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TOO MANY GRADUATES? A THEORY OF (EFFICIENT) EDUCATIONAL MISMATCH AND EVIDENCE FROM A QUASI-NATURAL EXPERIMENT

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Too Many Graduates? A Theory of (Efficient) Educational Mismatch and Evidence from a Quasi-Natural Experiment

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Abstract

This paper analyzes the process driving graduate workers in undergraduate jobs. Micro and macro perspectives are considered so that the interrelationships between individual mismatch and over-education at the aggregate level are analyzed. The theoretical model highlights that individual mismatch does not necessarily imply that the share of graduates exceeds what is optimally required at the aggregate level. The empirical investigation tests to what extent the individual probability of mismatch is related to the availability of graduates in the labor market. A structural estimation is implemented using a quasi-natural experiment ideally provided by an exogenous expansion of higher education that took place in some Italian regions in the mid '90s. Difference-in-Differences models show that in this country an increase in the supply of graduates has actually reduced the individual probability of mismatch which is an effect rationalized by the theory.

*Jel classification:* J24, J64, I23.

*Key Words:* Overeducation, mismatch, matching models, difference-in-differences.
1 Introduction

This work proposes an investigation of educational mismatch in the labor market focusing on the process that drives graduate workers in undergraduate jobs. The interest on this specific mismatch paradigm has been, and it is, very high among labor economists. In continental Europe the phenomenon involves a large part of skilled workers. In Germany, Italy, and Spain approximately 30% of graduates enter positions that do not require the acquired skills. In the UK and in the United States the share of mismatched workers ranges between 17% and 42% of the whole employed graduate labor force according to different measures and definitions.\footnote{See Bauer (2002), Budria and Moro-Egido (2008); Ordine and Rose (2009), and Tsai (2010) for evidence on these countries.} The European Union has also focused its interest on this topic considering that the identification of the source of disequilibria between labor supply and demand is important since these imbalances could potentially be very costly to the economy by restricting productivity growth (European Commission, 2008). Overall, these facts give rise to a number of important issues for policy. In particular, since mismatch may imply unproductive human capital investments, the wisdom of most OECD governments in pursuing high participation rates in higher education could be questioned.

The economic literature provides two different approaches to study educational mismatch. From an empirical perspective, educational mismatch is measured considering to what extent individuals possess a level of education in excess of that required in their specific job (seminal papers include Sicherman, 1991; and Cohn and Kahn, 1995). In this case, mismatched individuals are also named ‘overeducated’. Instead, the theoretical literature refers to educational mismatch as a phenomenon arising because of aggregate over-education, i.e., when the share of graduates overruns its optimal level (Moen, 1999; Charlot and Decreuse, 2005 and 2010). Both perspectives present strong and in some cases contrasting implications in terms of education policy and human capital accumulation. Indeed, the occurrence of over-education should imply that the society would fare better if individuals invested less in education. At the same time, as far as there is a significant non-negative return to education for mismatched workers, it would not be possible to unambiguously argue that there is a ‘surplus schooling’ requiring less investment in higher
The aim of this study is to reconcile these two views. We provide a continuous time matching model where workers differ in pre-university ability and choose whether to invest in human capital, while firms differ in technological endowment and choose whether to post graduate or undergraduate vacancies. The search market is segmented by job type, graduates can apply in both sectors, while undergraduates only apply for undergraduate jobs. As in Moen (1999) the matching technology allows for multiple matches and firms can rank applicants. Our theoretical model highlights that educational mismatch does not necessarily imply that the share of graduates exceeds what is optimally required at the aggregate level. Instead, we argue that it may only be due to search frictions in the labor market and, in this case, it may coexist with optimal human capital investments. Furthermore, the model considers the possibility that mismatch arises as an inefficient outcome in the presence of either over-education or (even) under-education at the aggregate level. These results are extremely relevant for policy since they relate the characteristics and the dimension of the tertiary system of education to the occurrence of mismatch. In order to empirically investigate the relevance of our insights, we implement a structural estimation using a quasi-natural experiment ideally provided by an exogenous expansion of higher education that took place in Italy in the mid ’90s. We use data from several waves of a survey conducted by the Italian National Institute of Statistics (ISTAT) on the labor market outcomes of university graduates interviewed three years after their graduation to implement Difference-in-Differences models supported by many robustness checks. We show that in this country a spreading out of higher education has generated a reduction of the individual probability of mismatch which is an effect rationalized by our theory and which supports the main idea of this work: The existence of educational mismatch does not necessarily imply that there are too many graduates in the labor market.

The outline of the article is as follows. In the next Section we discuss how our insights relate to the existing literature. In Section 3 we set up the theoretical model and in Section 4 we evaluate the equilibria and disentangle different scenarios. Section 5 contains our empirical investigation. Our concluding remarks are presented in Section 6.
2 Existing Background and Our Insights

The issue of educational mismatch has been extensively investigated in the economic literature. From an empirical perspective, existing works concentrate their efforts in assessing its relevance in determining wages broadening the human capital framework. Despite a very wide research extent, evidence on wage for overeducated is far from being well defined. Some authors argue that there is a wage penalization associated with mismatch (recent works include Lamo and Messina 2010; Ordine and Rose, 2011). Conversely, other studies report that, after controlling for individuals' ability and labor market opportunities, this negative effect becomes much smaller or totally disappears (Bauer, 2002; and Tsai, 2010). From a theoretical point of view, the debate involves a comparison between private and social returns to education implying different views on the role of tuition fees and public expenditure on higher education. Charlot and Deheure (2005, 2010), and Moen (1999) present models where over-education arises when the share of graduates exceeds its optimal level. This happens since the social return to education is lower than the private return because of various market imperfections inducing inefficient self-selection into education. Although these works differ in many respects, they share some common features. In particular, they discuss the presence of over-education and the right policy to reduce it by assuming that once the educational choice has been made, graduates' and undergraduates' labor markets are perfectly segmented (excluding ex-ante the possibility of mismatch). However, this assumption appears to be strongly counterfactual.

We present a theoretical framework where: i) There is strategic complementarity between individuals' educational choice and firms’ technological decision; ii) graduates may be employed in the undergraduate sector. In this setting, we show that an increase in the number of graduates may induce a rise in the technological endowment of firms via a tightness effect: The larger the pool of graduates the greater the probability that a firm fills graduate-complementary vacancies. However, this relation is concave since the presence of a too large share of graduates in the labor force implies a composition effect: The larger the share of graduates the lower their expected ability. In this context, the event of mismatch is not necessarily the result of inefficient educational choices. We prove that, under some circumstances it may mirror an efficient outcome in the sense that, given labor market frictions, the composition and the tightness effects perfectly
balance each other allowing for output maximization. In this case, the steady-state unemploy-
ment outflow of graduates towards undergraduate jobs only reflects (exogenous) search-frictions
and the fact that graduates benefit from a wider set of job opportunities. Notwithstanding,
there can be equilibria wherein either the composition or the tightness effect dominates inducing
inefficient outcomes. The selectivity of the higher education system and the characteristics of
firms in terms of technological endowment are crucial variables in this context since they shape
the specific form of mismatch arising in the labor market.

The theoretical specification calls for an empirical verification of the main hypothesis settled
in the model. In particular, it is crucial to understand whether the occurrence of individual
mismatch is related to the supply of graduates in the labor market and then to assess how
policy on higher education may deal with this phenomenon. To this aim, we implement a
structural estimation by using Italian data and by relying on an exogenous shock derived by a
policy measure targeted to realize an expansion of higher education in some (7 over 20) Italian
regions. This policy has led to a growth in the number of campuses in some regions of the
country and to a consequent rise in the supply of graduate workers. This scenario provides a
valuable quasi-experimental research design to test to what extent the individual probability of
mismatch is related to the availability of skilled workers in the labor market. The main empirical
strategy consists in comparing graduates’ labor market outcomes before and after the expansion
in areas where new university campuses were established and in areas where the number of
universities remained unchanged. The results highlight that after the reform graduates from
treated regions have a lower probability of being mismatched of about 5.0%. This result adds to
the existing literature on social returns to education. Acemoglu and Angrist (2000) and Moretti
(2004) argue that an increase in the share of educated workers may rise the returns to education
whenever spillover effects overrun the (decreasing marginal) productivity effect. These authors
find significant human capital spillover, i.e., a positive relation between the share of college
educated workers and individual wages. Here we show that the quality of individual match may
represent an additional channel through which human capital externalities show up.
3 The Model

3.1 Overview

We consider an economy characterized by a continuum of risk-neutral individuals and firms matching in the labor market following the lines set out by Diamond-Mortensen-Pissarides. Differently from standard matching models, we assume that i) the matching technology follows an urn-ball model as in Hall (1979); ii) before entering the job-market, firms and individuals have to make a technological and an educational choice respectively.\footnote{The urn-ball matching function has become extremely popular among labor economists. For interesting applications and some proofs concerning analytical derivations see Moen (1999) and Gavrel (2009).} We assume the mass and the distribution of agents remain constant over time. Therefore, although we do not explicitly model agents’ entry and exit, our results would remain unchanged in case we considered agents with exogenous exit rates replaced by before-choice identical alter-egos. Individuals/firms decide whether they want to enter the graduate/high-tech or the under-graduate/low-tech market respectively.\footnote{Moen (1999) only allows for educational choices and he does not address possible complementarity between educational and technological choices.} We assume that individuals are heterogeneous with respect to their pre-university characteristics which determine their productivity on the job and we consider the case where these attributes are inversely related to the cost (effort) of acquiring education. From now on, we simply refer to these individual characteristics as ability. On the demand side, each firm can post a limited number of vacancies, normalized to 1, and it sets production on the basis of a technological choice. In particular, a firm can choose the sector where posting a vacancy, i.e., it can choose to operate either within the high- or the low-technological sector. In order to simplify notation, from now on we refer to graduate versus undergraduate choice for both firms and individuals. However the reader should keep in mind that individuals make an educational choice while firms take a technological decision. Once these choices have been made, the pure matching-process starts. We assume that undergraduate individuals can only be matched with low-tech firms, while graduates can search in both high-tech and low-tech markets. Using this setting, we demonstrate how educational mismatch is a phenomenon that might characterize standard matching models, i.e., it might be attributable to a simple problem of frictional search.
in the labor market, arising even if the social outcome is efficient in terms of output. At the same time, we set out the conditions which may lead to inefficient mismatch related to either over- or under-education, calling for targeted educational policy to improve the overall economic performance.

