The supply of education quality in a spatial model with asymmetric moving costs

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Abstract

The paper analyses the characteristics of the supply of higher education in different geographical macroareas using a strategic interaction framework. It focuses on universities operating in centralised funding system that autonomously set the quality of education showing that in equilibrium it is inversely related to students’ moving costs across areas. We show that in the presence of asymmetric information about workers’ ability and asymmetric costs of moving, the only PBE consistent with forward induction involves that only high ability workers acquire education and the quality of education is lower in macroareas where the moving costs are higher. Our model predicts that in economies with centralised university funding, educational policies must be regulated according to the specific socioeconomic characteristics of the area. Direct subsidies to universities may be ineffective in improving the quality of education in the less developed areas. When regional disparities are not too big, efficiency gains may be obtained by reducing moving costs.

Keywords: Cost sharing funding; Perfect Bayesian equilibrium; Forward induction; Spatial models

1. Introduction

This paper is motivated by the need to find some possible explanations of quality differentials among universities considering that the supply of education may be determined by the degree of competition among them which in turn depends on students’ moving costs across areas. We focus on universities that operate in economies characterised by the so-called cost sharing funding system where the main sources of university funding are the government allowances, and the students’ tuition fees have only a marginal impact on the university resources. Many works (among others see De Fraja and Iossa (2002), Epple et al. (2000) and Rothschild and White (1995)) try to explain quality differentials among universities that autonomously set tuition fees. On the other hand, the problem of quality differentials in a centralised funding system has not been widely considered in the literature. In particular, the issue

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concerns many European countries where although universities basically operate with the same amount of funds, there exists evidence of different returns of higher education possibly related to differentials of educational quality supply. The existing debate in Europe considers that the reform of the educational system must confront the issues of the degree of governmental ownership, the degree of governmental subsidy, and the degree of competition (OECD, 2006). A crucial issue is then to evaluate if the government may affect quality by changing the level of spending for higher education. In particular, it is relevant to evaluate the right instruments to implement this policy: tax exemptions, educational vouchers or direct subsidies to universities. Moreover it is important to consider how subsidies have to be distributed among universities and geographical areas in order to improve and to standardise the quality of education.

In this work we analyse the characteristics of the supply of higher education in different geographical macroareas using a strategic interaction framework and we highlight how in the presence of asymmetric information on workers’ ability and asymmetric costs of moving, the only perfect Bayesian equilibrium (PBE) consistent with forward induction involves that only high ability workers acquire education and the quality of education is lower in macroareas where the mobility costs are higher. We consider explicitly the case where education works both as a signalling device for individuals’ ability and as an instrument that raises individuals’ human capital.

The model setup considers an economy that can be described as a circular city where a finite number of universities compete to attract students. The universities are located at the same distance from each other and give rise to different regional markets. In each market, there is a continuum of individuals choosing how much education to obtain conditional on their ability. In this economy two firms operate. They are exogenously located on the circle and set production on the basis of technology and compete to employ individuals. We model a strategic interaction process where there are strategic complementarities between the educational choice of individuals and the technological choices of firms. At the same time, we assume that the universities, choices in terms of quality spill over on productivity and wages. The use of a spatial setting allows us to consider a pool of individuals with different elasticities of substitution with respect to the supplied education quality(ies). The equilibrium is analysed considering both symmetric and asymmetric costs of moving. In the presence of symmetric costs, we find that the quality of education decreases as moving costs grow and education works as a signalling device for individuals’ ability. If we consider the case of a circular city with asymmetric costs of moving, we find that the only PBE is characterised by universities setting different levels of quality. In particular, the universities located in the areas where the moving costs are relatively higher set a lower quality of education. As a consequence, in order to signal ability a higher education level is required. The presence of moving costs reinforces the monopoly power of universities which prefer to lose part of the potential demand of education and lower the education quality. Even if the quality of education is observable by all the agents, high ability individuals located in these areas may decide to study in low quality universities although they know that this choice generates a low wage. Another implication of the model is that in the presence of asymmetric moving costs productivity differentials among firms may arise. Our model predicts that educational policies must be regulated according to the specific socioeconomic characteristics of the area. Moreover, in general direct subsidies to universities may be ineffective in improving the quality of education in the less developed areas. This conclusion is consistent with the results of Aghion et al. (2005) who highlight that regions investing more in higher education do not necessarily grow faster. In our model it is crucial how governmental subsidies are distributed in order to improve the quality of education, the firms’ productivity levels and the regional macroperformances. Further, although the degree of competition may be undoubtedly improved by raising the individuals’ information about universities’ characteristics and students’ performance in the labour market, we conclude that the monopoly power of local universities may be regulated more effectively by modifying the structural characteristics and the institutions operating in the regional economies. Inefficiencies in the credit market and liquidity constraints, for example, may induce high mobility costs among areas and consequently influence the supply of quality. The labour market performance and the local rate of unemployment also might strongly influence the individuals’ choice of moving and consequently the incentive of local universities to improve quality.

The paper is organized as follows. Section 2 contains the main features of the model. Sections 3 and 4 discuss the equilibria without and with symmetric moving costs respectively. Section 5 describes the equilibrium with asymmetric moving costs. Section 6 contains the main policy implications and Section 7 concludes the paper.

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1 There exists some evidence on the returns of education that appear to depend on the specific institution attended and on the location of the universities. See Brunello and Cappellari (2005), Makovec (2006) and Ordine and Rose (2007).
2. The model setup

We consider an economy that can be described as a “circular city” (Salop, 1979) where a finite number $J$ of universities, exogenously located along a circle of circumference $J$, compete to attract students supplying different levels of educational quality.

The universities are located at the same distance from each other and give rise to $J$ regional markets. In each market, there is a continuum of individuals whose total number is $M = \sum \gamma_k$, which are assumed to be located uniformly along the circle arch of length 1. An individual location is indexed by $z \in [0, 1]$ in each regional market. The individuals choose to invest in human capital and may incur in moving costs to reach the universities. Moving costs have been extensively used in the economic modelling in order to justify industry agglomeration or specific employment patterns (Fujita et al., 1999; Fujita and Thisse, 2000; Guiso et al., 2007). They can be thought of as a function of several components, including pecuniary and nonpecuniary costs. The former are related to distance, to the existence and availability of transportation networks, to accommodation costs and facilities in the area of destination. Nonpecuniary costs also determine the propensity to move and include individual/psychological factors, societal factors related to community norms and cultural values. Hence, they depend on the specific socioeconomic context in which decisions are taken. The state of the economy (agrarian, industrial, the level of development); the labour market conditions and the conditions of work; the ability of the economy to provide jobs and the type of jobs available; the number of industries; the ability of the national and local government to provide related infrastructure and the state of the credit market, interact with the individual and household relations and affect costs and decisions about moving. Evidence on the role of family ties for geographical mobility, for example, is illustrated in Alesina and Giuliano (2007) who predict low mobility in the presence of strong family ties. Cook (2006) reports evidence on the existence of asymmetries in housing markets in the UK while Nilssen (1997) discusses problems of location when transportation costs are asymmetric. The effects of differences in local financial development are studied in Guiso et al. (2004) and in Ordine and Rose (2008). In general, we assume that moving costs may be different depending on the individuals’ origin area with respect to the region of destination, since they may be influenced by the performance of the local labour market, the educational and cultural background of the local collectivity, the efficiency of the credit market, the state of the infrastructure and transportation networks, and the functioning of the housing market.

