WCLTA 2010

Overeducation and unemployment spells’ duration

Giuseppe Rose a,b *, Patrizia Ordine b

a Birkbeck College, London WC1E 7HX, United Kingdom
b University of Calabria, Arcavacata di Rende 87036, Italy

Abstract

This work proposes an investigation of educational mismatch focusing on the study of individuals' unemployment spells. We present evidence for Italy, showing that overeducation is basically an occurrence that follows long periods of unemployment and it is strictly related to individuals' innate ability. We build up a matching model coupled with endogenous educational and technological choices and we consider the role of university selectivity and individual innate ability in determining unemployment duration.

© 2010 Published by Elsevier Ltd.

Keywords: Overeducation; Unemployment Spells; Hazard Functions; Matching Models, University Selectivity.

1. Introduction

This work proposes an investigation of educational mismatch focusing on the study of individuals' unemployment spells. In the economic literature, educational mismatch, in the form of overeducation, describes the extent to which individuals possess a level of education in excess of that required in their specific job. The phenomenon significantly affects graduate workers. In particular, in the UK and in the United States it seems to involve a number of workers that ranges between the 17% and the 42% of the whole employed graduate labor force, while in Italy the share of overeducated workers is around 39% (McGuinness, 2006). The goal of this paper is twofold. Firstly, we present some evidence showing that overeducation is basically an occurrence that follows long unemployment spells. By using competing risk models, we show that the observed individuals' matching differences are due to heterogeneity in terms of their innate ability. Secondly, we build up a general equilibrium model able to fit our empirical findings.

The interest on the overeducation phenomenon has been, and it is, very high among labor economists. From an empirical perspective, almost all the existing works concentrate their efforts in assessing the relevance of the phenomenon in determining wages broadening the human capital framework (Bauer, 2000; McGuinness & Bennett, 2007). To the best of our knowledge, there are no works trying to analyze educational mismatch in terms of unemployment spells. In this context, by pointing out the peculiar unemployment history of workers that end up in positions that do not require their skills, our empirical analysis brings important new insights into the debate concerning the explanations for this phenomenon. From a theoretical point of view, a few papers investigate the
occurrence of educational mismatch leading to various explanations of the existing empirical evidence. Andolfatto & Smith (2001) present a model where educational mismatch arises during steady-state transitions due to technological change. The authors argue that "deadwoods" educated workers re-allocate within the unskilled sector while young educated workers enter the skilled sector directly. While their results explain the occurrence of overeducation, they consider overeducated individuals as old workers with a well-matched occupational history, which appears to be at odds with the results of many empirical studies. Moreover the authors assume firms technological decisions are not affected by individuals' educational choices. However, it should be recognized that in order to determine the extent of the phenomenon, the strategic complementarity between educational and technological choices might be relevant. Charlot & Decreuse (2005) and Moen (1999) go in this direction by presenting models where overeducation arises since self-selection into education is inefficient. We enter this literature by presenting a theoretical framework where: i) Education gives rise, besides monetary costs, to costs related to individual innate talent; ii) there is strategic complementarity between individuals' educational choice and firms' technological decision; iii) graduates may be employed in the undergraduate sector. In this setting, we show that an increase in the share of graduates may induce a rise in the technological endowment of firms via a tightness effect: The larger the pool of graduates the greater the probability that a firm fills graduate-complementary vacancies. However, this relation is concave since the presence of a too large share of graduates in the labor force implies a composition effect: the larger the share of graduates the lower their expected ability. In terms of policy, our model offers a different perspective with respect to existing studies relying on inefficient self-selection into education as the main cause at the root of educational mismatch/overeducation. Many papers enter the debate concerning the relevance of considering students heterogeneity in terms of ability when evaluating the effect of policies that promote college attendance, suggesting that overeducation may be reduced only by raising tuition fees (Charlot & Decreuse, 2005; Hendel et al., 2005). Indeed, we believe that public education should be evaluated in the light of the fact that liquidity constraints do exist. We believe that a useful policy instrument may reside in setting the appropriate level of selectivity of the higher education system since it shapes the correlation between educational choices and individual ability and it might eventually boost the creation of graduate-complementary job positions reducing educational mismatch and rising the overall expected output. The outline of the article is as follows. Section 2 contains the descriptive statistics and our empirical investigation. In Section 3 we set up the theoretical model, while in Section 4 we discuss our policy implications. Some concluding remarks are presented in Section 5.