3.2 Individuals

Consider a continuum of individuals of mass 1 characterized by heterogeneous ability $\theta$. This is distributed according to a continuous and strictly increasing cumulative distribution $\Gamma(\theta)$, whose density function is $\gamma(\theta)$, over a support $[\underline{\theta}, \overline{\theta}]$ where $1 \leq \underline{\theta} < \overline{\theta}$ (so $\Gamma(\underline{\theta}) = 0$ and $\Gamma(\overline{\theta}) = 1$). $\Gamma(.)$ and $\gamma(\theta)$ are both stationary over time. We indicate with $e = \{g, ug\}$ the educational choice undertaken in order to maximize expected discounted utility ($g$ stands for graduate while $ug$ stands for undergraduate). For simplicity we assume that an individual has no income if unemployed (no unemployment benefits). As a consequence, once the educational choice has been made, in each period the individual’s utility function $W(e)$ is given by:

$$W(e) = \begin{cases} 
0 & \text{if unemployed} \\
ug & \text{if undergraduate and employed} \\
og & \text{if graduate and employed in an undergraduate position} \\
g & \text{if graduate and employed in a graduate position}
\end{cases}$$

(1)

where $ug$ indicates wage for undergraduate workers, while $og$ and $g$ indicate wages for graduates employed in undergraduate and graduate positions respectively ($O$ stands for "overeducated" while $R$ stands for "right match"). The cost of acquiring education $ug$ is normalized to zero. When individuals decide to acquire education $g$, they have to sustain a cost $c(\theta, \mu) > 0$ related to their individual ability with $\frac{\partial c}{\partial \theta} < 0$ and to the monetary costs of education $\mu$ with $\frac{\partial c}{\partial \mu} > 0$. We assume that monetary costs are the same for all individuals while the effort required to achieve a degree qualification is determined by personal ability. We consider the cost of education $c(\theta, \mu)$ as a measure of the selectivity of the higher education sector, determining the share of graduates in the labor market and, consequently, the tightness of the two sectors.
3.3 Firms

Consider a continuum of firms of mass 1. We indicate with \( T = \{ g, ug \} \) firm’s investment in graduate and undergraduate vacancy respectively. The cost of entering the \( g \) sector is given by \( \delta > 0 \). The cost of entering the \( ug \) sector is normalized to zero.\(^4\) We crucially assume that firms are heterogeneous with respect to the cost they have to sustain in order to enter the \( g \) sector. In fact, in the growth theory literature, the cost of advanced technology has been considered typically related to the actual firm’s technological endowment. The closer is a firm to the technological frontier the lower is the cost it needs to sustain in order to update its technology. The concept of technological frontier has been introduced by Nelson and Phelps (1966). Acemoglu et al. (2006) study empirically the relation between R&D expenditure and the distance from the technological frontier and build up a model where firms differ in terms of costs to adopt new technologies. In our case, we assume that firms are distributed with a continuous and strictly increasing cumulative distribution \( \Phi(.) \) whose density function is \( \phi(.) \), over a support \( [\delta, \overline{\delta}] \) where \( \delta < 1 < \overline{\delta} \) (so \( \Phi(\delta) = 0 \) and \( \Phi(\overline{\delta}) = 1 \)). \( \Phi(.) \) and \( \phi(.) \) are stationary over time.

According to Acemoglu (1997), the production function is given by:

\[
y = y(e, T, \theta) = \overline{y}[\Theta(T \equiv g \text{ and } e \equiv g) \Phi(T \equiv ug \text{ or } e \equiv ug)]
\]

where \( \overline{y} > 0 \) is a constant. Relation (2) indicates that there is homogeneity in the undergraduate sector, i.e., when individuals work in the \( ug \) sector they produce an output \( \overline{y} \) independently on their ability and education. At the opposite, graduate technologies are complementary only to graduate workers and the intensity of such complementarity is given by individual’s ability \( \theta \).\(^5\)

\(^4\)This assumption may easily be justified by thinking that in order to enter the graduate sector, firms are required to have costly technological endowment that should be used by engineers, doctors, investors, etc.; while low-skills complementary machines are typically less costly. See Mokyr (1996) on this argument.

\(^5\)In fact, in eq. (2) we are assuming skill-ability complementary technologies. This conjecture regarding the centrality of the positive interaction between technologies and ability is largely consistent with empirical evidence. Among others, Bartel and Sicherman (1999) find that the education premium in the US over the period 1979-1993 is the result of an increase in demand for innate ability or other unobserved characteristics of more educated workers.
Finally, we indicate with $Q$ the cost of maintaining a vacancy $\forall T$, and we assume that in the steady-state vacancies yield zero profit (free-entry condition). Once the technological decision has been made, each firm realizes a profit $\Pi(T)$ in each period, indicated as follows:

$$
\Pi(T) = \begin{cases} 
-Q & \text{if unfilled vacancy } \forall T \\
\bar{y} - w_{ug} - Q & \text{if filled } ug \text{ vacancy with an undergraduate worker} \\
\bar{y} - w_{g} - Q & \text{if filled } g \text{ vacancy with a graduate worker} \\
\theta \bar{y} - w_{g}^R - Q & \text{if filled } g \text{ vacancy.}
\end{cases}
$$

\[3\]

### 3.4 Interaction Process and Bellman Equations

The process consists in the following two stages. At the first stage, individuals and firms conditional on their own type (ability and distance to the frontier) simultaneously decide the sector they want to enter, i.e., they choose between graduate and undergraduate sectors. Once the educational/technological choices have been made, individuals and firms enter the labor market as unemployed and with unfilled vacancies respectively, and then the matching process starts. As a consequence, the relative markets’ tightness in the present model is endogenous. Once a match is realized, we assume individual Nash-bargaining axiomatic solution for wage determination which is discussed in details in Section 4.

In order to solve the model we proceed backward: Firstly, we evaluate the actual expected value functions for individuals and firms using a standard dynamic programming method; secondly, by using the obtained results we proceed backward to find the Bayesian Nash Equilibrium (BNE) of the simultaneous game in which agents decide, conditional upon their own type, educational level and technological contents to maximize their expected steady-state payoffs. Since we assumed stationary density functions $\phi(\delta)$ and $\gamma(\theta)$ the BNE is also stationary over time, i.e., in equilibrium the share of agents in the two sectors would be stationary even if agents were exiting the market being replaced by identical alter-egos re-taking the educational/technological choice.

We will show that the BNE can be efficient in terms of the total expected output (minus costs) of the economy only conditional upon the appropriate selectivity $c(\theta, \mu)$ in the higher education

---

\[6\] We could assume $Q_g \neq Q_{ug}$. However, by assuming $Q_g = Q_{ug} = Q$ we simplify the notation and, because of free-entry condition, this does not affect our main results.
sector. In this case although mismatch arises, it is not related to an over-supply of skilled workers. Instead, it mirrors the presence of search-frictions and larger labor market opportunities for graduates.\(^7\)

### 3.4.1 The matching functions

We indicate with \(E_e\) the employment level per educational groups \((e = \{g, ug\})\). \(\dot{E}_e\) indicates the over-time variation of employment levels with:

\[
\dot{E}_e = H_e - bE_e
\]

where \(b > 0\) is the exogenous quitting rate and \(H_e\) is the number of hirings per educational level. Since \(H_g\) indicates the overall number of hirings for graduates, and since graduates can be matched in both sectors, we have that:

\[
H_g = H_g^R + H_g^O
\]

where \(H_g^R\) indicates the number of graduates hired in the graduate sector, and \(H_g^O\) indicates the number of graduates employed in undergraduate positions. By indicating with \(U_e\) the number of unemployed workers with education \(e\) and with \(V_T\) the number of posted vacancies per sector \(T\), we can write the hiring functions as follows:

\[
H_g^R = a^R_g(\lambda)U_g.
\]

\[
H_{ug} = KV_{ug}^{n_y}U_1^{1-n_y}
\]

\[
H_g^O = KV_{ug}^{n_y}U_1^{1-n_y}
\]

\(^7\)In this paper we do not cover explicitly the issue of on-the-job search. However, we remark that, albeit on-the-job search activities could be relevant for mismatched workers who could decide to quit their current position, the structure of the model is set so that there are no incentives to engage in this activity. Intuitively, when the wage setting procedure is solved, the surplus realized by a worker employed in a right-position is identical (conditional on worker’s characteristics) to that realized in a wrong position. This modeling choice, discussed in details in paragraph 4.1, ensures that there are no incentives for mismatched individuals to engage in on-the-job search activities.
where \( a_R^g(\lambda) \) is the unconditional probability that a graduate is employed in a right job expressed as a function of the tightness of the graduate sector \( \lambda = U_g/V_g \), while \( K \) is a constant and \( 0 < \eta < 1 \). Some important clarifications are in order. First of all, we need to explain the reason for using three hiring functions. To this end, indicate with \( M_g \) the total number of matches that characterize the graduate sector. Since only graduates can be matched in this sector we have that \( M_g = H_R^g \). Mutatis mutandis by indicating with \( M_{ug} \) the total number of matches in the undergraduate sector we have that \( M_{ug} = H_{ug} + H_Q^g \). In the AppendixA we prove that the hiring functions (7) and (8) derive from a unique CRTS Cobb-Douglas matching function given by:

\[
M_{ug} = KV_{ug}^\eta (U_{ug} + U_g)^{1-\eta}.
\]

From eq. (9) it appears that when seeking for undergraduate jobs, graduates and undergraduates create congestion effects for each other. By using hiring instead of matching functions we considerably simplify the illustration of the model. Now, we can turn our attention to the functional forms of the hiring process described in eqs. (6), (7) and (8). In undergraduate jobs, eqs. (7) and (8) are assumed to be standard Cobb-Douglas CRTS hiring functions. Microfoundations for the Cobb-Douglas matching function are discussed in details in Stevens (2007). In our paper, this modeling choice is due to tractability reasons. In particular, this assumption enables us to model the presence of a partially segmented labor market and to prove the existence of a steady-state unemployment equilibrium for all workers. Moreover, since there is homogeneity in the undergraduate sector, by using a Cobb-Douglas function we can rely on the parameter \( \eta \) to assess efficiency in this sector (Hosios, 1990). This implies that we can focus attention only on the \( g \) sector when evaluating the resulting mismatch in terms of efficiency. Turning to the graduate sector, since workers’ ability matters, the matching process is described as an urn-ball model where workers send applications and firms choose among applicants. This matching regime may be represented by a Poisson distribution as discussed in the next paragraph.\footnote{In the standard (one sector) urn-ball process, unemployed workers can send only one application each period. Here, we assume that graduates may send one application per sector each period, with a strict preference for graduate jobs. This implies that the urn-ball process in graduate sector is described by a Poisson distribution.}
3.4.2 The probability of getting a job/filling a vacancy

In Box 1 we set the notation for individuals and firms probabilities of getting a job and filling a vacancy respectively (in general \(a(.)\) refers to workers while \(\alpha(.)\) refers to firms). From Box 1, we need to define the probability for a graduate of getting a job and the probability for a firm of filling a vacancy (both conditional and unconditional to \(\theta\)).