In this economy two firms operate ($f = 1, 2$). The firms are exogenously located on the circle and set production on the basis of technology $T$ and compete to employ individuals. The economy lasts $p = 0, 1, 2, \ldots$ periods, agents are risk neutral and they discount the future at a given rate.

We model a strategic interaction process where there are strategic complementarities between the educational choice of individuals and the technological choices of firms.\footnote{Strategic complementarity is defined as in Acemoglu (1996).} At the same time, we assume that the universities’ choices in terms of quality spill over on productivity and wages.

Usually, the education quality is measured by some outcome or structure indicators such as the student–teachers ratios, the class size, the students’ marks or the proportion of students who graduate on time. However, these measures may contain some ambiguities. In our study we define the quality of education as the set of scientific and technical skills provided by the university that raises the individuals’ productivity and we avoid to consider students’ facilities.

2.1. The individuals

Individuals maximise a utility function, $u(\cdot)$, expressed in terms of wages $w(\cdot)$ and the cost of education $c(\cdot)$. Both functions $w(\cdot)$ and $c(\cdot)$ are determined by workers’ ability. We assume that there exist two types of workers with ability $\theta_i$, $i = h, l$ with $\theta_h > \theta_l$, and $\gamma = \text{prob} (\theta = \theta_h) \in (0, 1)$. The individual’s ability is his own private information as in the Spence model (Spence, 1973), but $\gamma$ is common knowledge. Hence:

$$u(w, e, q_j) = w(e, \theta_i, q_j, T) - c(e, \theta_i, q_j)$$  \hspace{1cm} (1)

with

$$u_{q_j}(\cdot) > 0, \quad u_{q_j|q_j}(\cdot) < 0$$
Fig. 1. The net effect of a decrease in education quality.

where $q_j \in [0, q_{\text{max}}]$, $q_{\text{max}} > 0$, indicates the quality of education supplied by the attended university $j$ ($j = 1, 2, \ldots, J$); $e$ is the educational level and $T$ represents the firm’s technological endowment.

In practice, before entering in the job market the individual can obtain a level of education $e \geq 0$, involving monetary and non-monetary costs represented by the twice differentiable function $c(e, \theta_i, q_j)$. We assume that:

$$c_q(\cdot) > 0, \quad c_e(\cdot) > 0.$$  \hspace{1cm} (2)

Moreover, we assume that the so-called “single cross property” holds only if $q > 0$. Hence:

$$c_e(e, \theta_l, q_j) > c_e(e, \theta_h, q_j) \text{ iff } q_j > 0.$$  \hspace{1cm} (3)

This implies that if the individual acquires education of quality $q = 0$ the single cross property disappears and the marginal cost of education is identical for individuals with different ability. We assume also that:

$$c_{qe}(e, \theta_l, q_j) > c_{qe}(e, \theta_h, q_j)$$  \hspace{1cm} (4)

where $c_{qe}$ is the cross partial derivative of the cost function with respect to the level and the quality of education. The implication of (4) is that a reduction in the quality of education lowers the indifference curves of low ability more than those of high ability individuals. The net effect of a decrease in $q$, illustrated in Fig. 1, results in the fact that the indifference curves become less distant from each other.

2.2. The firms

Each firm employs workers in order to produce the final output. Before hiring a worker the firm has to decide the technology to adopt. In particular, the firm can choose between high or low technologies. We indicate $T =$ \{HT, LT\} the firm’s investment in high or low technology respectively. The cost of technology HT is given by $\delta > 0$ while the cost of technology LT is normalized to zero. Following Acemoglu (1997), the average productivity per worker is given by:

$$y = y(e, \theta_i, q_j, T) = e_0 + e(q_j + \varepsilon \times 1_{\{\theta = \theta_h, T = HT\}})$$  \hspace{1cm} (5)

where $e_0$ is a constant and $\varepsilon > 0$. From (5) it appears that the high technology is complementary only to the high ability workers. As a consequence, firms need a credible signal on individuals’ ability in order to invest in high technology. The worker’s productivity is a linearly increasing function of education when $q > 0$. As illustrated in Fig. 2, Eq. (5) assumes a scenario where the effect of education on a worker’s productivity depends on the quality of education and on the match between high ability individuals and high technology. Ceteris paribus, a high ability worker from a university of high quality produces a higher output.
The firms’ profits per worker are given by the difference between workers’ productivity and wage minus the cost of the adopted technology. We assume a wage schedule that fixes the wage equal to the individual’s productivity \( y(\cdot) \) minus a rent \( \Delta(T, \theta_h) \) that the firm may capture from each worker conditional on its technological decision and individual’s ability. This rent is only due to the presence of mobility costs that make an individual strictly prefer to work for one firm even when wages are the same. Since technology \( HT \) costs \( \delta \), in order to be convenient for a firm to invest in \( HT \) when a high ability individual is observed, the following condition must hold:

\[
\Delta(HT, \theta_h) - \delta \geq \Delta(LT, \theta_h).
\]

Moreover, since we consider \( \Delta(\cdot) \) only as the result of non-perfect competition between firms, we must have that condition (6) is satisfied as an equality:

\[
\Delta(HT, \theta_h) - \delta = \Delta(LT, \theta_h). \tag{7}
\]

We can define \( \Delta \) as the net profit per worker due to the presence of mobility costs, where:

\[
\Delta = \Delta(HT, \theta_h) - \delta = \Delta(LT, \theta_h). \tag{8}
\]

It is important to note that, even if firms make the same profit \( \Delta (\cdot) \) per worker unconditional on his ability, there is still an incentive to invest in \( HT \) in the presence of a credible signal on individuals’ ability. In fact, competition between firms ensures that if a high ability individual is observed, a firm invests in \( HT \) in order to retain the worker. In this setup firms’ profits are given by \( \Delta \) times the number of individuals employed. As a consequence firms want to maximise the number of employees. Since firms are identical we have a symmetric equilibrium where each firm employs half of the individuals \( (JM/2\gamma) \). We should notice that in the infinitely repeated version of the game, firms might decide to collude and fix \( T = LT \). However, this collusion is not sustainable even if firms were to discount entirely the future. In fact, in the presence of (symmetric) firms’ collusion with \( T = LT \) the profit level for each firm would not change with respect to the profit arising in the case of competition and, as a consequence, a firm would always deviate and set \( T = HT \). \(^4\)

2.3. The universities

The theoretical framework of this paper considers economies that use a higher education financing method called cost sharing where the main sources of university funding are the government allowances and the students’ fees have

\(^3\) The same result can be achieved by modelling Bertrand competition in a wage-offer game between firms in the presence of mobility costs. In this case, in equilibrium a firm pays a wage equal to worker’s productivity minus a rents that is a function of individual’s mobility costs. The assumption of a wage schedule allows us to simplify the exposition of the agents’ strategies in equilibrium.