2. The Empirical Investigation

2.1. Data and Duration Patterns

As part of the motivation of our study, and in order to provide some facts describing the pattern of unemployment duration of individuals with different characteristics, we present some empirical evidence using data from a survey carried out by the Italian Institute for Vocational Training of Workers (ISFOL) containing information on the labor market outcomes of a sample of 8156 workers recorded in 2006. This survey provides information on workers' status (employed/unemployed) and on the length of their unemployment spells. The data set records the time needed to obtain the present job (in months) or the censored time for those still unemployed at the time of the interview. Only uninterrupted spells of unemployment are considered. In Table 1 we describe in details our variables. This data set provides indications to determine if workers are in job positions where the competencies acquired at school are effectively needed and allows us to evaluate the extent of educational mismatch and the unemployment spell duration associated to the characteristics of the job match. Some preliminary aspects of the duration pattern of unemployed spells may be gathered by inspecting the Kaplan-Meier estimated hazard functions. These functions reflect the percentage of spells ending into employment during time. As reported in Figure 1, the general pattern of the hazard functions is non-linear with an increasing exit rate at the beginning of the spell which declines with the elapsed time into unemployment. In Figure 1, panel a) depicts the empirical hazard functions for individuals with different education levels. The hazard function of graduates lies above that of their less educated counterpart. In general, this reflects a faster transition out of unemployment for more educated people. The same exercise has been done for individuals living in areas with a different level of economic development i.e., the less developed South of
Figure 1: Kaplan-Meier hazard functions estimates

- a) Estimates by education
- b) Estimates by education and region
- c) Estimates when the failure is right match
- d) Estimates when the failure is wrong match
- e) Estimates by education and region. Failure: right match
- f) Estimates by education and region. Failure: wrong match

Figure 1: Kaplan-Meier hazard functions estimates
Italy in comparison with the rest of the country. Interestingly, we find that a graduate from the South of Italy has a higher probability of finding a job than an undergraduate from the North only after about two years of unemployment spell duration (panel b). Further, we compare people with different education levels in terms of their unemployment spell duration to obtain a job where their competencies are effectively used. In panel c) we show the hazard function of individuals that report transitions toward occupations congruent with their education level. In this case, the hazard rate of graduates lies far above that of undergraduates. In contrast, panel d) refers to individuals who terminate their unemployment spells in a wrong match. These figures highlight that although unemployment duration is higher for individuals that exit toward bad occupations, differences between individuals with different education levels are not too pronounced and surprisingly the curve for graduates lies below that of undergraduates. This would imply that when graduates are overeducated they have a spell length higher than that of their undergraduate counterpart. This aspect is even more evident in the South of Italy, as reported in panel e) and f).

Although informative, this preliminary analysis based on the empirical estimates of the hazard functions does not control for possible factors at work in shaping duration dependence. In particular, the observed differences between individuals with right and wrong matches might be due to differences in their characteristics in terms of skills, innate ability or family background. These variables may affect the patterns of hazard probabilities and the duration dependence that we see in the data. In order to take these differences into account we evaluate the transitions out of unemployment by estimating proportional hazard competing risk models as discussed in the next paragraph.

### Table 1: Description of variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Employed</td>
<td>Variable indicating if the respondent is employed at the time of the interview.</td>
</tr>
<tr>
<td>Unemployed</td>
<td>Variable indicating if the respondent is unemployed at the time of the interview.</td>
</tr>
<tr>
<td>Unemployment Spells</td>
<td>Variable indicating the length of unemployment spell to find the present job if the respondent is employed or the length of unemployment spell since starting the job search process if the respondent is unemployed at the time of the interview. Duration is measured in months.</td>
</tr>
<tr>
<td>Female</td>
<td>Dummy variable indicating the respondent's sex, Female=1, 0 otherwise.</td>
</tr>
<tr>
<td>Age</td>
<td>Respondent's age at the interview.</td>
</tr>
<tr>
<td>Married</td>
<td>Dummy variable indicating if the respondent is married, Married=1, 0 otherwise.</td>
</tr>
<tr>
<td>South</td>
<td>Dummy variable indicating if the respondent is resident in the South of Italy, South=1, 0 otherwise.</td>
</tr>
<tr>
<td>Father education</td>
<td>Highest grade of years of school completed by respondent’s father.</td>
</tr>
<tr>
<td>Education: high-school</td>
<td>Dummy variable indicating if the respondent owns a high-school degree. High-School=1, 0 otherwise.</td>
</tr>
<tr>
<td>Education: graduate</td>
<td>Dummy variable indicating if the respondent is a graduate. Graduate=1, 0 otherwise.</td>
</tr>
<tr>
<td>Degree subject</td>
<td>A vector of 4 0-1 dummy variables indicating degree subjects: 1) Science=1 if mathematics, science, chemistry, pharmacy, geo-biology, agrarian and engineering; 2) Medicine=1 if medicine; 3) Econ.&amp;Law=1 if political science, economics, statistics, law; 4) Humanities=1 if humanities, linguistic, teaching, psychology.</td>
</tr>
<tr>
<td>Secondary sch. leaving grade</td>
<td>Dummy variable for final score at secondary school SS Score=1 if secondary school final score is medium-high; SS Score=0 otherwise.</td>
</tr>
<tr>
<td>High sch. leaving grade</td>
<td>Dummy variable for final score at high school HS Score=1 if high school final score&gt;55/60 or high school final score=90/100; HS Score=0 otherwise.</td>
</tr>
<tr>
<td>University leaving grade</td>
<td>Dummy variable for final score at university. University Score=1 if university score≥110/110; University Score=0 otherwise.</td>
</tr>
<tr>
<td>Degree on time</td>
<td>Dummy variable indicating if the degree is completed on time (adjusted for course duration), Degree on time=1, 0 otherwise.</td>
</tr>
<tr>
<td>Overeducation</td>
<td>Dummy variable for the answer to the question: &quot;Is your degree a required qualification for your job?&quot; Overeducation=1 if the answer is not, 0 otherwise.</td>
</tr>
</tbody>
</table>