<table>
<thead>
<tr>
<th>Box 1: Notation for individuals and firms probability of being employed/filled</th>
</tr>
</thead>
</table>

**Individuals**

\[ a_{u^g} = \frac{H_{u^g}}{U_{u^g} + U_g} \Rightarrow \text{prob. that an undergraduate is employed} \]

\[ a_{O^g} = \frac{H_{O^g}}{U_{u^g} + U_g} \Rightarrow \text{prob. that a graduate is employed in an undergraduate position} \]

\[ a_{R^g}(\lambda, \theta) \Rightarrow \text{conditional prob. that a graduate with ability } \theta \text{ is employed in a right position} \]

\[ a_{R^g}(\lambda) \Rightarrow \text{unconditional prob. that a graduate is employed in a graduate position} \]

**Firms**

\[ \alpha_{u^g} = \frac{H_{u^g} + H_{O^g}}{V_{u^g}} \Rightarrow \text{prob. that an undergraduate vacancy is filled} \]

\[ \alpha_{g}(\lambda, \theta) \Rightarrow \text{conditional prob. that a } g \text{ vacancy is filled with a } \theta \text{-type worker} \]

\[ \alpha_{g}(\lambda) \Rightarrow \text{unconditional prob. that a } g \text{ vacancy is filled} \]

The probability that an unemployed graduate with ability \(\theta\) receives a job offer from a \(g\) firm is given by:

\[ a_{g}^{R}(\lambda, \theta) = \exp^{-[1 - G(\theta)]} \lambda. \quad (10) \]

In eq. (10), the probability of receiving a job offer increases with individual ability \((\partial a_{g}^{R} / \partial \theta > 0)\) and if \(\theta = \theta^*\) then \(a_{g}^{R}(\lambda, \theta)\) has a unit value since \(\theta^\ast\)-types get any job they apply for. By integrating \(a_{g}^{R}(\lambda, \theta)\gamma(\theta)\) over \([\theta^*, \overline{\theta}]\), whose lower bound \(\theta^*\) is the threshold-ability determined \(ex-ante\) in the BNE, we obtain the unconditional probability of being hired in a \(g\) position, called \(a_{g}^{R}(\lambda)\) with:

\[ a_{g}^{R}(\lambda) = \int_{\theta^*}^{\overline{\theta}} a_{g}^{R}(\lambda, \theta)\gamma(\theta)d\theta \quad (11) \]

11
and this explains eq. (6). Now, we can turn our attention to the urn-ball process from firms’ perspective. We can write the probability that a $g$ firm hires a $\theta$-type graduate as follows:

$$\alpha_g(\lambda, \theta) = \exp^{-[(1 - \Gamma(\theta))\lambda]} \gamma(\theta) \lambda.$$  

(12)

Eq. (12) contains the probability that a firm does not meet any applicant of ability greater than $\theta$ times the probability of matching a worker with ability $\theta$. When integrating $\alpha_g(\lambda, \theta)\gamma(\theta)$ over $[\theta^*, \bar{\theta}]$ we get the unconditional probability that a $g$ vacancy is filled defined as follows:

$$\alpha(\lambda) = \int_{\theta^*}^{\bar{\theta}} \alpha_g(\lambda, \theta)\gamma(\theta) d\theta.$$  

(13)

3.4.3 The value functions

We set the notation for actual expected values as indicated in Box 2. By indicating with $r > 0$ the intertemporal interest rate, we can write down the value functions as follows:

- Undergraduate individuals:

  $$rV_{ug}^E = w_{ug} - b(V_{ug}^E - V_{ug}^U)$$  

  (14)

  $$rV_{ug}^U = a_{ug}(V_{ug}^E - V_{ug}^U).$$  

  (15)

- Graduate individuals:

  $$rV_{g}^R = w_{g}^R - b(V_{g}^R - V_{g}^U)$$  

  (16)

  $$rV_{g}^O = w_{g}^O - b(V_{g}^O - V_{g}^U)$$  

  (17)

  $$rV_{g}^U = a_{g}^R(V_{g}^R - V_{g}^U) + a_{g}^O(V_{g}^O - V_{g}^U).$$  

(18)

- Firms with undergraduate job-positions:

  $$rV_{ug}^{Fug} = \bar{y} - w_{ug} - Q - b(V_{ug}^{Fug} - V_{ug}^V)$$  

  (19)

  $$rV_{ug}^{Fg} = \bar{y} - w_{g}^O - Q - b(V_{ug}^{Fg} - V_{ug}^V)$$  

(20)
\[ rV_{ug}^V = -Q + \alpha_{ug}(V_{ug}^{Fug})_{1(\epsilon=x)} - V_{ug}^V. \] (21)

- Firms with graduate job-positions:

\[ rV_{g}^{F} = \theta y - w_{g}^{R} - Q - b(V_{g}^{F} - V_{g}^{V}) \] (22)

\[ rV_{g}^{V} = -Q + \alpha_{g}(\cdot)(V_{g}^{F} - V_{g}^{V}). \] (23)

Box 2: Notation for actual expected values

<table>
<thead>
<tr>
<th>Firms</th>
<th>Individuals</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{g}^{F} ) ( \Rightarrow ) filled ( g ) position;</td>
<td>( V_{ug}^{E} ) ( \Rightarrow ) empl. ( ug ) individual;</td>
</tr>
<tr>
<td>( V_{g}^{V} ) ( \Rightarrow ) vacant ( g ) position;</td>
<td>( V_{ug}^{U} ) ( \Rightarrow ) unempl. ( ug ) individual;</td>
</tr>
<tr>
<td>( V_{ug}^{Fug} ) ( \Rightarrow ) filled ( ug ) position with a ( ug ) individual;</td>
<td>( V_{g}^{R} ) ( \Rightarrow ) empl. ( g ) individual in a right position;</td>
</tr>
<tr>
<td>( V_{ug}^{Fr} ) ( \Rightarrow ) filled ( ug ) position with a ( g ) individual;</td>
<td>( V_{g}^{O} ) ( \Rightarrow ) empl. ( g ) individual in a over position;</td>
</tr>
<tr>
<td>( V_{ug}^{V} ) ( \Rightarrow ) vacant ( ug ) position;</td>
<td>( V_{g}^{U} ) ( \Rightarrow ) unempl. ( g ) individual.</td>
</tr>
</tbody>
</table>

Some of the relations above are pretty standard value functions, while others, in particular eq. (18) and eqs. (19)-(21), deserve some clarifications. Eq. (18) indicates that the value of being an unemployed graduates with ability \( \theta \) includes both the probability of being employed in a graduate position (\( a_{g}^{R}(\cdot) \)) and the probability of entering an undergraduate position (\( a_{g}^{O}(\cdot) \)). Eqs. (19) and (20), and as a consequence eq. (21), disentangle the value of a filled \( ug \) position conditional upon the education of the matched worker. In fact, even if we assumed homogeneity in the \( ug \) sector, namely all individuals realize the same output independently on their characteristics, education matters in wage determination in \( ug \) position since graduates have a larger set of job opportunities than their undergraduate peers. This steams out from the use of individual Nash bargaining solution as discussed in the next Section.
4 The Equilibria

4.1 Equilibrium Wages

In order to solve the model, we use individual Nash-bargaining solution, hence we impose that when a match is realized, the generated surpluses for firm and worker must be equal conditional upon agents’ characteristics and labor market opportunities. We can solve the bargaining situations as follows. Consider the undergraduate sector. In this case, firms can match with \(ug\) and \(g\) individuals hence our bargaining equations are:

\[
V^{E}_{ug} - V^{U}_{ug} = V^{F}_{ug} - V^{V}_{ug}
\]  
\(24\)

\[
V^{O}_{g} - V^{U}_{g} = V^{F}_{ug} - V^{V}_{ug}.
\]  
\(25\)

In order to have both graduate and undergraduate workers employed in the \(ug\) sector we must impose that, conditional on individual’s ability \(\theta\), in equilibrium the following condition must hold:

\[
V^{F}_{ug} - V^{V}_{ug} = V^{F}_{ug} - V^{V}_{ug}.
\]  
\(26\)

The rationale behind eq. (26) is as follows. Consider the case of an undergraduate firm matched with a graduate worker. Since this worker has more job-market opportunities than his undergraduate peer, the Nash bargaining solution should imply that \(w^{O}_{g} > w_{ug}\) for all \(\theta\). However, in this case the firm would have a strict preference for a \(ug\) worker. Crucially, this is not irrelevant in the bargaining process between the \(ug\) firm and the \(g\) worker. In particular, the impasse point arising in case of a bargaining failure should be set considering that the firm has a probability of matching a \(ug\) worker who has less outside options than the current \(g\) worker. This improves the bargaining position of the \(ug\) firm bargaining with the \(g\) worker and reduces \(w^{O}_{g}\). The same argument implies that when there is a match between \(ug\) types, the worker’s bargaining position embodies that in case of a bargaining failure the firm has a probability of matching a \(g\) worker who has more labor market opportunities than the current \(ug\) worker. This improves the bargaining position of the \(ug\) worker and raises \(w_{ug}\). By adding relation (26) to the other standard individual bargaining equations we impose that, conditional upon \(\theta\), the results of the bargaining
processes lead to the same surplus for a ug firm independently on the education of the matched worker. This ensures the existence of a steady-state equilibrium in which both graduates and undergraduates are employed in the ug sector.