\(^4\) Collusion does not raise firms’ profits with respect to the payoff of the only Nash equilibrium of the single move game. See Mas-Colell et al. (1995, p. 404–405) for a detailed discussion on this issue.
only a marginal impact on the university resources (for a detailed discussion see Johnstone (2003)). The existing literature on education supply discusses when it is appropriate to introduce profit maximising behaviour for the objective functions of universities.

We assume that the university’s objective function derives from the strategic behaviour of the internal political membership in period \( p \), who maximises the probability \( \rho(0 \leq \rho < 1) \) of being re-elected in period \( p + 1 \). This probability may be expressed as:

\[
\rho_{p+1} = h(\alpha q_{j,p}, \pi_{j,p}(q_{j,p}))
\]

where \( h(\cdot) \) is a generic “well-behaved” function, \( \alpha > 0 \) is a parameter, \( q_{j,p} \) is the quality fixed by university \( j \) in period \( p \) and:

\[
\pi_{j,p}(q_{j,p}) = [S - C(q_j)]X_j(q_j, q_{-j})
\]

is the revenue generated by the university’s governance, with \( X_j(\cdot) \) representing the enrollment demand for university \( j \), \( C(\cdot) \) the unit cost per student and \( S \) is the government subsidy per student.

In Eq. (9) we are assuming that the probability of being re-elected in period \( p + 1 \) is a function of the quality \( q \) fixed in period \( p \). An increase in \( q \) obtained by increasing the quality of the academic staff raises the utility of high quality professors since they prefer to be employed in a high quality institution and, consequently they will sustain a membership investing in high quality. In this case, the higher the quality set in period \( p \), the larger the political consensus that the membership will have in period \( p + 1 \). At the same time, we assume that the probability of being re-elected is a function of the additional resources \( \pi(\cdot) \) that the membership is able to generate given the subsidy \( S \). The membership can spend this surplus in activities (larger administrative staff, standard teaching activities) that do not directly affect the education quality but generate a general consensus and increase the probability of re-election. Hence, from (9):

\[
\frac{\partial \rho_{p+1}}{\partial q_p} > 0 \iff \frac{\partial h}{\partial q_p} > -\frac{\partial h}{\partial \pi_p} \frac{\partial \pi_p}{\partial q_p}
\]

In this context we assume that each university has the following objective function:

\[
\max_{q_j} \Pi^u_j = [S - C(q_j)]X_j(q_j, q_{-j}) + \alpha q_j
\]

s.t.

\[
S - C(q_j) \geq 0 \quad \text{and} \quad X_j(q_j, q_{-j}) > 0
\]

where the cost function is increasing in quality, and the demand is a continuous function of the quality \( q_j \) and of the quality supplied by other universities \( q_{-j} \) operating in the whole economy:

\[
C_q > 0 \quad C_{qq} \geq 0 \quad \frac{\partial X_j}{\partial q_j} > 0 \quad \frac{\partial X_j}{\partial q_{-j}} < 0.
\]

Notice that given \( \alpha \) the quality choice \( q_j^* \) that maximises \( \Pi^u_j \) depends on the size of \( \pi_{j,p}(q_{j,p}) \).

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5 The cost-sharing funding method became widespread in the European economies during the last ten years. The impact of students’ fees on the total university funding is less than 15% on average (see OECD (2004)).

6 Rothschild and White (1995) consider the issue of competitive pricing and allocative efficiency for higher education. At the same time, Winston (1999) puts forward the limits of the use of standard profit maximising theory for the analysis of education supply. In any case, these approaches refer to universities that can autonomously set fees and admission rules. On the other hand, Dranove and White (1994) present theoretical arguments suggesting that non-profit organizations behave as profit maximisers and Kemnitz (2003) considers the universities as profit-maximiser agents because of the intent of having additional resources for teaching or research expenditures.

7 The use of political economy models to justify the adoption of inefficient policies and inefficiencies in the bureaucratic organization of the state is not new in the literature. Acemoglu et al. (2006) show that the expansion of the size of bureaucracy may be an instrument to gain additional votes.
2.4. The timing of the model

The strategic interaction process is analysed considering sequential moves that lead education to be a signal of individuals’ ability unless the quality is set to zero.

Following the Harsanyi (1967/68) approach, we assume that the timing is as follows. Each period $p$ is characterised by $t = 0, 1, 2, 3, 4$ instants of time. At time $t = 0$, Nature chooses the ability of the individuals, at $t = 1$ elections take place in each university and then the elected membership sets the quality $q$ that all agents can observe. At $t = 2$, given the quality choices of the universities, the individuals decide where they want to study and the level of education, considering the distance from the universities. At $t = 3$, the firms observe the level and quality of education of the individuals and make the technological choices. At $t = 4$ the individual chooses the firm in which he will work and then payoffs realize. The game is repeated for $p = \infty$ periods. As it is well known, when the game is repeated we can have collusion among agents instead of competition and this may generate an infinite number of equilibria conditional to the agents’ discount rate. In our case the only agents that may collude are universities. In this respect we should notice 3 elements. First, any credible punishment arising from “Nash reversion strategies” in the presence of moving costs always gives rise to positive surplus/profits for universities. Consequently, in this setup in order to have collusion we should observe that universities do not discount the future too much. Secondly, the incentive of the university’s membership to deviate from a collusion is larger than that in a normal case of a repeated game simply because in our case the membership is not sure to be in charge of the next stage of the game because it has only a probability of being re-elected. Using the well-known Selten’s argument (Selten, 1975) this uncertainty about the future affects the discount rate (the uncertainty makes an agent not to care much about the future) and this makes more difficult to have collusion. Third, in order to have collusion in the presence of a number of universities larger than 2, the discount rate should require a very high care about future payoffs. For all these reasons, we exclude the possibility that universities may decide to cooperate in setting the quality of education. As a consequence we focus on the equilibrium of the one shot game illustrated in Fig. 3.
The equilibrium concept used to solve the model is the Perfect Bayesian Equilibrium (PBE) consistent with forward induction reasoning (Kohlberg and Mertens, 1986) that allows us to exclude PBE characterised by unreasonable beliefs. This is also consistent with the Intuitive Criterion (Cho and Kreps, 1987).