#### 2.2 The Estimates

We undertake the empirical analysis using a competing risks model (CRM) where we assume that each subject has an underlying failure time that may be of \( m \) different types given by the set \( J = \{1, \ldots, m\} \). We assume a competing risks formulation in which independent competing risks determine the duration of unemployment. In our case we suppose that unemployment may terminate by exiting toward employment with a right or a wrong educational match. So in our case \( m = 2 \). Table 2 reports the results of the estimated models. We implement a competing risk analysis where we distinguish two separate destination states: right match job and wrong match job. For comparison, we also report the estimates referred to transition toward employment without separating the
destination states. In Table 2, Risk1 refers to exit towards employment, while Risk2 and Risk3 refer to exits towards a right and a wrong match respectively.

<table>
<thead>
<tr>
<th>Table 2: Hazard Rate. Competing and Independent Risk estimates of Cox Model</th>
<th>Risk1</th>
<th>Risk2</th>
<th>Risk3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age</td>
<td>-0.026***</td>
<td>-0.032***</td>
<td>-0.023***</td>
</tr>
<tr>
<td>(0.002)</td>
<td>(0.004)</td>
<td>(0.003)</td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>-0.098***</td>
<td>-0.122***</td>
<td>-0.085***</td>
</tr>
<tr>
<td>(0.046)</td>
<td>(0.069)</td>
<td>(0.061)</td>
<td></td>
</tr>
<tr>
<td>Graduate</td>
<td>0.452***</td>
<td>1.174***</td>
<td>-0.335</td>
</tr>
<tr>
<td>(0.075)</td>
<td>(0.098)</td>
<td>(0.129)</td>
<td></td>
</tr>
<tr>
<td>Graduate*South</td>
<td>-0.075***</td>
<td>-0.024</td>
<td>-0.388***</td>
</tr>
<tr>
<td>(0.012)</td>
<td>(0.142)</td>
<td>(0.199)</td>
<td></td>
</tr>
<tr>
<td>University leaving grade</td>
<td>0.034</td>
<td>0.103</td>
<td>-0.165</td>
</tr>
<tr>
<td>(0.089)</td>
<td>(0.106)</td>
<td>(0.164)</td>
<td></td>
</tr>
<tr>
<td>High sch. leaving grade</td>
<td>0.251***</td>
<td>0.662***</td>
<td>-0.043</td>
</tr>
<tr>
<td>(0.078)</td>
<td>(0.113)</td>
<td>(0.110)</td>
<td></td>
</tr>
<tr>
<td>Secondary sch. leaving grade</td>
<td>0.089***</td>
<td>0.316***</td>
<td>-0.049</td>
</tr>
<tr>
<td>(0.051)</td>
<td>(0.087)</td>
<td>(0.062)</td>
<td></td>
</tr>
<tr>
<td>Degree in Humanities</td>
<td>-0.319***</td>
<td>-0.570***</td>
<td>0.185</td>
</tr>
<tr>
<td>(0.096)</td>
<td>(0.121)</td>
<td>(0.162)</td>
<td></td>
</tr>
<tr>
<td>Degree in Science</td>
<td>-0.094</td>
<td>-0.147</td>
<td>0.096</td>
</tr>
<tr>
<td>(0.133)</td>
<td>(0.162)</td>
<td>(0.236)</td>
<td></td>
</tr>
<tr>
<td>Degree in Medicine</td>
<td>0.429***</td>
<td>0.653***</td>
<td>-1.728</td>
</tr>
<tr>
<td>(0.177)</td>
<td>(0.183)</td>
<td>(1.006)</td>
<td></td>
</tr>
<tr>
<td>South</td>
<td>-0.547***</td>
<td>-0.520***</td>
<td>-0.560***</td>
</tr>
<tr>
<td>(0.055)</td>
<td>(0.093)</td>
<td>(0.068)</td>
<td></td>
</tr>
<tr>
<td>Father education</td>
<td>0.287***</td>
<td>0.256***</td>
<td>0.320***</td>
</tr>
<tr>
<td>(0.081)</td>
<td>(0.107)</td>
<td>(0.125)</td>
<td></td>
</tr>
<tr>
<td>Married</td>
<td>0.024</td>
<td>-0.093</td>
<td>0.108</td>
</tr>
<tr>
<td>(0.058)</td>
<td>(0.093)</td>
<td>(0.076)</td>
<td></td>
</tr>
</tbody>
</table>

Notes: i) Risk1 is failure toward employment; Risk2 is failure toward right match; Risk3 is failure toward wrong match; ii) Standard Error in parenthesis; iii)**1% significant, **5% significant, *10% significant.