Now, we can turn our attention to the solutions of the different bargaining processes. By using eqs. (24), (25), and (26) and by combining the relative value functions we obtain the following wage expressions for undergraduates and overeducated graduates:

\[ w_{ug} = \frac{\bar{y}(r + b + a_{ug})}{a_{ug} + a_{ug} + 2b + 2r}. \] (27)

\[ w_{Og}^{O} = \frac{1}{a_{Rg}^{O}(\cdot) + r + b} \left[ a_{Rg}^{O}(\cdot) w_{Rg}^{O} + \frac{\bar{y}(r + b + a_{Rg}^{O}(\cdot) + a_{Og}^{O}(\cdot)(r + b))}{a_{ug} + a_{ug} + 2b + 2r} \right]. \] (28)

Similarly, by imposing \( V_{g}^{R} - V_{g}^{U} = V_{g}^{F} - V_{g}^{V} \), after implementing some tedious algebra we obtain the wage of graduates employed in right position:

\[ w_{Rg}^{R} = \frac{\theta \bar{y} [r + b + a_{g}^{O} + a_{g}^{R}(\cdot)] (r + b) + a_{g}^{O} [r + b + a_{g}(\cdot)] w_{g}^{O}}{(r + b) [2r + 2b + a_{g}(\cdot) + a_{g}^{O} + a_{g}^{R}(\cdot)] + a_{g}^{O} [r + b + a_{g}(\cdot)]}. \] (29)

Some issues are worth noting. Firstly, when comparing eqs. (27) and (28) it appears that if \( a_{g}^{R}(\cdot) = 0 \) wages \( w_{ug} \) and \( w_{Og}^{O} \) are equal to each other. In words, if graduates could not search in an additional market, wages for graduates and undergraduates working in the undergraduate sector would be the same. In our model, wage differentials across ug positions are related to individuals’ education and ability only because of heterogeneous labor market opportunities. Put differently, even if ability and education do not play any role in determining the output level of a ug position, they are important in determining wage differentials between overeducated individuals and their co-workers with the required educational level. Although this is not the main punch-line of our paper, we remark this point. Indeed, a large part of the empirical literature argues that overeducated workers derive some wage-benefit from surplus education with respect to their co-workers and this evidence has been interpreted as reflecting human capital differentials across workers. Here, we highlight a different channel (possibly complementary to the former) that may lead to reward graduates more than undergraduates even if human capital does not affect productivity directly. Secondly, notice that if graduates could not search in the ug sector
eq. (29) would describe a standard wage equation for matching model as that in eq. (27). In Appendix A we set the conditions under which eqs. (27), (28), and (29) give rise to a steady-state equilibrium in the matching process in which $a_g^O > 0$. In words, we prove that there exists a steady-state characterized by a flow of graduates going towards undergraduate jobs. We now proceed to evaluate the simultaneous decision of individuals and firms concerning educational level and technological sector respectively.

### 4.2 The Entry Decision

Individuals and firms have to decide, conditional on their ability and distance to the frontier, the sector they want to enter. We assume that agents ground their decisions on the parameters $a_g^O, a_{ug}, a_g^R(\cdot), \alpha_{ug}$, and $\alpha_g(\cdot)$ as if they were at their steady-state value. Put differently, we are assuming agents choose their strategy in order to maximize the payoffs they would obtain in the steady-state. Once they make their choice, they enter labor markets as unemployed and vacant and then the matching process starts. In Figure 1 we describe the interaction process using a standard game in normal form. The game is Bayesian since each agent knows his own type (ability/distance to the frontier) and just the distribution of types of player to whom he may be matched. Since individual’s ability is revealed only when a match is realized, in the figure $E[V^V_g|\theta]$ indicates the expected payoff of a $g$ firm that matches a $g$ worker. Moreover, since $g$ firms and $ug$ workers cannot match each other we set $V^V_g = -\delta$ and $V^U_{ug} = 0$ when $e = ug$ and $T = g$. Notice that, in this interaction process we look for pure strategies of firms and individuals that are best responses to each other, conditional on the type of players. As a consequence, the BNE gives us the shares of individuals and firms that acquire higher education and invest in graduate positions respectively and it provides a measure of the relative tightness of the two sectors in steady-state.

**Proposition 1** It exists a unique BNE of the game in Figure 1 in which only individuals with ability $\theta \geq \theta^\ast$ set $e = g$ and only firms with $\delta \leq \delta^\ast$ set $T = g$. 

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Figure 1: The individual-firm Bayesian game in normal form.

**Proof.** Consider the firm’s choice. Indicate with $p$ the probability that the individual sets $e = g$. In this case, a firm invests in $g$ position only if:

$$\delta \leq pE[V^V_g | \theta] - V^V_{ug}. \quad (30)$$

Given our assumption on the monotonicity of $\Phi(\cdot)$, we can indicate with $\delta^*$ the cutoff level of distance to the frontier for which relation (30) is satisfied. Now, indicate with $p'$ the probability that a firm sets $T = g$ and consider the individual’s educational choice. Setting $e = g$ is optimal for an individual only if:

$$c(\theta, \mu) \leq p'V^U_g. \quad (31)$$

Given our assumption on the monotonicity of $\Gamma(\cdot)$, given $\mu$ and given that $\frac{\partial c}{\partial \mu} < 0$, we can indicate with $\theta^*$ the cutoff ability level for which relation (31) is satisfied. Hence, the following
pair characterizes the BNE of the game in Figure 1:

\[
\begin{align*}
p &= 1 - \Gamma(\theta^*) \\
p' &= \Phi(\delta^*).
\end{align*}
\] (32)

Intuitively, a firm invests in a g position only if the associated expected payoff is greater than that associated with a u\_g position. Crucially, this depends on the distribution of \( \theta \) within individuals that decide to acquire education \( g \), on the relative markets' tightness, and on firm's distance to the technological frontier (eq. 30). At the same time, worker's decision of investing in education \( g \) is a function of the number of firms that decide to create \( g \) positions and of his own ability (eq. 31). Relation (32) contains the shares that are best response to each other and these can be considered as the shares of agents that represent the unique steady-state of the interaction process.

4.3 Equilibrium Efficiency

In order to establish the equilibrium efficiency, consider the undergraduate sector first. Since this market is characterized by a CRTS Cobb-Douglas matching function, labor market frictions may induce efficient unemployment only if the elasticity of matching with respect to unemployment is equal to workers' bargaining power (Hosios, 1990). In AppendixA we prove that the steady-state of the undergraduate sector is consistent with all value of \( \eta \) such that 0 < \( \eta < 1 \). Hence we prove that the resulting unemployment generated by frictions in the u\_g sector is consistent with the efficient scenario where \( \eta = 1/2 \). This implies that we need to focus our efficiency analysis on the graduate sector only. To this aim, the cutoff level \( \delta^* \), i.e., the one satisfying relation (30) as an equality, has to be evaluated. In fact, \( \delta^* \) approximates the share \( \phi(\delta^*) \) of firms creating graduate-complementary positions. As it appears from eq. (2) since a g firm realizes a greater output than its u\_g counterpart, the cutoff level \( \delta^* \) represents the value at which the aggregate expected output net of costs is maximized. Therefore, the greater \( \delta^* \), the larger the share of \( g \) firms, and the better is the performance of the considered economy in terms of expected output. To evaluate \( \delta^* \) we need to make explicit relation (30). As illustrated in AppendixA, by combining
eqs. (21) and (23) we can write the cutoff level \( \delta^* \) in relation (30) as follows:

\[
\delta^*(\theta^*) = \Gamma(\theta^*) \frac{Q}{r} + \frac{1}{rA} \frac{1 - \Gamma(\theta^*)}{\gamma(\lambda)} \left[ E[\theta|\theta \geq \theta^*] \left( C - \frac{\alpha^O_{g,\theta^*}}{B} \right) - \frac{A}{F} \right] - \frac{\alpha_{ug}\gamma}{rD}
\]  

(33)

where \( A, B, C, D, \) and \( F \) summarize strictly positive constants.\(^9\) Relation (33) represents the best response function in terms of share of firms investing in graduate positions. Since we are evaluating the best response when the share of graduates is \( \gamma(\theta^*) \), eq. (33) describes the BNE of the game. Notice that in eq. (33) we have that:

\[
\gamma(\theta^*) = \frac{\int_{\theta^*}^\infty \theta \gamma(\theta) d\theta}{1 - \Gamma(\theta^*)} 
\]

(34)

and, as a reminder \( \alpha_g(\lambda) = \int_{\theta^*}^\infty \alpha_g(\lambda, \theta) \gamma(\theta) d\theta. \) We can now evaluate how the share \( \delta^* \) changes in equilibrium as \( \theta^* \) changes. By differentiating eq. (33) with respect to \( \theta^* \) using the Leibniz rule for differentiation of definite integrals we get:

\[
\frac{\partial \delta^*}{\partial \theta^*} = \frac{1}{r} \left( \gamma(\theta^*) \left[ Q + \frac{\alpha^O(\lambda)\gamma C}{A[1 - \Gamma(\theta^*)]} \right] \int_{\theta^*}^\infty \theta \gamma(\theta) d\theta - \theta^* \right) + \gamma(\theta^*) \left[ \alpha(\lambda) + \alpha(\gamma(\lambda), \theta^*) [1 - \Gamma(\theta^*)] \right] 
\]

(35)

>0 composition effect

\[
\frac{\gamma(\theta^*)}{A} \left[ \left( \alpha(\gamma, \theta^*) \gamma(\theta^*) \right) \theta^* + \left( \frac{\alpha^O(\lambda)}{B[1 - \Gamma(\theta^*)]} \right) \left( \gamma(\theta^*) \alpha(\lambda) + \alpha(\lambda, \theta^*) [1 - \Gamma(\theta^*)] \right) \right] 
\]

>0 composition effect

\[
- \frac{\gamma(\theta^*)}{A} \left[ E[\theta|\theta > \theta^*] C (\gamma(\theta^*) \alpha(\lambda) + \alpha(\lambda, \theta^*) [1 - \Gamma(\theta^*)]) + \frac{\alpha(\lambda)}{B[1 - \Gamma(\theta^*)]} \int_{\theta^*}^\infty \theta \gamma(\theta) d\theta \right] 
\]

<0 tightness effect

Relation (35) indicates how a variation in the best response in terms of share of graduates

\(^9\) \( A = (r + b)[2r + 2b + \alpha^O_{g} + \alpha^R_{g}(\cdot)] + \alpha^O_{g}(r + b + \alpha_{g}(\cdot)); B = [2r + 2b + \alpha_{g}(\cdot) + \alpha^R_{g}(\cdot)]; C = (r + b + \alpha^O_{g}); D = (\alpha_{ug} + \alpha_{ug} + 2b + 2r); F = [2r + 2b + \alpha_{g}(\cdot) + \alpha^R_{g}(\cdot)][2r + 2b + \alpha_{g}(\cdot) + \alpha_{ug}]. \)
(\(\theta^*\)) affects in equilibrium the share of firms investing in graduate positions. The first two lines indicate that firms’ expectation positively depends on \(\theta^*\): The higher the cutoff ability level, the higher is the expected productivity of graduates and this induces a composition effect which fosters firms’ investment in graduate jobs. Conversely, the bottom line of eq. (35) shows the negative effect that a rise in \(\theta^*\) has on firms’ expectation: In this case, as the cutoff point \(\theta^*\) rises, the probability of filling a vacancy reduces, inducing a tightness effect that limits the creation of graduate-complementary positions. Assuming satisfied second order conditions, we can indicate with \(\theta^{**}\) the share of graduates that ceteris paribus maximizes firms’ investments in graduate positions, i.e., the share of graduates balancing tightness and composition effects:

\[
\frac{\partial \delta^*}{\partial \theta^*} |_{\theta^* = \theta^{**}} = 0.
\] (36)

It is important to note that only the appropriate selectivity level \(c(\theta, \mu)\) can ensure that \(\theta^{**}\) is actually achieved in equilibrium. If this is the case, the resulting steady-state is characterized by educational mismatch even if self-selection into education is fully efficient allowing for a perfect balance between tightness and composition effects (\(\theta^* = \theta^{**}\)). In this case, given exogenous labor market frictions and agents distributions in terms of individual ability and distance to the technological frontier, there is no higher education policy that could either avoid educational mismatch or improve the overall economic performance in terms of produced output.