3. The equilibrium without moving costs

We first consider the case where the individuals do not have costs of moving. We define the Bayesian equilibrium as the assessment \( \sigma^* = (\sigma_J^*, \sigma_M^*, \sigma_f^*, b(e)) \) where \( \sigma_J^*, \sigma_M^*, \sigma_f^* \) represent the strategies of universities, individuals and firms respectively and \( b(e) \in [0, 1] \) is the firms’ belief function, i.e. the probability that a worker with education \( e \) is a high ability type derived using the Bayes rule.

**Proposition 1.** The following profile of strategies and beliefs \( \sigma^* \) is the only PBE of the game described in Fig. 3 consistent with reasonable beliefs:

\[
\begin{align*}
\sigma_J^* &= \{ q_j = q_{\text{max}} \quad \forall j \} \quad (15) \\
\sigma_M^* &= \begin{cases} 
  d = d_{\text{min}}; & e = e^s \quad \text{iff} \quad \theta_i = \theta_h \\
  e = 0; & d = 0; \quad e = 0 \quad \text{iff} \quad \theta_i = \theta_l \\
  \text{work in the closest firm} \quad \forall \theta
\end{cases} \quad (16) \\
b(e) &= \begin{cases} 
  0 & \text{iff} \quad e < e^s \\
  1 & \text{iff} \quad e \geq e^s
\end{cases} \quad \forall f \quad (17) \\
\sigma_f^* &= \begin{cases} 
  T = HT & \text{iff} \quad e \geq e^s \\
  T = LT & \text{iff} \quad e < e^s
\end{cases} \quad \forall f \quad (18)
\end{align*}
\]

where \( e^s \) indicates the generic level of education consistent with a separating equilibrium as shown in Fig. 4.

The proof of Proposition 1 is given in the Appendix. Intuitively, Bertrand competition among universities drives education quality at its maximum level. The single cross property is preserved and a separating signalling equilibrium, where only high ability individuals’ invest in human capital, arises. In this case, the maximum level of education quality is determined by the amount of subsidy \( S \), since the Bertrand competition among universities implies a zero profit equilibrium with \( S = C(q_j) \) in (12). Since the quality of education is identical in all universities, high ability individuals are indifferent about location. We assume that they choose the closest university so that \( d = d_{\text{min}} \). All
firms generate the highest level of productivity given the share of high ability individuals on the total labour force $\gamma$, and the amount of subsidy $S$.

4. The equilibrium with symmetric moving costs

We consider the equilibrium when individuals incur in moving costs. Since in this economy $J > 2$ universities operate, we consider a circular city model as in Salop (1979). In Fig. 5, we describe a location of firms and universities when $J = 3$. The total cost of acquiring education in university $j$, for an individual located at a distance $d$, is $c(e, \theta_i, q_j) + \tau d$ (with $\tau, d > 0$) where $\tau$ can be thought of as the cost per unit of distance travelled by the individual in going to university $j$’s location.

The use of a spatial setting allows us to consider a pool of individuals with different elasticities of substitution with respect to the supplied education’s quality. We assume that the cost per unit of distance is identical for each individual and it is independent of the area where he is located. At the same time, the net wage obtained in firm $f$ is $y(e, \theta_i, q_j, T) - \Delta(T, \theta_i)$. In order to find the PBE of the game we need to find the Nash equilibrium in the simultaneous move game among universities. To keep things treatable, we assume that it is always worthwhile for a high ability individual to acquire education so that:

$$w(e, \theta_h, q_j, HT) - c(e, \theta_h, q_j) - \tau d > 0. \quad (19)$$

**Proposition 2.** The following profile of strategies and beliefs $\sigma^*$ is the only PBE of the game described in Fig. 3 consistent with reasonable beliefs in the presence of symmetric moving costs:

$$\sigma^*_j = \begin{cases} q_j = q^* < q^{\text{max}} & \forall j \end{cases} \quad (20)$$

$$\sigma^*_M = \begin{cases} d = d^{\text{min}}, & e = e^s \quad \text{iff} \quad \theta_i = \theta_h \\ d = 0; & e = 0 \quad \text{iff} \quad \theta_i = \theta_l \end{cases} \quad \text{work the closest firm} \quad \forall \theta \quad (21)$$

$$b(e) = \begin{cases} 0 & \text{iff} \quad e < e^s \\ 1 & \text{iff} \quad e \geq e^s \end{cases} \quad \forall f \quad (22)$$

$$\sigma^*_f = \begin{cases} T = HT \quad \text{iff} \quad e \geq e^s \\ T = HT \quad \text{iff} \quad e < e^s \end{cases} \quad \forall f \quad (23)$$

where $q^*$ is the level of quality satisfying the following condition:

$$[S - C(q^*)]u_qM - 2\tau MC_q + 2\alpha\tau = 0. \quad (24)$$

---

8 We should notice that without moving costs we lose one of the elements that prevent collusion among universities. However we may still exclude collusion in the presence of a large number of universities.
The expression in (24) represents the best response function in the simultaneous move game among universities in a symmetric equilibrium. We derive analytically (24) in the proof given in the Appendix.

Using the implicit function theorem, it is possible to show that \( \partial q^*/\partial \tau < 0 \) and \( \partial q^*/\partial S > 0 \). This means that the quality level set by universities decreases as the cost of moving rises. Moreover, with symmetric moving costs, the quality of education increases with subsidies. It is interesting to notice that it may be more effective a reduction in moving costs than an increase in subsidies as long as \( |u_q(\cdot)| < \left| -C_q + \frac{2q}{M} \right| \) since in this case we have that \( |\partial q^*/\partial \tau| > |\partial q^*/\partial S| \). As shown in the Appendix, in the PBE equilibrium \( 0 < q^* < q^{\text{max}} \). From (24) it may be easily seen that \( q = q^{\text{max}} \) only if \( \tau = 0 \). Each university has a demand equal to \( M \) and we observe high ability individuals uniformly distributed among universities. The presence of travel costs introduces differentiation between the two firms since individuals may now strictly prefer to work in one of the two firms even when wages are the same. In this type of economy the level of productivity depends on \( q^* \). The level of subsidies and the dimension of the travelling costs are the crucial determinants of firms’ productivity and individuals’ utility. However, the equilibrium levels of these variables are always inferior to those reached in the case of absence of moving costs.

5. The equilibrium with asymmetric moving costs

We now discuss the case when the costs of moving depend on the origin area of the individuals. We assume that there are \( J = 3 \) universities as described in Fig. 5. We indicate the area delimited by universities \( j = 1 \), and \( j = 2 \) as area \( A_1 \); the area delimited by universities \( j = 2 \), and \( j = 3 \) as area \( A_2 \); the area delimited by universities \( j = 1 \), and \( j = 3 \) as area \( A_3 \). In particular we assume that individuals located in the area \( A_1 \) incur in higher moving costs per unit of distance travelled, denoted by \( \xi \), than people located in the area \( A_2 \) and \( A_3 \), which have costs indicated by \( \tau \), with \( \tau < \xi \). We first assume that firms are located symmetrically with respect to \( A_1 \). Later we relax this assumption and discuss the implications of an asymmetric firms’ positioning.