We introduce explanatory variables which are likely to influence the educational choices as well as the availability and the characteristics of jobs. Hence, our empirical analysis puts special emphasis on the role played by two order of pushing factors in unemployment outflows: personal ability, and local economic characteristics. In our econometric exercise we notice that, when separating by exit type transitions, graduates have an extremely significant advantage in finding a job congruent with their competencies with respect to their undergraduate peers. Conversely, when transitions towards mismatched occupations are considered, having a university degree does not provide a preferential track. Further, graduates located in less developed areas of the country are significantly penalized in exiting the unemployment pool if they compete with undergraduate job searchers. In our model, individual ability is represented, in line with the existing literature, by pre-college and pre-high-school leaving grades (Cappellari & Lucifora, 2010). Interestingly, high and secondary school leaving grades are significant in determining transitions towards well matched occupations but they have no role in speeding up unemployment outflows towards wrong matched positions. In line with some previous results (among others see Ordine & Rose, 2009), the university leaving grade is not significant in determining labor market transitions. We know that university grades may be biased by the so called "grade inflation" phenomenon, and in Italy many universities tend to inflate their final marks. Family background may also impact the individuals’ choices in many ways (Checchi, 2003). Our results point out that the father education is an important variable in determining employment transitions. Even for mismatched workers, employment opportunities are significantly influenced by family background and this may be in support of a "family network" hypothesis. We are also aware of the existing sharp South-North labor market differences in Italy (Brunello et al., 2001). The situation is even more severe when considering youth unemployment. Our findings reveal that the process of job finding is more difficult and long lasting in the South of Italy. An implication of our results is that mismatched individuals wait for a job position consistently more that their
well matched peers and this labor market segmentation is relevant especially for graduate individuals. As far as we want to enter the debate on overeducation as a "stepping stone" towards well matched occupation, and as a temporary occurrence in the individual's working life, we must really be very cautious in clasping this hypothesis.

3. The Theoretical Model

In this section we figure out a possible explanation of the empirical evidence presented so far. In particular we aim at analyzing theoretical underpinnings linking the duration of unemployment to individuals' ability conditional to job match characteristics. Consider an economy characterized by a continuum of risk-neutral individuals and firms, who match in the labor market following the lines set out by Pissarides (1985). Without loss of generality we normalize to 1 the continuum of both individuals and firms and we assume the mass of agents remains constant over time. Differently from standard matching models, we assume that i) the matching technology follows an urn-ball model as in Moen (1999); ii) before entering the job-market, firms and individuals have to make a technological and educational choice: In particular, individuals/firms decide whether they want to enter the graduate/high-tech or the under-graduate/low-tech market respectively. We assume that individuals are heterogeneous with respect to their innate talent (ability), which determines their productivity on the job and we consider the case in which innate ability is inversely related to the cost (effort) of acquiring education. On the demand side, each firm can post a limited number of vacancies, normalized to 1, and it sets production on the basis of a technological choice \( T \). In particular, a firm can choose the sector where posting a vacancy i.e., it can choose to operate either within the high- or the low-technological sector. In order to simplify notation, from now on we refer to graduate versus undergraduate choice for both firms and individuals. However the reader should keep in mind that individuals make an educational choice while firms make a technological choice. Once the educational/technological choices have been made, the pure matching-process starts. We assume that undergraduate individuals can only be matched with low-tech firms, while graduates can search in both high-tech and low-tech markets. Using this setting, we demonstrate how overeducation is a phenomenon that might characterize standard matching models i.e., it might be attributable to a simple problem of frictional search in the labor market, arising even when self-selection into education is efficient. However, in this case we should not observe any relationship between innate ability and labor market outcomes. In contrast, such a relation may arise when educational choices are socially inefficient. In this case, we highlight the importance of considering, among other policy instruments, the selectivity intensity of the university system in order to re-establish social efficiency and to reduce mismatch.

3.1 Individuals

Consider a continuum of individuals of mass 1. We assume individuals characterized by heterogeneous innate ability \( \theta \). Innate ability is distributed according to a continuous and strictly increasing cumulative distribution \( \Gamma(\cdot) \), whose density function is \( \gamma(\cdot) \), over a support \([\theta, \overline{\theta}]\) where \( 1 \leq \theta < \overline{\theta} \). We indicate with \( e = \{g, ug\} \) the educational choice made by individuals in order to maximize their expected discounted utility (\( g \) stands for graduate while \( ug \) stands for undergraduate). For simplicity we assume that an individual has no income if unemployed. As a consequence, once the educational choice has been made, in each period the individual's utility function \( W(e) \) is given by:

\[
W(e) = \begin{cases} 
0 & \text{if unemployed} \\
\bar{w}_{ug} & \text{if employed in a } ug \text{ position} \\
\bar{w}_{g} & \text{if employed in a } g \text{ position}
\end{cases}
\]  