### 4.4 Two Types of Inefficient Mismatch

Consider Figure 2 where we draw the best response function \(\delta^*(\theta^*)\) (it represents the set of all possible BNE) and the ability cumulate distribution \(\Gamma(\theta)\). Notice that the pair \((\delta^*_{\text{max}}, \theta^{**})\) represents the only efficient BNE. In the particular case depicted in Figure 2, we characterize an equilibrium with inefficient educational mismatch since \((\delta^*, \theta^*) \neq (\delta^*_{\text{max}}, \theta^{**})\). Indeed, since we draw \(\theta^* > \theta^{**}\) we are representing inefficiency due to a tightness problem which implies undereducation. In words, few individuals have access to the higher education system and this constraints both the creation of graduate complementary jobs and the overall output level. In this case, as illustrated in Figure 2, a higher education expansion implemented through a reduction in \(\mu (c(\theta, \mu) \downarrow)\) induces a rise in the share of graduates \((\theta^* \downarrow)\) that in turn induces an increase in
the share of firms investing in graduate positions. The overall expected output of the economy increases. Furthermore, a crucial point should be remarked. Albeit in this scenario both demand and supply of graduates increase, it is possible to show that when more individuals gain access to the g sector the individual probability of being mismatched \((a^O_g)\) decreases. The intuition is as follows. When a share of vacancies moves from the ug to the g sector, competition for these vacancies actually reduces since only graduate workers may apply for them. This is true even if the number of graduate increases since all workers could search for vacancies posted in the ug sector. Therefore, employment probability for graduates unambiguously increases. Further, since the availability of jobs increases only in the graduate sector, the probability of being employed with a right match increases \(a^R_g(\cdot) \uparrow\), while the probability of being mismatched actually falls down since more graduates have less ug firms to search for \((a^O_g \downarrow)\). When educational mismatch is associated with under-education, an expansion of the share of graduates reduces the individual probability of being mismatched.

Now, consider Figure 3. Here, we again draw an inefficient BNE, i.e., a scenario where
(\(\delta^*, \theta^*\)) \(\neq (\delta^*_{\text{max}}, \theta^{**})\) but, differently from the previous case, we consider \(\theta^* < \theta^{**}\). This equilibrium hides a composition problem which implies overeducation: A large number of individuals acquire education implying low expected productivity of graduate labor force. This curbs the creation of graduate jobs and the overall performance in terms of output. In this case an expansion of the higher education sector \((c(\theta, \mu) \downarrow)\) induces a rise in the share of graduates \((\theta^* \downarrow)\) and generates a reduction in the share of firms investing in graduate positions. In this case, the policy induces a decrease in output and, simultaneously, a reduction in the probability of having a right match \((a_g^R(\cdot, \downarrow))\) since more graduates have less \(g\)-type firms looking for them in the labor market. In addition, since the number of competitors for \(ug\) vacancies remains unchanged while the share of these vacancies increases, the probability of being mismatched increases \((a_g^O \uparrow)\). When mismatch is associated with over-education, an expansion of the share of graduates increases the individual probability of being mismatched.
Higher Education Expansion and Mismatch: An Empirical Investigation

5.1 Target and Methodology

The theoretical model puts forward that the occurrence of mismatch is not necessarily related to over-education at the aggregate level. Indeed, there can be stable equilibria where mismatch arises in economies with either efficient education level or (even) under-education. Therefore, the educational policy required to improve efficiency depends on the specific equilibrium reached in the labor market. In particular, an expansion of the tertiary system of education could reduce mismatch in economies characterized by a low share of graduate workers. In order to test this hypothesis we provide an empirical analysis using an exogenous supply shock that took place in Italy during the period 1998-1999. In these years, the number of public campuses has significantly risen and this happened only in some specific regions of the country. As reported in the Ministry of Education and Research Development Plan (1997) the reason of this tertiary education expansion resided in the need of providing accessibility to university homogeneously across Italian regions. Indeed, 7 over 20 Italian regions increased their supply by means of the institution of new campuses. This exogenous policy shock represents a valuable quasi-natural experiment to set out how educational mismatch is related to the supply of graduates. The case of Italy is also particularly interesting to our aim. Indeed this country is characterized by a ‘puzzling’ scenario since it records a widespread incidence of mismatch and low rates of participation to tertiary education. These are well known characteristics of the Italian labor market attested by the fact that albeit in this country a significant number of graduates seem to enter job positions that do not require their skills, the European Union often calls for a rise in the share of educated labor force in order to achieve levels similar to those of other developed countries (OECD, 2012).

5.2 Data

The empirical investigation presented in this study is based on data from three repeated cross-sections coming from surveys carried out by the Italian National Statistical Institute (ISTAT)
on the labor market outcomes of representative samples of graduate workers. Observations cover 73,088 individuals owning a university degree obtained after a 4/5 years course of study. These are all university graduates who entered the labor market in 1998, 2001 and 2004 and were interviewed three years later. Hence the surveys have been collected in 2001, 2004 and 2007 respectively.\textsuperscript{10} For those individuals who are employed, the survey records whether they are dependent workers or self-employed and for the former it records the type of job contract, plant dimension, industry sector, firm’s ownership (private/public) and the date of job start (year and month). Moreover, these surveys give information on high school performance of individuals (final mark and type of school) and on their family background (parents’ education).\textsuperscript{11} We rely on these specific repeated cross-sections for three main reasons.

Firstly, these surveys allow for the implementation of an experimental design, graphically summarized in Box 3. In particular, consider individuals from the 2001 wave (first line in Box 3). These are all individuals who graduate in 1998 and search in the labor market till the time of survey in 2001. Since new campuses started their activities in either 1998 or 1999, graduates from these new universities cannot be in labor market till 2002. This implies that the 2001 survey contains graduates who are unaffected by the institution of new campuses. Now, consider the 2007 survey (bottom line in Box 3). Individuals in this survey completed university in 2004, hence their labor market outcome is affected by the presence of more graduates from new campuses. By comparing the labor market outcome of individuals in the 2007 wave in those regions affected by the reform with that of individuals in the same regions in the 2001 survey,

\textsuperscript{10}From now on we refer to these samples as 2001, 2004 and 2007. However, the reader should keep in mind that the date refers to the date of the interviews while workers entered the labor market three years earlier. It is important to remark that the 2007 survey explicitly separates those graduates who, after the 3+2 university reform implemented in 2001, enrolled at universities under the new regime. Indeed, since at that time the old regime was in charge along with the new one, the ISTAT survey collected two separated representative samples for both the old and the new regime. We use only the survey covering the old regime which is fully comparable with the previous ones (similar number of graduates, majors, years of education, etc.). Moreover the survey which refers to the new university-regime contains only graduates with a three-years degree since 5 years were not elapsed since the higher education reform. We also remark that in our analysis we exclude individuals from region Valle D’Aosta because of a limited number of observations due to its small geographical dimension.

\textsuperscript{11}In Appendix B, Table B1 defines our variables while Table B2 and Table B3 contain some representative statistics of our samples in terms of academic/personal characteristics and labor market outcomes respectively.
Box 3: Time-prospect of the quasi-experimental design

<table>
<thead>
<tr>
<th>Year</th>
<th>New Campuses Start</th>
<th>New Graduates in Labor Market</th>
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<tbody>
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<table>
<thead>
<tr>
<th>Year</th>
<th>Survey</th>
<th>Year of Enrollment</th>
<th>Year of Graduation</th>
<th>Year of Interview</th>
</tr>
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<tbody>
<tr>
<td>01</td>
<td>survey</td>
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<tr>
<td>04</td>
<td>survey</td>
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<tr>
<td>07</td>
<td>survey</td>
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</tbody>
</table>

Notes: The surveys contain individuals with a time-to-degree delay of no more than 4 years with respect to the institutional term.

and by differentiating out the difference for individuals in untreated regions, we can estimate the effect of the higher education expansion on mismatch. Furthermore, consider the 2004 survey (medium line in Box 3). This is composed by individuals who graduated in 2001 that have been only in part affected by the reform because some of them have been employed before graduates from new campuses entered the labor market. Since the data record the date of job start, we can actually untangle treated and untreated individuals. Notice that this procedure requires to single out treated individuals according to the specific year of university expansion (1998 or 1999) in the region of residence. The use of the 2004 survey yields the opportunity to test the common time trend assumption providing a robust estimate of the higher education expansion on the probability of mismatch.

A second reason of why we rely on these surveys is that they report for each individual information concerning: i) The region where the attended university is located and ii) the region where the individual is actually working. This set of information is crucial to our aim since it makes possible to address an obvious caveat arising when separating treated and untreated individuals, i.e., the presence of mobility flows across regions. Indeed, we will show that workers’
mobility flows across regions are particularly low in Italy, and they remain almost unchanged along our surveys and across treated and untreated groups.

Finally, for those individuals who were actually employed at the time of the interview all surveys report a proxy to assess the occurrence of educational mismatch as defined in our theory. We consider as mismatched those individuals who declare that neither their specific degree nor any other academic tertiary qualification was required to apply for their job. This definition is the usual subjective assessment of mismatch which has been used by Sicherman (1991) and Cohn and Khan (1995) among others. Many studies report that no consistent difference arises when assessing the extent of mismatch among graduate workers by relying on subjective measures rather than professional assessment of job positions (see McGuinness, 2006; p.p. 396-399 for a review of this literature).