**Proposition 3.** The following profile of strategies and beliefs \( \sigma^* \) is the only PBE of the game described in Fig. 3 consistent with reasonable beliefs:

\[
\begin{align*}
\sigma^*_j &= \begin{cases} 
q_j = \hat{q} & \text{iff } j = 1, 2 \\
q_j = q^*_j > \hat{q} & \text{iff } j = 3
\end{cases} \\
\sigma^*_M &= \begin{cases} 
d = d^\text{min} & e = e^{ij} & \text{iff } (\theta_l = \theta_h \text{ in } A_1) \\
d = d^\text{min} & e = e^{ij} & \text{iff } (\theta_l = \theta_h, z \in \left[0, \frac{1}{2}\right] \cup \left[\frac{1}{2}, 1\right] \text{ in } A_2 \text{ and } A_3) \\
d = d^{\text{max}} & e = e^{ij} & \text{iff } (\theta_l = \theta_h, z \in \left[\frac{1}{2}, 1\right] \text{ in } A_2 \text{ and } A_3) \\
d = 0 & e = 0 & \text{iff } \theta_l = \theta_l \text{ work for the closest } \forall \theta
\end{cases}
\end{align*}
\]

\[b(e) = \begin{cases} 0 & \text{iff } e < e^{ij} \\
1 & \text{iff } e \geq e^{ij} \forall f
\end{cases}\]

\[\sigma^*_f = \begin{cases} T = HT & \text{iff } e \geq e^{ij} \\
T = LT & \text{iff } e < e^{ij} \forall f
\end{cases}\]

where \( e^{ij} \) indicates the level of education that generates a separating equilibrium with:

\[e^{i3} < e^{i2} \text{ and } e^{i2} = e^{i1}\]

and \( d^{\text{max}} \) represents the maximum distance between the individuals’ location and the contiguous universities.

The interesting result derives from the analysis of the equilibrium strategies of the universities. As shown in the Appendix, the quality level set by the universities located in the area where individuals face high travelling costs is lower than the quality level set elsewhere. Moving costs give monopoly power to universities which prefer to lose part of the potential demand of education and lower the education quality. This implies a separating equilibrium in which high ability individuals need to acquire a higher education level in order to signal their ability.
6. Educational policies implications

An interesting exercise is to evaluate the impact of different policy measures in order to raise quality in different areas in the presence of asymmetric moving costs. In what follows we compare the marginal impact on education quality of changing governmental subsidies or moving costs. This comparison does not necessarily imply welfare considerations since we do not control for the costs of different policies, and we do not have a welfare aggregate function to make predictions on the overall effect of increasing the quality of education. We just aim at evaluating the right instruments to obtain ceteris paribus educational quality improvements.

First of all, we may show that an increase in governmental subsidies $S$, is more effective in increasing the education quality in universities facing the demand of individuals from lower moving costs area. As proved in the Appendix we have that:

$$\frac{\partial q^*_3}{\partial S} > \frac{\partial \hat{q}}{\partial S}. \quad (29)$$

At the same time we show that rising subsidies or changing moving costs may have an impact on quality which depends on the relative size of moving costs and on the margin of profits per student of the universities. We show that it is worthwhile raising subsidies instead of reducing moving costs only if:

$$\xi^2 + \tau \xi - \tau [S - C(q)] > 0. \quad (30)$$

The values of $\xi$ for which condition (30) is satisfied are graphically illustrated in Fig. 6 where $\xi_0 = \frac{-\tau + \sqrt{\tau^2 + 4\tau[S - C(q)]}}{2}$. Since $\xi > \tau$ must hold, two scenarios may arise:

- $\xi_0 \leq \tau \Rightarrow$ It is always worthwhile reducing the marginal moving cost instead of increasing the direct subsidy to the universities.
- $\xi_0 > \tau \Rightarrow$ In the case of extremely wide regional disparities ($\xi > \xi_0$) an increase in $S$ generates a greater improvement in education quality than a reduction in $\xi$. In the case of small regional disparities ($\tau < \xi < \xi_0$) reducing the marginal moving cost leads to a greater quality improvement than the one obtained by increasing the direct subsidy.

On top of that, the symmetry in firms’ location with respect to the area with the higher travel costs generates no differences in firms’ productivity. If we remove this symmetry assumption, the market power of the firm closer to the highest travel costs area can be used only on individuals that have a lower productivity, because of the low education quality received. In this case a firm always performs at low productivity.
7. Conclusions

In this paper we analyse the supply of higher education considering the implications of individuals’ moving costs and firms’ technological choices in determining the quality of education. We define the quality of education as the set of scientific and technical skills that raise the individuals’ productivity. We consider moving costs as determined by the characteristics of the local socioeconomic environment so that these costs may be influenced by the degree of efficiency of the local credit market, the educational and cultural background of the local collectivity, the performance of the regional labour market. We show that the quality of education and the average productivity of workers is lower in the presence of moving costs. Moreover, we find that if moving costs are asymmetric, in the areas where they are relatively higher, the quality of education is lower than the quality supplied elsewhere. In these areas high ability individuals need to acquire a higher education level in order to signal their ability. Moving costs in these areas generate a sort of stronger monopoly power of universities which prefer to lose part of the potential demand of education and lower the education quality. In our model this implies that the level of firms’ productivity in less developed areas with high moving costs is also low. These results are relevant when discussing educational and growth policies of economies with regional disparities and where there exist a high degree of governmental ownership and a centralised funding based on cost-sharing formula. This is the case of many European economies. We argue that when the system of funding is centralised and based on a fixed amount of subsidy per student, increasing this subsidy may not be effective in improving the education quality in less developed areas. The right instruments to be used in order to raise the quality of higher education and then to promote local growth and to reduce regional disparities depend on a set of variables which include relative moving costs across areas and the margin of profit per student. An increase in governmental subsidies might just raise the monopoly power of local institutions and might not be useful in reducing disparities. This conclusion is consistent with the results of Aghion et al. (2005) who highlight that regions investing more in higher education do not necessarily grow faster. In our model it is crucial how governmental subsidies are distributed in order to improve the quality of education, the firms’ productivity levels and the regional macroperformances. Further, although the degree of competition may be undoubtedly improved by raising the individuals’ information about universities’ characteristics and students’ performance in the labour market, we conclude that the monopoly power of local universities may be regulated more effectively by modifying structural characteristics and institutions operating in the regional economies. Inefficiencies in the credit market and liquidity constraints, for example, may induce high mobility costs among areas and consequently influence the supply of quality. The labour market performance and the local rate of unemployment also might strongly influence the individuals’ choice of moving and consequently the incentive of local universities to improve quality.