(1)

where \( \bar{w}_{g} \) and \( \bar{w}_{ug} \) indicate wages for graduate and undergraduate positions respectively. The cost of acquiring education \( ug \) is normalized to zero while, when individuals decide to acquire education \( g \), on top of monetary costs, they have to sustain a cost \( c(\theta) > 0 \), with \( \partial c / \partial \theta < 0 \), related to their innate ability. We assume that monetary costs
are the same for all the individuals, while the effort required to achieve a degree qualification is determined by personal ability. From now on, we refer to $|\dot{c}/\dot{\theta}|$ as a measure of the selectivity of the higher education sector. In words, the more the cost of education rises when ability decreases the more selective may be considered the higher education sector.

### 3.2 Firms

Consider a continuum of firms of mass 1. We indicate with $T = \{g, ug\}$ firm's investment in graduate (high-tech) and undergraduate (low-tech) vacancy respectively. The cost of entering the $g$ sector is given by $\delta > 0$. The cost of entering the $ug$ sector is normalized to zero. We crucially assume that firms are heterogeneous with respect to the cost they have to sustain in order to enter the $g$ sector. In fact, in the growth theory literature, the cost of advanced technology has been considered typically related to the actual firm's technological endowment. The closer is a firm to the technological frontier the lower is the cost it needs to sustain in order to update its technology. The concept of technological frontier has been introduced by Nelson and Phelps (1966). In our case, we assume that firms are distributed with a continuous and strictly increasing cumulative distribution $\Phi(\cdot)$ whose density function is $\phi(\cdot)$, over a support $[\delta, \delta']$ where $\delta < 1 < \delta'$. Following Acemoglu (1997), the production function is given by:

$$ y = y(e, T, \theta) = \tilde{y}[\theta]^{\{e=g and T=g\} \{1\} e=ug or T=ug] $$  

where $\tilde{y}$ is a constant. Relation (2) indicates that there is homogeneity in the undergraduate sector i.e., when individuals work in the $ug$ sector they produce an output $\tilde{y}$ independently on their ability and education. On the other hand, graduate technologies are complementary only to graduate workers and the intensity of such complementarity is given by individual's innate ability. Finally, we indicate with $Q$ the cost of maintaining a vacancy for all $T$, and we assume that vacancies can be destroyed at no cost. Once the technological decision has been made, each firm realizes a profit $\Pi(T)$ as follows

$$ \Pi(T) = \begin{cases} 
- Q & \text{if unfilled vacancy} \\
\tilde{y} - w_{ug} - Q & \text{if filled ug vacancy} \\
\tilde{y} - w_{g} - Q & \text{if filled g vacancy} 
\end{cases} $$  

### 3.3 Interaction process and Bellman equations

The process consists in the following two stages. Firstly, individuals and firms conditional on their own type (ability and distance to the frontier) decide simultaneously the sector they want to enter i.e., they choose between graduate and undergraduate sectors. Once the educational/technological choices have been made, individuals and firms enter the labor market as unemployed and with unfilled vacancies respectively, and then the matching process starts. As a consequence, the relative markets' tightness in the present model is endogenous. Once a match is realized, we assume a standard Nash-bargaining (axiomatic) solution for wage determination. In order to solve the model we proceed backward: Firstly we evaluate the actual expected value for individuals and firms using a standard dynamic programming method; secondly, by using the obtained results we proceed to find the Bayesian Nash Equilibrium (BNE) of the simultaneous first-stage game in which agents decide educational level and technological contents. We will show that the BNE can be efficient in terms of the total expected output of the economy conditional upon the appropriate level of selectivity $|\dot{c}/\dot{\theta}|$ in the higher education sector. Intuitively, when the mass of graduates increases, firms raise their investment in graduate-complementary positions since the probability of filling a vacancy increases (tightness effect). However, when the number of graduates rises, the average innate ability of the graduate labor force decreases. As a consequence, there exists a cutoff level in the mass of graduates above which firms find optimal to reduce their investment in graduate-complementary jobs (composition effect). In this respect, policy designed to promote college attendance may induce an improvement in the overall efficiency only when the tightness effect dominates the composition effect. However, we prove that
under some reasonable assumptions, only when the composition effect dominates, we should observe unemployment spells’ duration of graduates in right positions being inversely related to their innate ability.