5.3 The Identification Strategy

The identification strategy presented in this study is funded on an exogenous policy shock introducing 9 new campuses across 7 Italian regions. These have been established homogeneously across the country involving Southern regions (Puglia and Sicilia), Central regions (Molise and Marche) as well as Northern regions (Lombardia, Piemonte and Trentino Alto Adige). In this context, the basic framework of our empirical strategy consists in the following steps. Firstly, we consider as treated those graduates from regions where new campuses have been established ($G_i = 1$ in case of treated graduates; $G_i = 0$ in case of untreated where $i$ indicates the generic individual). Secondly, we divide the time period according to the before and after policy implementation. In particular we separate graduates according to date at which the survey has been collected ($A_i = 1$ if the individual labor market outcome has been recorded after graduates from new campuses enter the labor market; $A_i = 0$ otherwise). Then, we implement a Difference-in-Differences approach in a Probit model where the dependent variable is a binary variable $\Lambda_i$ equal to 1 in case of mismatch. By indicating with $X_i$ the set of covariates that may affect $\Lambda_i$, we consider as treated those graduates from regions where new campuses have been established ($G_i = 1$ in case of treated graduates; $G_i = 0$ in case of untreated where $i$ indicates the generic individual). Secondly, we divide the time period according to the before and after policy implementation. In particular we separate graduates according to date at which the survey has been collected ($A_i = 1$ if the individual labor market outcome has been recorded after graduates from new campuses enter the labor market; $A_i = 0$ otherwise). Then, we implement a Difference-in-Differences approach in a Probit model where the dependent variable is a binary variable $\Lambda_i$ equal to 1 in case of mismatch. By indicating with $X_i$ the set of covariates that may affect $\Lambda_i$,  

12 These campuses are: Univ. of Piemonte Orientale (Piemonte) 1998; Univ. Milano Bicocca (Lombardia) 1998; Univ. of Insubria (Lombardia) 1999, Univ. of Bolzano (Trentino Alto Adige) 1999; Univ. of Piceno - campus of Ascoli Piceno - (Marche) 1998; Univ. of Molise - campus of Isernia - (Molise) 1998; Univ. of Foggia (Puglia) 1998; Univ. of Enna (Sicilia) 1999; Univ. of Catania - campus of Siracusa - (Sicilia) 1999.
we can write the model to be estimated as follows:

\[
E [A_i | X_i, G_i, A_i] = N (\beta X_i + \beta_0 G_i + \beta_1 A_i + \beta_2 G_i * A_i)
\]  

(37)

where \(N(\cdot)\) is the conditional distribution function of the standard normal distribution and \(\beta, \beta_0, \beta_1\) and \(\beta_2\) are parameters. Our parameter of interest is \(\beta_2\) since the associated marginal effect gives us the sign and the extent of the treatment effect, as shown in details in Phuani (2012). As robustness check, we also estimate eq. (37) by means of a linear probability model and by clustering standard errors at the regional level. The results are, as expected, not affected by our modeling choice, hence we report only our main specification’s results.

5.4 Addressing some caveats

The approach highlighted in the previous paragraph is, however, not straightforward. A first problem arises since workers’ mobility may affect our results. Mobility issues, if present, may undermine the identification strategy along many dimensions leading to cast doubts on the interpretation of the results. To deal with this issue, we provide evidence concerning the presence of a very low mobility for individuals in our samples. Moreover, we show that mobility across regions also remained constant over time and does not seem to be affected by the creation of new campuses. In Figure 4 we report three panels providing information on the share of individuals according to their region of work and region of study for all our surveys. It unambiguously appears that graduates’ mobility is a rare phenomenon in Italy since in each panel the diagonal - containing individual whose region of work is the same of that of study - embodies almost 97.0% of employed graduates for all waves. In addition, in Figure 5 panel a), b) and c) we report differences of mobility flows across surveys disentangling treated and untreated regions. These panels show a variability of mobility flows that is almost zero for all regions ranging from -3.5% to 1.5%. Grounding on this evidence we argue that mobility does not represent a serious caveat that may undermine our results’ interpretation hence we estimate our model considering no movers only. However, since we lose very few observations, estimates remain unchanged when considering the whole sample.

A second drawback may derive from the fact that the occurrence of mismatch may be recorded
Figure 4: Share of employed individuals according to region of study and region of work for treated (black) and untreated (white) regions. Region Valle d’Aosta excluded from our sample.
Figure 5: Difference in the share of employed individuals according to region of study and region of work for treated (black) and untreated (white) regions. Region Valle d’Aosta excluded from our sample.
only for employed workers. In practical terms, in eq. (37) we observe the dependent variable \( \Lambda_i \) only if the individual is actually employed. Since the creation of new campuses may also affect labor market participation, by ignoring this potential source of selection bias, we could confound the effect of the policy on employment probability with its effect on the probability of mismatch. To tackle this issue, we estimate the so called Averaged-Log-Likelihood-Function accounting for the probability of being mismatched and for the probability of being employed. In this case, the effect of the policy on employment probability is controlled by including variables \( A_i \), \( G_i \) and \( G_i * A_i \) in the employment equation too, while identification problems are solved by means of exclusion restrictions related to job characteristics which are not included in the selection equation.

5.5 Results

5.5.1 First check: Double differences across samples

We estimate eq. (37) by carrying out a preliminary pair comparison between the 2001 and the 2007 samples. The dependent variable \( \Lambda_i \) is equal to 1 if individual \( i \) declares to be mismatched, i.e., he declares neither his degree or any other degree qualification is required to apply for his job. The sample considers only full-time non-movers dependent workers and in this case it consists of about 14,000 individuals. In the RHS of eq. (37), \( X_i \) indicates a set of 20 control variables (age dummies, gender, marital status, 4 major dummies, university leaving grade, a dummy indicating the time to degree, high school leaving grade by 5 types of high school, parents’ education, a multilevel firm size dummy, a dummy for the public sector and a multilevel dummy for industries) plus 18 regional dummies. \( G_i = \{0,1\} \) indicates the ‘treatment’ and takes the value of 1 if individual \( i \) is working in a region that has been involved in the higher education expansion while \( A_i = \{0,1\} \) indicates the before/after period and it takes the value 1 for individuals from the 2007 sample. Our parameter of interest is \( \beta_2 \) which measures the relative variation in the probability of being mismatched for workers in treated regions after the reform compared to workers in untreated regions. As reported in column (1) of Table 1, the estimated value for \( \beta_2 \) is statistically significant and is about \(-13.8\%\) with a corresponding marginal effect of about \(-5.0\%\).
Table 1: Double Differences Estimates with Multiple Periods and Groups

<table>
<thead>
<tr>
<th>Method</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Dependent variable</td>
<td>Latent variable equal to one in the case of mismatch</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent Variable</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$G \times A$</td>
<td>$-0.136^{**}$ (0.005)</td>
<td>$-0.049^{**}$ (0.005)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$(G \times January02/03$ December07$)$</td>
<td>No</td>
<td>$-0.098^{**}$ (0.026)</td>
<td>$-0.032^{**}$ (0.026)</td>
<td>$-0.082^{**}$ (0.007)</td>
</tr>
<tr>
<td>$(G \times January01$ December01/02$)$</td>
<td>No</td>
<td>$-0.005$ (0.291)</td>
<td>$-0.007$ (0.291)</td>
<td>$-0.001$ (0.345)</td>
</tr>
<tr>
<td>Job start-year Fixed effects (9)</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Job start-year Fixed effects (9)</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$G=$Treated region Fixed Effects (2)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Sample-year Fixed effects (2)</td>
<td>Yes (2)</td>
<td>Yes (3)</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Clustered S.E.</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Control Var. (20)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Regional Dumm. (18)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Obs.</td>
<td>13,934</td>
<td>24,202</td>
<td>24,202</td>
<td>24,202</td>
</tr>
</tbody>
</table>

Notes: Maximum Likelihood estimates (BHHH procedure applied). Robust p-values in parentheses. The dependent variable is a 0-1 latent variable taking the value 1 in case of mismatch. In all columns only dependent workers with a 4/5 years degree qualification are considered and region Valle D’Aosta has been excluded. In column (1) only 2001 and 2007 surveys used; $G = 1$ if the individual attended university in a treated region (Piemonte, Lombardia, Trentino Alto-Adige, Marche, Molise, Puglia and Sicilia) and $A = 1$ if the individual is from the 2007 survey. In columns (2)-(4) $G = 1$ if the individual attended university in a treated region; $January02/03$ $December07$ is a dummy variable equal to 1 if the individual has been employed after December 2001 or after December 2002; $January01$ $December01/02$ is a dummy variable equal to 1 if the individual has been employed from January 2001 to December 2001 or December 2002. In column (3) Job start-year fixed effects used instead of Sample-year fixed effects. In column (4) the same specification of column (3) is estimated and treated region fixed effects for each Job start-year have been included.
Table 2: Double Differences Estimates with Correction for Sample Selection with Multiple Periods and Groups.

<table>
<thead>
<tr>
<th>Main Equation</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent Variable</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$-0.088^{**}$</td>
<td>$(0.050)$</td>
<td>$-0.031^{**}$</td>
<td>$(0.050)$</td>
</tr>
<tr>
<td>$(G \times January02/03 _December07)$</td>
<td>No</td>
<td>$-0.104^{**}$</td>
<td>$(0.020)$</td>
<td>$-0.034^{**}$</td>
</tr>
<tr>
<td>$(G \times January01 _December01/02)$</td>
<td>No</td>
<td>$-0.001$</td>
<td>$(0.150)$</td>
<td>$-0.008$</td>
</tr>
<tr>
<td>(Job start-year) * $(G)$ Fixed Effects (9)</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Job start-year Fixed Effects (9)</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>$G$</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Sample-year Fixed effects</td>
<td>Yes (2)</td>
<td>Yes (3)</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td>Control Var. (20)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Regional Dumm. (18)</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Selection Equation</th>
<th>Latent variable equal to one in the case of employment</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$-0.043$</td>
</tr>
<tr>
<td>$(G \times 2004)$</td>
<td>No</td>
</tr>
<tr>
<td>$G$</td>
<td>Yes</td>
</tr>
<tr>
<td>Sample-year Fixed effects</td>
<td>Yes (2)</td>
</tr>
<tr>
<td>Control Var. (14)</td>
<td>Yes</td>
</tr>
<tr>
<td>Regional Dumm. (18)</td>
<td>Yes</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.810$^{***}$</td>
</tr>
<tr>
<td>Obs.</td>
<td>21,088</td>
</tr>
</tbody>
</table>

Notes: Maximum Likelihood estimates of the Averaged Log-likelihood functions (BHHH procedure applied). Robust p-values in parentheses.
For the main equation see notes in Table 1. $\rho$ is the estimated correlation index between residuals in the main and in the selection equation. In the selection equation $(G \times 2004)$ and $(G \times 2007)$ interact individuals from treated regions with the 2004 and 2007 survey respectively, hence the reference category are individuals from treated regions from the 2001 survey.
5.5.2 Second Check: Double differences with multiple groups and time periods

In this paragraph we construct an empirical strategy in order to be able to estimate a DD model and, simultaneously, to use all available data sets. This procedure makes possible to test the common time trend assumption, i.e., to test if prior to the reform no significant differences arise in the probability of being mismatched for individuals from both treated and untreated regions.