Appendix

Proof of Proposition 1. First we show that $q_j = q_{\text{max}}$ for all $j$ is a Nash equilibrium in the simultaneous move game among universities. At the level of quality $q_{\text{max}}$ all the universities earn zero profits. No university can gain by lowering quality $0 < q_j < q_{\text{max}}$ because it will lose all students due to the assumption that $u_{q_j}(\cdot) > 0$. We need to show that setting $q_j = 0$ also implies that university $j$ loses all students. Suppose that $q_j = 0$. Since $q_j$ is common knowledge, education acquired in university $j$ is not a signal of individuals’ ability. When firms observe an individual with education $e$ from university $j$, they choose $T = HT$ only if the expected profit from this choice is higher than the profit when $T = LT$. Hence, the following condition must hold:

$$
\gamma [\Delta(\theta_h, HT) - \delta] + (1 - \gamma) [\Delta(\theta_l, HT) - \delta] \geq \Delta(\theta_l, LT) \quad \forall i
$$

(31)

where the LHS represents expected profits in the case of $T = HT$ while the RHS are profits in the case of $T = LT$. Since relation (5) implies that $\Delta(\theta_l, LT) = \Delta(\theta_l, HT)$, we can write (31) as follows:

$$
\gamma \Delta(\theta_h, HT) + (1 - \gamma) \Delta(\theta_l, HT) - \delta \geq \Delta(\theta_l, HT).
$$

(32)

In the absence of moving costs firms make no profit per worker ($\Delta = 0$). Using relations (7) and (8) we have that firms invest in $HT$ only if:

$$
\delta(\gamma - 1) \geq 0.
$$

(33)
Since $\gamma < 1$ and $\delta > 0$, firms never choose to invest in $HT$ when individuals are from university $j$. Consequently, as shown in Fig. 7 no one chooses to attend university $j$. The single cross property holds and the other strategies and beliefs of Proposition 1 represent a separating equilibrium similar to that discussed in Spence (1973). □

**Proof of Proposition 2.** Consider the simultaneous move game among universities. The universities are located at the same distance from each other along the circle of length $J$. In this spatial model the universities compete only locally. We first consider competition between contiguous universities as in a linear model. Given the assumption of symmetry, we can extend the essential features of the results to the case in which $J > 2$. Indicating with $\tilde{z}$ the location of an individual who is indifferent between studying in university $j$ or $-j$:

$$u(\cdot, q_j) - \tilde{z} \tau = u(\cdot, q_{-j}) (1 - \tilde{z}) \tau$$

we have:

$$\tilde{z} = \frac{u(\cdot, q_j) - u(\cdot, q_{-j}) + \tau}{2\tau}$$

and we can write the demand for universities $j$ as follows:

$$X_j (q_j, q_{-j}) = \begin{cases} 0 & \text{if } \tilde{z} < 0 \\ M \tilde{z} & \text{if } \tilde{z} \in [0, 1] \\ M & \text{if } \tilde{z} > 1 \end{cases}$$

which implies

$$X_j (q_j, q_{-j}) = \begin{cases} 0 & \text{if } u(\cdot, q_j) < u(\cdot, q_{-j}) - \tau \\ M \tilde{z} & \text{if } u(\cdot, q_j) \in [u(\cdot, q_{-j}) - \tau; u(\cdot, q_{-j}) + \tau] \\ M & \text{if } u(\cdot, q_j) > u(\cdot, q_{-j}) + \tau. \end{cases}$$

Mutatis mutandis the demand for university $-j$ is given by:

$$X_{-j} (q_j, q_{-j}) = \begin{cases} 0 & \text{if } u(\cdot, q_{-j}) > u(\cdot, q_j) - \tau \\ M \tilde{z} & \text{if } u(\cdot, q_{-j}) \in [u(\cdot, q_j) - \tau; u(\cdot, q_j) + \tau] \\ M & \text{if } u(\cdot, q_{-j}) > u(\cdot, q_j) + \tau. \end{cases}$$

Note that in searching for the best response to quality choice of its rival $\tilde{q}_{-j}$, each university $j$ can restrict itself to the interval $[u(\cdot, q_{-j}) - \tau; u(\cdot, q_{-j}) + \tau]$ since in the other scenarios it can never raise profits. Thus the best response to $\tilde{q}_{-j}$ solves the following maximisation problem:

$$\max_{q_j} (S - C(q_j)) M \frac{u(\cdot, q_j) - u(\cdot, \tilde{q}_{-j}) + \tau}{2\tau} + \alpha q_j$$
s.t.

\[ u(\cdot, q_j) \in [u(\cdot, q_j) - \tau; u(\cdot, q_j) + \tau]. \]

The necessary and sufficient Kuhn–Tucker FOC for (37) are:

\[
\begin{align*}
- \frac{M}{2\tau} C_{q_j} [u(\cdot, q_j) - u(\cdot, q_j - \gamma) + \tau] \\
+ \frac{M}{2\tau} (S - C(q_j)) u_{q_j} + \alpha \begin{cases} \\
\leq 0 & \text{if } u(\cdot, q_j) = u(\cdot, q_j - \gamma) - \tau \\
= 0 & \text{if } u(\cdot, q_j) \in [u(\cdot, q_j - \gamma) - \tau; u(\cdot, q_j - \gamma) + \tau] \\
\geq 0 & \text{if } u(\cdot, q_j) = u(\cdot, q_j - \gamma) + \tau.
\end{cases}
\end{align*}
\]

(38)

Since we look for a symmetric equilibrium where \( q_j^* = q_{-j}^* = q^* \) we can see from (38) that it may be reached only in the middle case. So \( q^* \) is the solution of:

\[
\frac{1}{2\tau} [(S - C(q)) u_q M - \tau M C_q + 2\alpha \tau] = 0
\]

(39)

with:

\[
\frac{\partial q^*}{\partial \tau} = - \frac{-MC_{q^*} + 2\alpha}{u_{q^*} M (S - C(q^*)) - C_{q^*} u_{q^*} M - \tau M C_{q^*} q^*} < 0
\]

(40)

\[
\frac{\partial q^*}{\partial S} = - \frac{M u_{q^*}}{u_{q^*} M (S - C(q^*)) - C_{q^*} u_{q^*} M - \tau M C_{q^*} q^*} > 0
\]

(41)

because of the assumptions on first and second order derivatives and:

\[
-M C_{q^*} + 2\alpha = -(S - C(q^*)) u_{q^*} M \tau
\]

(42)

To show that Proposition 2 contains the only equilibrium of the interaction process, we need to show, as in the case without moving costs, that if quality is set equal to zero no one invests in human capital and the demand for education collapses to zero. Suppose that \( q_j = 0 \). As in the previous case, if firms observe an individual with education \( \epsilon \) from university \( j \), they choose \( T = HT \), only if the expected profit from this choice is higher than the profit when \( T = LT \):

\[
\gamma \Delta(\theta_\epsilon, HT) + (1 - \gamma) \Delta(\theta_\epsilon, HT) - \delta \geq \Delta(\theta_\epsilon, HT).
\]

