	26	30				
5					5	5	5		5		5	5	5	5
	0				7	9	10		23		29	31	33	35
6				6	6	6		6	6	6	6	6	6	6
	0			4	5	7		17	19	21	23	25	27	29
7														
	0													

* Players   ** Initial position

TABLE 7 (Continued)

	*	**	1	2	3	4	5	6	7	8	9	10	11	12	13
RACE 5	**1**		**1**	**1,3**	**1,3,6**	**1,3,6**	**1,3,6**	**1,3,6**	**1,3,6**	**1,3,6**	**1,3,6**				
		0	1	6	14	20	26	32	38	44	50				
	2		**2**	**2**	**2**	**2**	**2**	**2**	**2,4,5,7**	**2,4,5,7**	**2,4,5,7**				
		0	2	4	6	8	10	12	34	42	50				
	3		**3**												
		0	2												
	4		**4,5**	**4,7**	**4,5**	**4,5**	**4,5**	**4,5,7**							
		0	4	8	12	16	20	26							
	5			**5,6**											
		0		8											
	6		**6**												
		0	1												
	7		**7**		**7**	**7**	**7**								
		0	2		8	10	12								

	*	**	1	2	3	4	5	6	7	8	9	10	11	12	13
RACE 6	**1**		**1,7**	**1,7**	**1,7**	**1,7**	**1,7**	**1,7**	**1,7**	**1,7**	**1,7**	**1,3**	**1,7**		
		0	5	7	11	15	19	23	27	31	35	43	47		
	2		**2**	**2,6**	**2,6**	**2,6**	**2,6**	**2,6**	**2,6**	**2,6**	**2,6**	**2**	**2,6**		
		0	2	6	10	14	18	20	24	28	32	34	38		
	3		**3,4,5**	**3,4,5**	**3,4,5**	**3,4,5**	**3,4,5**	**3**	**3,4,5**	**3,4,5**	**3,4,5**		**3**		
		2	9	12	18	21	27	28	34	37	43		45		
	4							**4,5**				**4,5**	**4,5**		
		1						27				47	51		
	5														
		0													
	6		**6**									**6**			
		0	2									34			
	7											**7**			
		3										37			

	*	**	1	2	3	4	5	6	7	8	9	10	11	12	13
RACE 7	**1**		**1**	**1,3**	**1,3**	**1,3**	**1,3**	**1,3**	**1,3**	**1**	**1,2**	**1,2**	**1,2**	**1,2**	
		0	2	6	10	14	18	22	24	26	38	42	46	50	
	2		**2**	**2**	**2**	**2**	**2**	**2,4**	**2,4,5**	**2,3**					
		0	2	3	4	4	5	24	30	34					
	3		**3**								**3,5**	**3,5**	**3,4**	**3,4,5,7**	
		0	2								38	42	42	50	
	4		**4,5,6**	**4,5,6**	**4,5**	**4,5**	**4**			**4,5**	**4**	**4**			
		1	9	15	17	19	20			32	33	34			
	5						**5**	**5**					**5**		
		2					21	23					44		
	6				**6**	**6**	**6,7**	**6,7**	**6,7**	**6,7**	**6,7**	**6,7**	**6,7**	**6**	
		3			17	19	23	25	29	33	37	39	43	45	
	7		**7**	**7**	**7**	**7**									
		0	1	3	3	5									

TABLE 7 (Continued)

RACE 8

Row	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1,3,5	1,3,5	1,3,5	1,3,5	1,3,5	1,3,5	1,3,5	1,3,5					
	6	12	18	24	30	36	42	48					
2	2,4,6	2,4,6	2,4,6,7	2,4,6,7	2,4,6,7	2,4,6,7	2,4,7	2,4,7					
	9	15	23	31	39	47	47	50					
3													
4													
5													
6							6	6					
							49	50					
7													

RACE 9

Row	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1	1,3,5	1,3,5	1,3	1,3	1	1	1	1,3	1,3	1,3	1,2,3,5,6	
	2	8	14	16	20	22	24	26	34	38	42	52	
2	2,6	2,6	2	2	2,5	2,3	2,3	2,3	2	2	2,5,6		
	3	5	7	9	16	24	26	30	32	34	40		
3	3												
	2												
4	4,7	4,7	4,7	4,7	4,7	4,7	4,7	4,7	4,7	4,7	4,7	4,7	
	6	10	14	18	22	26	30	34	38	42	46	50	
5	5			5		5	5,6	5,6	5,6	5,6			
	2			16		18	22	26	30	34			
6			6	6	6	6							
			7	9	11	13							
7													

RACE 10

Row	1	2	3	4	5	6	7	8	9	10	11	12	13
1	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,4	1,7	
	6	10	14	18	22	26	30	34	38	42	46	53	
2	2,3	2,3	2,3	2,3	2,3	2,3	2,3	2	2,3	2,3	2,3,7	2,3	
	4	8	12	14	18	22	26	28	32	36	49	53	
3								3					
								28					
4													
5	5,6	5,7	5,7	5,7	5,7	5,7	5,7	5,7	5,7	5,7	5	5	
	7	11	15	19	23	27	31	35	39	43	45	45	
6		6	6	6	6	6	6	6	6	6	6	6	
		9	11	12	14	16	18	20	22	24	26	26	
7	7												
	2												