We apply a DD strategy according to the following framework:

\[ E[A_{isj}|X_{isj}, G_{isj}, A_{isj}] = N \left( X_{isj}\beta + \xi_s + \chi_j + \beta_0 G_{isj} + \right. \]
\[ \left. \beta_1 (G \ast January01_{01-02})_{isj} + \beta_2 (G \ast January02_{03-07})_{isj} \right) \]

where \( i \) corresponds to individuals, \( s \) to the year in which the individual \( i \) has been interviewed and \( j \) indicates groups. \( \xi_s \) are sample fixed effects (2001, 2004 and 2007). \( \chi_j \) is a dummy indicating groups fixed effects for workers in treated and untreated regions. Only non-movers full-time dependent workers are considered and in this case we are using about 24,000 observations. \( G_{isj} \) is a dichotomous variable taking the value 1 if the individual works in a region that has been characterized by higher education expansion. \( X_{isj} \) contains the 18 regional dummy variables plus the 20 control variables as described in previous paragraph. Variable \( (G \ast January01_{01-02})_{isj} \) is a dummy taking the value 1 if the individual is resident in a region where a new campus was established in 1998/1999 and he has found a job in the period January 2001-December 2001/2002. Variable \( (G \ast January02_{03-07})_{isj} \) is a dummy taking the value of 1 if the individual is resident in a region where a new campus was established in 1998/1999 and he has found a job after December 2001/2002. Therefore, the reference dummy variable considers individuals from treated regions whose occupation starts between January 1998 and December 2000. It is worth noting that the introduction of the variable \( (G \ast January01_{01-02})_{isj} \) allows us to test the common time trend assumption, i.e., prior to the reform there should be no significant differences in the probability of being mismatched for individuals from both treated and untreated regions. As in paragraph 5.5.1 the coefficient of main interest is \( \beta_2 \). In column (2) of Table 1 \( \beta_2 \) is equal to \(-9.8\%\) (which corresponds to a marginal effect of about \(-3.2\%\)) and it is statistically significant. This means that graduates who work in regions where new campuses have been created have a lower probability.
of being mismatched compared with their colleagues employed in the 1998-2000 period. The common time effect assumption is verified being $\hat{\beta}_1$ not statistically different from zero, as reported in column (2). In column (3) of Table 1 we present additional estimates derived including among regressors year fixed effects instead of survey fixed effects, using information concerning the date of job start for each employed individual. Our results appear to be robust according to this additional specification too. Furthermore, in column (4) we report more robust estimates obtained after including among regressors time-varying regional specific effects (9). This approach has the advantage of taking into account the concerns raised by Conley and Taber (2011) about the inconsistency of the difference-in-differences estimation when the treated group and the number of policy changes are small. Accounting for time-varying treated-region specific effects is perfectly in line with the solution proposed by these authors. As in the previous case only the coefficient associated to $(G * January02/03 _December07)_{isj}$ is statistically significant with a marginal effect of $-4.8\%$.

5.5.3 Accounting for sample selection

In this section we address concerns related to possible sample selection bias which can be a serious obstacle when dealing with educational mismatch whose occurrence is recorded for employed workers only. This is particularly true when estimating a policy effect in a DD framework. Indeed, higher education expansion may have changed the employment probability instead of that of being well matched and, in this case, we would confound the effect of the reform on mismatch with that on employment. Put differently, if the creation of new campuses has reduced the individual probability of being employed (for instance by boosting participation into post-graduate education), we could detect significant effect of the reform on mismatch only because there are less individuals in the labor force within the treated group. To tackle this issue we estimate a bivariate Probit model, i.e., we estimate simultaneously the interest and the selection equations by means of maximum likelihood estimator. Control variables in the selection equation are all those included in eq. (38) but we exclude variables related to job characteristics, hence the model is identified. On top of that, variables $\xi_x$, $\chi_j$, $G_{isj}$ are all included in the selection equation. Since in the employment equation we cannot use information concerning the job-start date for unemployed individuals, we construct two interaction dummies $(G * 2004)_{isj}$
and \((G_{2007})_{i,s,j}\) taking the value of 1 if individual \(i\) is resident in a treated region and he is recorded in the 2004 or in the 2007 survey respectively. Therefore, in the employment equation the parameter associated to \((G_{2007})_{i,s,j}\) gives us a measure of the effect (if any) of the reform on the employment probability. Results are reported in Table 2, according to our previous exercises and robustness checks. The total number of observations we are using in these cases ranges from 21,000 to 33,000. As it appears from Table 2, the issue of sample selection is relevant when dealing with individual mismatch since a positive and significant correlation between the residuals of the two equations is reported. Notwithstanding, the estimates concerning the causal effect of the creation of new campuses on mismatch are entirely not affected by the new specification, confirming that in Italy graduates who work in regions involved in an expansion of the tertiary system of education have actually seen a reduction in their probability of being mismatched of about 5.0%.

6 Conclusions

This paper considers the issue of educational mismatch, a phenomenon affecting almost all developed countries. We undertake the analysis with the intent of offering a comprehensive theoretical framework to reconcile different interpretations on its occurrence and consequences. We link the extent of mismatch to many factors: The distribution of pre-college individual ability, the distribution of firms in terms of technological contents and the selectivity level of the higher education sector. These variables interact each other along several dimensions shaping the specific form of mismatch. We highlight the possibility that educational mismatch arises as an efficient outcome: In the presence of search-frictions, it may be the consequence of a wider set of labor market opportunities for graduate workers. Nevertheless, it may be an inefficient outcome too, resulting from the interaction process underlining educational and technological investments. In this case, we disentangle two different scenarios requiring different higher education policy measures. In particular, we argue that the provision of incentives for participating to higher education may affect mismatch depending on the dominance of tightness or composition scenario. In the former case, since the number of graduates is low, a higher education expansion generates positive externalities reducing mismatch. We present some evidence for Italy, a country characterized
by a high incidence of mismatch and, simultaneously, a very low rate of workers with tertiary education. We provide evidence that individuals resident in regions which have expanded their supply of education have actually seen a reduction in the probability of being mismatched. This finding attests that the presence of graduates in undergraduate jobs does not imply that there are too many educated people in the labor market. We argue that the characterization of the specific scenario where mismatch takes place should be attentively considered by policy makers in order to boost the creation of graduate-complementary job positions and to rise efficiency.

References


AppendixA

Proof of a the existence of a unique matching function for the undergraduate sector: Here we prove that equations (7) and (8) derive from equation (9). Using the fact that 

\[
\alpha_{ug} = \frac{H_{ug} + H^O_g}{V_{ug}}
\]

after summing up eqs. (7) and (8) and re-arranging we get:

\[
H_{ug} + H^O_g = \left[ \frac{K}{\alpha_{ug}} \right]^{1/1-\eta} [U_{ug} + U_g]^{1/1-\eta}.
\]  

(39)
By taking exponential of both sides and re-arranging we get:

\[ e^{H_{ug} + H_g^O} = e^\left[ \frac{\alpha_{ug}}{w_{ug}} \right]^{1/1-\eta} U_{ug} + \left[ \frac{\alpha_{ug}}{w_{ug}} \right]^{1/1-\eta} U_g. \] (40)

By taking logs of both sides, using the expression for \( \alpha_{ug} \) and re-arranging we get:

\[ M_{ug} \equiv H_{ug} + H_g^O = KV_{ug}^O (U_{ug} + U_g)^{1-\eta}. \quad Q.E.D. \]

**Proof of the existence of a steady-state employment level for graduate and undergraduate workers:** We divide the proof in two parts. In Part 1 we set the existence of a steady-state (s.s.) employment level for undergraduate workers for all \( \eta \in (0, 1) \). In Part 2 we set the conditions under which when undergraduates’ employment is in a s.s. graduates employment level is in a s.s. too with \( a_g^O > 0 \), i.e., with a positive outflow of unemployed graduates towards undergraduate jobs.

Part 1. From eqs. (14) and (15) we know that:

\[ V_E^{ug} - V_U^{ug} = \frac{w^{ug}}{r + \alpha_{ug}} + b. \] (41)

Since in equilibrium \( V_E^{ug} - V_U^{ug} = V_F^{ug} - V_{ug}^V \), using eq. (27) we can write eq. (21) as follows:

\[ rV_{ug}^V = -Q + \alpha_{ug} - \alpha_{ug} + 2b + 2r. \] (42)

Since we assumed the free entry condition, in a s.s. where \( \dot{E}_{ug} = 0 \) we must have \( rV_{ug}^V = 0 \).

Here we prove that eq. (42) is strictly decreasing in \( E_{ug} \) with a positive value in \( E_{ug} = 0 \) and a negative value in \( E_{ug} = \Gamma(\theta) \), i.e. it must exist an employment level \( E_{ug} \) in which \( rV_{ug}^V = 0 \). In equation (42) consider \( a_{ug} = \frac{H_{ug}}{U_{ug} + U_g} \). In a s.s. we must have that \( bE_{ug} = H_{ug} \) hence we can write \( a_{ug} \) as follows:

\[ a_{ug} = \frac{bE_{ug}}{1 - E_{ug} - E_g}. \] (43)

with: i) \( \lim_{E_{ug} \to 0} a_{ug} \to 0 \); ii) \( \lim_{E_{ug} \to \Gamma(\theta)} a_{ug} \to \frac{H(\theta)}{1 - \Gamma(\theta) - E_g} \); iii) \( \frac{\partial a_{ug}}{\partial E_{ug}} > 0 \).
Now, consider \( \alpha_{ug} = \frac{H_{ug} + H^O_{ug}}{V_{ug}} \). Since \( bE_{ug} = H_{ug} \) and \( H^O_g = KV_{ug}U_1^{-\eta} \) we can write \( \alpha_{ug} \) as follows:

\[
\alpha_{ug} = \frac{bE_{ug}}{V_{ug}} + KV_{ug}^{\eta-1}U_1^{-\eta}. \tag{44}
\]

By using the fact that from eq. (7) \( V_{ug} = \left( \frac{bE_{ug}}{KV_{ug}} \right)^{1-\eta} \) we can write eq. (44) as:

\[
\alpha_{ug} = (bE_{ug})^{\frac{n-1}{n}} K [\Gamma(\theta) - E_{ug}]^{1-n} \left[ 1 + \frac{K [1 - \Gamma(\theta) - E_{ug}]^{1-\eta}}{K^{\eta-1} [\Gamma(\theta) - E_{ug}]^{\eta-1}} \right]. \tag{45}
\]

with: i) \( \lim_{E_{ug} \to 0} \alpha_{ug} \to +\infty \); ii) \( \lim_{E_{ug} \to \Gamma(\theta)} \alpha_{ug} \to 0 \) (\( \forall \eta \in (0,1) \)); iii) \( \frac{\partial \alpha_{ug}}{\partial E_{ug}} < 0 \).