3.5 The matching functions

We indicate with $E^e$ the employment level per educational groups. $E^e$ indicates the over-time variation of employment levels with $\dot{E}^e = H^e - bE^e$ where $b > 0$ is the exogenous quitting rate and $H^e$ is the number of hirings per educational level. Since $H^g$ indicates the overall number of hirings for graduates, and since graduates can be matched in both sectors, we have that $H^g = H^g_R + H^g_O$ where $H^g_R$ indicates the number of graduates hired in the graduate sector ($R$ stands for "right match"), and $H^g_O$ indicates the number of graduates employed in undergraduate positions ($O$ stands for "overeducated"). By indicating with $U^e$ the number of unemployed workers with education $e$ and with $V_T$ the number of posted vacancies per sector $T$, we can write the hiring functions as follows:

$$H^g = K \gamma^\eta U^g_\omega \eta$$

$$H^g_O = K \gamma^\eta U^g_\omega \eta$$

$$H^g_R = a^g(t) U^g_\omega$$

Some clarifications are in order. On one side, since worker's ability is irrelevant in $ug$ jobs there is no room for worker's ranking in this sector and, as a consequence, eqs. (4) and (5) are assumed to be standard (Cobb-Douglas CRTS) matching functions ($K$ is a constant and $0 < \eta < 1$). On the other side, since ability matters in the $g$ sector, in eq. (6) the matching process is described, in line with the existing literature, as an urn-ball model where workers send applications and firms select the best candidates they receive. The urn-ball process is a convenient tool for describing the labor market when workers are heterogeneous and in our model it makes possible to specify a graduate's exit rate from unemployment toward a right-match as a function of his characteristics. In order to understand where eq. (6) comes from, indicate with $V_T$ the tightness of the graduate sector. Indicate with $\gamma^\omega T V_T R^g$ the probability that an unemployed graduate with ability $\gamma^\omega T$ receives a job offer from a $g$ firm. The probability $a^g_R(t, \theta)$ is given by:

$$a^g_R(t, \theta) = \exp\left(\frac{1 - \Gamma(\theta)}{t}\right) \gamma(\theta)$$

with $\frac{\partial a^R_g}{\partial \theta} > 0$. By integrating $a^g_R(t, \theta)$ over $[\theta^*, \overline{\theta}]$, whose lower bound $\theta^*$ is the threshold-ability determined ex-ante in the BNE, we obtain the overall probability of being hired in a $g$ position, called $a^g_R(t)$ with:

$$a^g_R(t) = 1 - \exp\left(-\frac{1}{t}\right)$$

and this explains eq. (6). Now, we can set the following formalism:

$$a^g_{ug} = H^g_{ug} / (U^g_{ug} + U^g_g) \rightarrow \text{prob. that an undergraduate is employed;}$$

$$a^g_O = H^g_{ug} / (U^g_{ug} + U^g_g) \rightarrow \text{prob. that a graduate is employed in an undergraduate position;}$$

$$a^g_R(t, \theta) \rightarrow \text{prob. that a graduate with ability } \theta \text{ is employed in a right position;}$$

By indicating with $\alpha_T$ the probability that a $T$ vacancy is filled and using an argument similar to that made when explaining eq. (6), we can write the following expressions:

$$a^g_{T}(t, \theta) = \exp\left(-\left(1 - \Gamma(\theta)\right)t\right) \gamma(\theta) t \rightarrow \text{prob. that a } g \text{ vacancy is filled with a type-}\theta \text{ worker;}$$
\( \alpha_g(t) = \int_{\theta}^\infty \alpha_g(t, \theta) d\theta = 1 - \exp(-t) \to \text{prob. that a } g \text{ vacancy is filled;} \\
\alpha_{ug} = (H_{ug} + H_g^O) / V_{ug} \to \text{prob. that an undergraduate vacancy is filled.} \\

Finally, by indicating with \( r > 0 \) the intertemporal interest rate and considering the notation for the actual expected values as indicated in Table 3, we can write down the value functions as follows:

**3.6 Solving the model**

The use of Nash-bargaining solution imposes that when a match is realized, the generated surpluses for firm and worker must be equal. Hence by imposing \( V_{ug}^E - V_{ug}^U = V_{ug}^E - V_{ug}^U \) we obtain the following expression for the undergraduate workers’ wage:

\[
w_{ug} = \frac{-\bar{y}(r + b + \alpha_{ug})}{\alpha_{ug} + \alpha_{ug} + 2r + 2b} \]

This expression, that should look familiar to the reader used with matching models, allows us to derive the actual expected value of workers that decide to acquire education \( ug \). Similarly, by imposing \( V_g^R - V_g^U = V_g^R - V_g^U \) and

| Table 3: Notation for actual expected values |
|------------------|------------------|
| **Firms**        | **Workers**      |
| \( V_{ug}^E \)   | \( V_{ug}^E \)   |
| \( V_{ug}^U \)   | \( V_{ug}^U \)   |
| \( V_{ug}^F \)   | \( V_{ug}^F \)   |
| \( V_{ug}^V \)   | \( V_{ug}^V \)   |
| \( V_g^R \)      | \( V_g^R \)      |
| \( V_g^O \)      | \( V_g^O \)      |
| \( V_g^V \)      | \( V_g^V \)      |
| \( V_g^U \)      | \( V_g^U \)      |