(43)

Using relation (7) we have that firms invest in \( HT \) only if:

\[
\delta(\gamma - 1) \geq 0.
\]

(44)

Since \( \gamma < 1 \) and \( \delta > 0 \), firms never choose to invest in \( HT \) when individuals are from university \( j \). Consequently we must have \( q^* > 0 \). If we consider the circular city we have that each university has a demand equal to \( M \). \( \square \)

**Proof of Proposition 3.** Consider first the simultaneous move game among universities. We show that the quality level of education of universities \( j = 1 \) and \( j = 2 \) is lower than the level of quality supplied by university \( j = 3 \). Indicate with \( \tilde{z}_{An}, n = 1, 2, 3; \) the location of an individual located in area \( An \) who is indifferent between studying in university \( j \) or \( -j \), defined as follows:

\[
\begin{align*}
&u(\cdot, q_1) - \tilde{z}_{A1} \xi = u(\cdot, q_2) - (1 - \tilde{z}_{A1}) \xi \\
&u(\cdot, q_1) - \tilde{z}_{A3} \tau = u(\cdot, q_3) - (1 - \tilde{z}_{A3}) \tau \\
&u(\cdot, q_2) - \tilde{z}_{A2} \tau = u(\cdot, q_3) - (1 - \tilde{z}_{A2}) \tau.
\end{align*}
\]

(45)

(46)

(47)

Hence:

\[
\begin{align*}
\tilde{z}_{A1} &= \frac{u(\cdot, q_1) - u(\cdot, q_2) + \xi}{2\xi} \\
\tilde{z}_{A2} &= \frac{u(\cdot, q_2) - u(\cdot, q_3) + \tau}{2\tau} \\
\tilde{z}_{A3} &= \frac{u(\cdot, q_1) - u(\cdot, q_3) + \tau}{2\tau}.
\end{align*}
\]

(48)

(49)

(50)
Consider the demand for education of university \( j = 1 \). Since this university competes to attract students from areas \( A1 \) and \( A3 \) we may have the following seven scenarios:

\[
X_1 (q_1, q_2, q_3) = 0 \quad \text{if} \quad \begin{cases} u (\cdot, q_1) < u (\cdot, q_2) - \xi \\ u (\cdot, q_1) < u (\cdot, q_3) - \tau \\ \end{cases}
\]

\[
= M \quad \text{if} \quad \begin{cases} u (\cdot, q_1) > u (\cdot, q_2) + \xi \\ u (\cdot, q_1) < u (\cdot, q_3) - \tau \\ \end{cases}
\]

\[
= M \quad \text{if} \quad \begin{cases} u (\cdot, q_1) < u (\cdot, q_2) - \xi \\ u (\cdot, q_1) > u (\cdot, q_3) + \tau \\ \end{cases}
\]

\[
= 2M \quad \text{if} \quad \begin{cases} u (\cdot, q_1) > u (\cdot, q_2) + \xi \\ u (\cdot, q_1) > u (\cdot, q_3) + \tau \\ \end{cases}
\]

\[
= \tilde{z}_{A1} M \quad \text{if} \quad \begin{cases} u (\cdot, q_1) \in [u (\cdot, q_2) - \xi; u (\cdot, q_2) + \xi] \\ u (\cdot, q_1) < u (\cdot, q_3) - \tau \\ \end{cases}
\]

\[
= \tilde{z}_{A3} M \quad \text{if} \quad \begin{cases} u (\cdot, q_1) < u (\cdot, q_2) - \xi \\ u (\cdot, q_1) \in [u (\cdot, q_3) - \tau; u (\cdot, q_3) + \tau] \\ \end{cases}
\]

\[
= \tilde{z}_{A3} M + \tilde{z}_{A1} M \quad \text{if} \quad \begin{cases} u (\cdot, q_1) \in [u (\cdot, q_2) - \xi; u (\cdot, q_2) + \xi] \\ u (\cdot, q_1) \in [u (\cdot, q_3) - \tau; u (\cdot, q_3) + \tau] \\ \end{cases}
\]

Note that in searching for the best response to the quality choice of its rivals, university \( j = 1 \) can restrict itself to the interval \( [u (\cdot, q_3) - \tau; u (\cdot, q_3) + \tau] \) and \( [u (\cdot, q_2) - \xi; u (\cdot, q_2) + \xi] \). The maximisation problem of university \( j = 1 \) considers only the last scenario described in (51). Since universities \( j = 1,\) and \( j = 2 \) are symmetric mutatis mutandis the same scenarios are faced by university \( j = 2 \). For university \( j = 3 \) the only relevant scenario to consider is the following:

\[
X_3 (q_1, q_2, q_3) = (1 - \tilde{z}_{A3}) M + (1 - \tilde{z}_{A2}) M \quad \text{if} \quad \begin{cases} u (\cdot, q_3) \in [u (\cdot, q_1) - \tau; u (\cdot, q_1) + \tau] \\ u (\cdot, q_3) \in [u (\cdot, q_2) - \tau; u (\cdot, q_2) + \tau] \\ \end{cases}
\]

where, in equilibrium, \( \tilde{z}_{A3} = \tilde{z}_{A2} \).

The maximisation problems are given as follows:

\[
\max_{q_1} (S - C(q_1)) \left[ \frac{M u (\cdot, q_1) - u (\cdot, \bar{q}_2) + \xi}{2\xi} + \frac{M u (\cdot, q_1) - u (\cdot, \bar{q}_3) + \tau}{2\tau} \right] + \alpha q_1 \quad \text{s.t.} \quad \begin{cases} u (\cdot, q_1) \in [u (\cdot, \bar{q}_2) - \xi; u (\cdot, \bar{q}_2) + \xi] \\ u (\cdot, q_1) \in [u (\cdot, \bar{q}_3) - \tau; u (\cdot, \bar{q}_3) + \tau] \\ \end{cases}
\]

\[
\max_{q_2} (S - C(q_2)) \left[ \frac{M u (\cdot, q_2) - u (\cdot, \bar{q}_1) + \xi}{2\xi} + \frac{M u (\cdot, q_2) - u (\cdot, \bar{q}_3) + \tau}{2\tau} \right] + \alpha q_2 \quad \text{s.t.} \quad \begin{cases} u (\cdot, q_2) \in [u (\cdot, \bar{q}_1) - \xi; u (\cdot, \bar{q}_1) + \xi] \\ u (\cdot, q_2) \in [u (\cdot, \bar{q}_3) - \tau; u (\cdot, \bar{q}_3) + \tau] \\ \end{cases}
\]

\[
\max_{q_3} (S - C(q_3)) \frac{M}{2\tau} [u (\cdot, q_3) - u (\cdot, \bar{q}_1) + u (\cdot, q_3) - u (\cdot, \bar{q}_2) + 2\tau] + \alpha q_3 \quad \text{s.t.} \quad \begin{cases} u (\cdot, q_3) \in [u (\cdot, \bar{q}_1) - \tau; u (\cdot, \bar{q}_1) + \tau] \\ u (\cdot, q_3) \in [u (\cdot, \bar{q}_2) - \tau; u (\cdot, \bar{q}_2) + \tau] \\ \end{cases}
\]

where \( \bar{q}_{-j} \) indicates the rivals’ choice in each \( q_j \) best response search process. We indicate with \( q^*_j \) the best response to \( \bar{q}_{-j} \).
we derive the following expressions: 

\[ \text{maximum distance \( j\) individuals located in the interval } \tilde{\text{z}} \text{ in university one and three is} \]

and we have only separating equilibria. Intuitively, the location of an individual who is indifferent between studying 

\[ q \]

demand for education collapses to zero. Consequently we must have 

\[ u \]

given the assumptions on first and second order derivatives of \( u(\cdot) \) and \( C(\cdot) \). The LHS can be negative only if:

\[ (S - C(q^*_3)) \frac{M}{\tau} u_{q^*_3} < (S - C(\hat{q})) \left[ \frac{M}{2\xi} + \frac{M}{2\tau} \right] u_{\hat{q}}. \]

Since:

\[ (S - C(q^*_3)) > (S - C(\hat{q})) \]

and

\[ \frac{M}{\tau} > \frac{M}{2\xi} + \frac{M}{2\tau} \]

we should have that \( u_{q^*_3} < u_{\hat{q}} \). Since \( q^*_3 < \hat{q} \) we have that \( u_{q^*_3} > u_{\hat{q}} \) and we have a contradiction.

Consider now \( \sigma^*_M \). As in the previous cases if quality is set equal to zero no one invests in human capital and the demand for education collapses to zero. Consequently we must have \( q^*_3 > \hat{q} > 0 \). Single cross property always holds and we have only separating equilibria. Intuitively, the location of an individual who is indifferent between studying in university one and three is \( \tilde{z}_{A3} < 1/2 \) due to quality differences between universities. In fact, in order to choose the low quality university (\( j = 1 \)) the individual should be located closer to it. The same holds for \( \tilde{z}_{A2} \). As a consequence, individuals located in the interval \( (\tilde{z}_{A3}, 1/2) \) and \( (\tilde{z}_{A2}, 1/2) \) choose to study in the university that is located at the maximum distance (\( j = 3 \)) where a higher quality is supplied. \( \square \)

**Proof of (29).** In order to prove (29) we derive the following expressions:

\[ \frac{\partial q^*_3}{\partial S} = -\frac{u_{q^*_3} M}{\frac{M}{\tau}} \]

\[ = -\frac{u_{q^*_3} M}{\frac{M}{\tau}} \left[ u_{q^*_3} - u_{\hat{q}} + \tau \right] - u_{q^*_3} \frac{M}{\tau} C_{q^*_3} + u_{q^*_3} \left[ \frac{M}{\tau} (S - C(q^*_3)) \right] - C_{q^*_3} u_{q^*_3} \frac{M}{\tau} \]
\[ \frac{\partial \hat{q}}{\partial S} = -\frac{u_{\hat{q}} \left[ \frac{M}{2\xi} + \frac{M}{2\tau} \right]}{-C_{\hat{q}\hat{q}} \left[ M + \frac{M}{2\tau} \left[ u_{\hat{q}} - u_{q_{\hat{q}}} \right] \right] - u_{\hat{q}} \frac{M}{\tau} C_{\hat{q}} + u_{\hat{q}} \frac{M}{2\xi} + \frac{M}{2\tau} \left( S - C \left( \hat{q} \right) \right) - C_{\hat{q}} u_{\hat{q}} \left[ \frac{M}{2\xi} + \frac{M}{2\tau} \right]. \]  

(64)

Ignoring high order derivatives we may show that:

\[ \frac{u_{\hat{q}} \frac{M}{\tau} C_{\hat{q}}}{u_{\hat{q}} \left[ \frac{M}{2\xi} + \frac{M}{2\tau} \right]} < \frac{u_{\hat{q}} \frac{2M}{\tau} C_{\hat{q}}}{u_{\hat{q}} \left[ \frac{M}{2\xi} + \frac{M}{2\tau} \right] C_{\hat{q}}}. \]  

(65)

This implies that:

\[ \frac{\partial q_{\hat{q}}^*}{\partial S} > \frac{\partial \hat{q}}{\partial S}. \quad \Box \]  

(66)

**Proof of (30).** In order to prove (30) we compare \( \frac{\partial \hat{q}}{\partial S} \) with \( \frac{\partial \hat{q}}{\partial \hat{S}} \).

We derive the following expressions:

\[ \frac{\partial \hat{q}}{\partial S} = -\frac{u_{\hat{q}} \left[ \frac{M}{2\xi} + \frac{M}{2\tau} \right]}{-C_{\hat{q}\hat{q}} \left[ M + \frac{M}{2\tau} \left[ u_{\hat{q}} - u_{q_{\hat{q}}} \right] \right] - u_{\hat{q}} \frac{M}{\tau} C_{\hat{q}} + u_{\hat{q}} \frac{M}{2\xi} + \frac{M}{2\tau} \left( S - C \left( \hat{q} \right) \right) - C_{\hat{q}} u_{\hat{q}} \left[ \frac{M}{2\xi} + \frac{M}{2\tau} \right]} \]

and

\[ \frac{\partial \hat{q}}{\partial \xi} = \frac{u_{\hat{q}} \left( S - C \left( \hat{q} \right) \right) \frac{M}{2\xi}}{-C_{\hat{q}\hat{q}} \left[ M + \frac{M}{2\tau} \left[ u_{\hat{q}} - u_{q_{\hat{q}}} \right] \right] - u_{\hat{q}} \frac{M}{\tau} C_{\hat{q}} + u_{\hat{q}} \frac{M}{2\xi} + \frac{M}{2\tau} \left( S - C \left( \hat{q} \right) \right) - C_{\hat{q}} u_{\hat{q}} \left[ \frac{M}{2\xi} + \frac{M}{2\tau} \right]}. \]

When:

\[ \frac{\xi^2 + \xi \tau - \tau \left( S - C \left( \hat{q} \right) \right)}{2\xi^2 \tau} > 0 \]

i.e.

\[ \left( S - C \left( \hat{q} \right) \right) < \frac{\xi^2 + \xi \tau}{\tau} \]  

(67)

we have:

\[ \frac{\partial \hat{q}}{\partial S} > \frac{\partial \hat{q}}{\partial \xi}. \quad \Box \]  

(68)

**References**