Notice that, apart from eq. (15) these are pretty standard value functions. Eq. (15) indicates that the value of being an unemployed graduates with ability \( \theta \) includes the probability of being employed in a graduate position (\( a^R(\theta) \)) and in an undergraduate position (\( a^O \)).
combining the relative value functions we obtain the following expression for the graduate workers' wage employed in a right position:

\[
\begin{align*}
\omega_g &= \gamma \frac{\theta(r + b + a_g^R) + a_g^R(r + b + \alpha_g)}{(a_g + a_{ug} + 2r + 2b)((r + b + a_g^R) + (2 + 2b + \alpha_g)(a_g^R + r + b))}
\end{align*}
\]

We now proceed (backward) in evaluating the simultaneous decision of individuals and firms concerning educational level and technological sector respectively. We assume that at this stage, agents ground their decisions considering the parameters \( a_g^R, \alpha_g, \) and \( a_{ug} \) as if they were at their steady-state value.

Individuals and firms have to decide, conditional on their ability and distance to the frontier, the level of education and the technology they want to acquire respectively. Once they make their choice, they enter the labor market as unemployed individuals and as firms with unfilled vacancies and then the matching process starts. Notice that, in this interaction process we look for pure strategies of firms and individuals that are best responses to each other, conditional to the type of players. As a consequence, the BNE gives us the shares of individuals and firms that acquire higher education and invest in graduate positions respectively and it provides a measure of the relative tightness of the two sectors.

**Proposition** It exists a unique BNE in which only individuals with ability \( \theta \geq \theta^* \) set \( e = g \) and only firms with \( \delta \leq \delta^* \) set \( T = g \).

**Proof** Consider the firm's choice first. Indicate with \( \gamma \) the probability (it is a density) that the individual sets \( e = g \). In this case, a firm invests in \( g \) position only if:

\[
\delta \leq \gamma \mathbb{E}[V_g | \theta] - V_{ug}^f
\]

Given our assumption on the monotonicity of \( \Phi(\cdot) \), we can indicate with \( \delta^* \) the cutoff level of distance to the frontier for which relation (20) is satisfied. Now, indicate with \( \phi \) the probability that a firm set \( T = g \) and consider the individual's educational choice. Setting \( e = g \) is optimal for an individual only if:

\[
c(\theta) \leq \phi V_{ug}^f
\]

Given our assumption on the monotonicity of \( \Gamma(\cdot) \) and given that \( \partial c / \partial \theta < 0 \) we can indicate with \( \theta^* \) the cutoff ability level for which relation (21) is satisfied. Hence, the following pair characterizes the only BNE:

\[
\{ \gamma = 1 - \Gamma(\theta^*); \phi = \Phi(\delta^*) \}
\]

In order to evaluate the equilibrium efficiency, we focus on the cutoff level \( \delta^* \) i.e., the share of firms that satisfies relation (20) as an equality. In fact, \( \delta^* \) indicates the share of firms that create graduate-complementary positions and, as it appears from eq. (2) the greater \( \delta^* \) the greater the expected output produced in the economy, since \( \text{ceteris paribus} \) a \( g \) firm realizes a greater output than its \( ug \) counterpart. By using eqs. (12) and (18) we can explicitate relation (20) as follows:

\[
\delta^*(\theta^*) = \Gamma(\theta^*)O + \frac{\gamma}{r} \left[ 1 - \Gamma(\theta^*) \right] \left[ \frac{FE[\theta | \theta > \theta^*]G - (r + b + a_{ug})a^O}{r} - a_{ug} \right]
\]

where \( F, G, \) and \( P \) summarize strictly positive constants. We can now evaluate how the share \( \delta^* \) changes in equilibrium as \( \theta^* \) changes. By differentiating eq. (22) with respect to \( \theta^* \) we can evaluate how a variation in the best response in terms of share of graduates (\( \theta^* \)) affects in equilibrium the share of firms investing in graduate positions. In is easy to check, that relation (22) is strictly concave and it presents a unique relative maximum. We indicate with \( \theta_{nm} \) the share of graduates that ceteris paribus maximizes firms' investments in graduate positions, i.e. the share of graduates maximizing \( \delta^* \) in eq. (22). It is important to note that only the appropriate selectivity level \( | \partial c / \partial \theta | \) can
ensure that \( \theta^{*}_{wm} \) is actually achieved in equilibrium. On the contrary, if this is not the case, we can have equilibria characterized by \( \theta^{*} \neq \theta^{*}_{wm} \). In particular, two possible scenarios that may arise:

Case a) - TIGHTNESS DOMINANCE: \( \theta^{*} < \theta^{*}_{wm} \). In this case a reduction in the selectivity level of the higher education sector \( (\partial c/\partial \theta | \downarrow) \) induces a rise in the share of graduates \( \theta^{*} \downarrow \) that in turn induces an increase in the share of firms investing in graduate positions. The overall expected output of the economy increases but the effect on overeducation cannot be uniquely defined since, in this scenario both demand and supply of graduates increase.

Case b) - COMPOSITION DOMINANCE: \( \theta^{*} > \theta^{*}_{wm} \). In this case an increase in the selectivity level of the higher education sector \( (\partial c/\partial \theta | \uparrow) \) induces a reduction in the share of graduates \( \theta^{*} \uparrow \) and this generates an increase in the share of firms investing in graduate positions. Differently from before, here the effect of such a policy induces an improvement in the overall expected output and, simultaneously, a reduction in overeducation since less graduates have more firms looking for them in the labor market.

4. Policy implications

According to our model, in order to suggest the right policy to maximize output and reduce overeducation, it is crucial to figure out the specific scenario that characterizes the presence of overeducation. To this end, assume that individuals' innate ability raises production up to a given level over which the output remains constant. This implies that from firms' perspective the distribution of individuals \( \Gamma(\cdot) \) has a mass point over which a rise in innate ability does not affect production. Now, assume that this mass point is in \( \theta^{*}_{wm} \), namely all individuals with ability \( \theta^{*} > \theta^{*}_{wm} \) have the same ability \( \theta^{*}_{wm} \). These assumptions imply that the economy is characterized by a mass of equally "high-productive" individuals and this mass is large enough so that below \( \theta^{*}_{wm} \), the tightness effect is always dominated by the composition effect. In this scenario, we can write the probability of being employed in a right position for a graduate with ability \( \theta \) as follows:

\[
a^{R}_{g}(t, \theta) = \begin{cases} 
\exp\left(\frac{-1-\Gamma(\theta)}{t}\right) \frac{\gamma(\theta)}{t} & \text{if } \theta^{*} \leq \theta < \theta^{*}_{wm} \\
\frac{\gamma(\theta^{*}_{wm})}{t} & \text{if } \theta \geq \theta^{*}_{wm}
\end{cases}
\]

where the bottom line of eq. (23) indicates that individuals with ability \( \theta \geq \theta^{*}_{wm} \) have a probability of being employed in a right position that is constant with respect to \( \theta \) since they are identical in terms of productivity. In contrast, the first line relates the exit rate of graduates towards right matched positions following the urn-ball process as in eq. (9) since when \( \theta^{*} \leq \theta < \theta^{*}_{wm} \) workers' ranking applies. Now, consider the case of "composition dominance". Here, in equilibrium graduates are heterogeneous in terms of their productivity since \( \theta^{*} < \theta^{*}_{wm} \). By approximating the average individual's unemployment spell terminated in a right match with \( S^{R}_{g} = 1/a^{R}_{g}(t, \theta) \); and by using eq. (23) we have that \( \partial S^{R}_{g}/\partial \theta < 0 \) with an absolute minimum in \( \theta^{*}_{wm} \). In words, in the presence of "composition dominance" we actually find differences in graduates' unemployment spells due to individuals' innate ability. When the selectivity level of the university system is too low, the unemployment spells' length terminating in a right match is inversely related to individuals' ability. According to the empirical evidence presented in Section 2, we may consider the scenario of "composition dominance" as that describing the possible causes at the root of educational mismatch that characterizes the Italian labor market. Indeed, the duration of unemployment terminated in a right match seems to be related to individuals' innate ability. In this scenario we point out the importance of considering, among other instruments, the selectivity level of the university system to implement policies targeted to reduce possible inefficiencies arising from self-selection into education. In this view, many papers (among others Charlot & Decreuse, 2005; Hendel et al., 2005) stress the importance of raising tuition fees in order to limit access
to higher education and to raise welfare. In our framework, we highlight the possibility of thinking about an alternative policy instrument, whose effectiveness in shaping the correlation between educational choices and individual ability should not depend on the presence of liquidity constraints or heterogeneity in the households' wealth.

5. Conclusions

In this study we undertake an analysis of overeducation through an investigation of individuals' unemployment spells. We start presenting some evidence on unemployment spells duration of Italian workers and we highlight that hazard rates of graduates are higher than those of undergraduates only for transitions towards occupations that require the competencies provided by the universities and this process is strictly related to innate ability. We build up a matching model coupled with endogenous educational and technological choices and we highlight the role of university selectivity in determining unemployment duration of mismatched workers in two different scenarios. In the case of "tightness dominance" a reduction in the selectivity level of the higher education system may induce a rise in the share of graduates, leading to an increase in the share of firms investing in graduate positions. In contrast, in the case of "composition dominance" a reduction in the selectivity level of the higher education sector induces an increase in the share of graduates and this might generate a reduction in the share of firms investing in graduate positions because of a too low expected productivity of graduates. We show that overeducation may characterize both scenarios. However, only in the latter case we should observe an inverse relationship between graduates' ability and the duration of unemployment spells terminating in a right match. Overall, we point out the importance of considering the selectivity of the higher education sector as a policy instrument that may affect both the extent of educational mismatch and the overall performance of the economy in terms of output and productivity.

References