Now, we can evaluate eq. (42) as a function of \( E_{ug} \). Given our results concerning \( a_{ug} \) and \( \alpha_{ug} \) we have that: i) \( \lim_{E_{ug} \to 0} rV^V_{ug} \to \bar{y} - Q \); ii) \( \lim_{E_{ug} \to \Gamma(\theta)} rV^V_{ug} \to -Q \); iii) \( \frac{\partial V^V}{\partial E_{ug}} < 0 \). As a consequence, it exists a level \( E_{ug} \) in which eq. (42) is equal to zero and this value is a s.s. \( Q.E.D. \).

It is important to note \( a_{ug} \) and \( \alpha_{ug} \) are both functions of \( E_g \) as well. As a consequence, the s.s. value of \( E_{ug} \) is a function the employment level in the graduate sector \( E_g \). In particular, from eqs. (43) and (45) we have that \( \frac{\partial a_{ug}}{\partial E_g} < 0 \) and \( \frac{\partial \alpha_{ug}}{\partial E_g} > 0 \). As a consequence, from eq. (42) it is easy to check that \( \frac{\partial V^V}{\partial E_{ug}} < 0 \). Using the implicit function theorem we have that in the s.s. \( \frac{\partial E_{ug}}{\partial E_g} < 0 \). In words, since graduates can be employed in the undergraduate labor market, the larger the share of employed graduates, the lower the share of employed undergraduates. In Figure 6, we draw the line representing the s.s. \( (\dot{E}_{ug} = 0) \) in the undergraduate labor market and we indicate with \( E_{ug} \) the s.s. of employed undergraduates when all graduates are employed.

Part 2. Graduates can be in a s.s. only if \( \dot{E}_g = 0 \). We know that

\[
\dot{E}_g = H^R_g + H^O_g - bE_g \tag{46}
\]

with \( H^R_g = a^R_g(\lambda)U_g \) and \( H^O_g = a^O_g(U_{ug} + U_g) \). We can write (46) as follows:

\[
\dot{E}_g = a^R_g(\lambda)(1 - \Gamma(\theta) - E_g) + a^O_g(1 - E_g - E_{ug}) - bE_g \tag{47}
\]
which implies that $\dot{E}_g = 0$ only if:

$$E_{ug} = 1 + \frac{1 - \Gamma(\theta)}{a_g^O} - E_g \left[ \frac{1 + b + a_g^R(\lambda)}{a_g^O} \right]$$

(48)

where:

$$\lim_{E_g \to 0} E_{ug} = 1 + \frac{1 - \Gamma(\theta)}{a_g^O} \left( \frac{1 - \Gamma(\theta)}{V_g} \right)$$

(49)

$$\lim_{E_g \to 1 - \Gamma(\theta)} E_{ug} = 1 - \frac{1 - \Gamma(\theta)}{a_g^O} (1 + b)$$

with $M > m \forall a_g^O > 0$. Since eq. (48) describes a continuous function, if $m$ is greater than $E_{ug}$, there must exist at least one point (in Figure 5 we represent the case of a single point SS) representing a pair $(E_g, E_{ug})$ that is a s.s. for both markets. Moreover, if

$$a_g^O \frac{\partial a_g^R}{\partial E_g} (1 - E_g) < \frac{\partial a_g^O}{\partial E_g} \left[ a_g^R(\lambda) + (1 + b + a_g^R(\lambda))E_g \right] + a_g^O (1 + b + a_g^R(\lambda))$$

eq (48)

is monotonically decreasing and the s.s. is unique. Q.E.D.

**Analytical derivation of eq. (33):** By using eqs. (21), (26), and (27) we have that:

$$rV_{ug} = -Q + \alpha_{ug} \frac{\bar{y}}{a_{ug} + a_{ug} + 2b + 2r}$$

(50)

By implementing some algebra on eqs. (19)-(21) using eq. (26) we have the following result:

$$V_g^R - V_g^U = \frac{(r + b + a_g^O)w_g^R - a_g^O w_g^O}{(r + b)(r + b + a_g^R(\lambda) + a_g^O)}$$

that can be substituted into eq. (23). After this substitution, by taking expectations of both sides of eq. (23) conditional upon $\theta > \theta^*$ and by substituting into that the wage expressions for $w_g^R$, and $w_g^O$ i.e., eqs. (29) and (28) we obtain the following expression:

$$rE[V_g^V|\theta > \theta^*] = -Q + \frac{\alpha_g(\lambda)\bar{y}}{A} \left[ E[\theta|\theta > \theta^*] \left( C - \frac{a_g^O a_g^R(\cdot)}{B} \right) \right] - \frac{A}{F}$$

(51)
Figure 6: A steady-state equilibrium in graduate and undergraduate labor markets.

where \( A, B, C, \) and \( F \) summarize strictly positive constant as indicated in the main text. By using eqs. (50) and (51) we can write the cutoff level \( \delta^* \) in eq. (30) as indicated in eq. (33).

**Proof of the effect of variation of \( c(\theta, \mu) \) on \( a^R_g(.) \) and \( a^O_g \) in tightness-dominance scenario:** In this part we show that in the tightness-dominance scenario a reduction in the selectivity level of education \( \theta^* \) generates an increase in the individual probability of having a right match \( (a^R_g(.)) \uparrow \) and a reduction of the probability of having a wrong match \( (a^O_g(.) \downarrow) \). We know that if \( \theta^* \downarrow \) we have more \( g \) workers and more \( g \) firms. Since the mass of firms and workers is constant, the economy is characterized by more \( g \) workers and less \( ug \) firms and, consequently, the probability that a graduate worker is employed in a \( ug \) firm decreases \( (a^O_g(.) \downarrow) \). Now, consider \( a^R_g(.) \). Since both \( g \) workers and \( g \) firms increase the effect on \( a^R_g(.) \) may be ambiguous since both demand and supply side increased. Here we prove that \( a^R_g(.) \) must raise. By contradiction, assume \( a^R_g(.) \) decreases. Since \( a^O_g(.) \) decreases too, in the s.s. the unemployment level of \( g \) workers increases \( (U_g \uparrow) \). Unemployed undergraduate must decrease \( (U_{ug} \downarrow) \) otherwise we would not be in a s.s.. If \( U_{ug} \downarrow \), since \( V_{ug} \) decreased too, we have that the number matches between \( ug \) workers and \( ug \) firms falls down, i.e., \( H_{ug} = KV_{ug}U_{ug}^{1-n} \) reduces. Since in a s.s. \( H_{ug} = bE_{ug} \) and \( b \) is constant, \( E_{ug} \) must decrease as well. If \( E_{ug} \downarrow \) then \( U_{ug} \uparrow \) and we have a contradiction. Q.E.D.
Appendix B

Table B1: Description of Variables

<table>
<thead>
<tr>
<th>Individual and Household</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Female</td>
<td>Dummy variable indicating the respondent’s sex, Female=1, 0 otherwise.</td>
</tr>
<tr>
<td>Age</td>
<td>Respondent’s age at the interview in four classes.</td>
</tr>
<tr>
<td>Employed</td>
<td>Dummy variable indicating if the respondent is working at the interview, Employed=1, 0 otherwise.</td>
</tr>
<tr>
<td>Wage</td>
<td>Monthly wage of full-time workers.</td>
</tr>
<tr>
<td>Parents education</td>
<td>Two dummy variables indicating if the respondent’s parents have a university degree. Father education=1 if the father has a university degree, 0 otherwise; Mother education=1 if the mother has a university degree, 0 otherwise.</td>
</tr>
<tr>
<td>Regional dummies</td>
<td>20 dummy variables indicating the respondent’s region of residence according to the ISTAT classification.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Education</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Degree subject</td>
<td>A vector of 6 0-1 dummy variables indicating degree subjects: 1) Science=1 if mathematics, science, chemistry, pharmacy, geo-biology, agrarian; 2) Medicine=1 if medicine; 3) Engineering=1 if engineering, architecture; 4) Econ.&amp;Law=1 if political science, economics, statistics, law; 5) Humanities=1 if humanities, linguistic, teaching, psychology; 6) Sport Science=1 if sport science.</td>
</tr>
<tr>
<td>High School Grade</td>
<td>Final score (scale from 36 to 60) by type of high school: Lyceum; Teaching; Accountancy; Vocational.</td>
</tr>
<tr>
<td>University Grade</td>
<td>Final score (scale from 66 to 110).</td>
</tr>
<tr>
<td>Time to degree</td>
<td>Multiple dummy variable indicating the number of years in excess with respect to the institutional course duration.</td>
</tr>
<tr>
<td>Mismatch</td>
<td>Dummy variable for the answer to the question: &quot;Is your degree or any other university degree a required qualification for your job?&quot;, Mismatch=1 if the answer is not, 0 otherwise.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Job</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permanent job</td>
<td>Dummy variable indicating if the respondent has a temporary or a permanent contract at the interview, Permanent job=1, 0 otherwise.</td>
</tr>
<tr>
<td>Para-subordinate job</td>
<td>Dummy variable indicating if the respondent has a para-subordinate temporary contract (contratto a progetto) at the interview, Para-subordinate job=1 if yes, 0 otherwise.</td>
</tr>
<tr>
<td>Self-employed</td>
<td>Dummy variable indicating if the individual is either self-employed or he has a subordinate/para-subordinate job; Self-employed=1 if self-employed, 0 otherwise.</td>
</tr>
<tr>
<td>Firm size</td>
<td>Multilevel dummy variable indicating plant size according to the number of employed worker. Firm size=0 if employees≤ 5; Firm size=1 if 5 &lt;employees&lt; 15; Firm size=2 if 15 ≤employees&lt; 50; Firm size=3 if 50 ≤employees&lt; 100; Firm size=4 if employees≥ 100.</td>
</tr>
<tr>
<td>Industry</td>
<td>A multilevel dummy variable (6 levels) indicating the industry sector for employed individuals.</td>
</tr>
<tr>
<td>Firm ownership</td>
<td>A dummy variable indicating if the firm ownership is public or private, Public=1, 0 otherwise.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>2001</th>
<th>2004</th>
<th>2007</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency</td>
<td>Percentage</td>
<td>Frequency</td>
</tr>
<tr>
<td><strong>Individual Features</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>20,844</td>
<td>100.0%</td>
<td>25,674</td>
</tr>
<tr>
<td>Female</td>
<td>11,148</td>
<td>54.6%</td>
<td>12,925</td>
</tr>
<tr>
<td>Male</td>
<td>9,273</td>
<td>45.4%</td>
<td>12,152</td>
</tr>
<tr>
<td>Mean Age class</td>
<td>2.8</td>
<td></td>
<td>2.6</td>
</tr>
<tr>
<td>Married</td>
<td>6,202</td>
<td>29.7%</td>
<td>7,432</td>
</tr>
<tr>
<td>Single</td>
<td>14,642</td>
<td>70.3%</td>
<td>18,360</td>
</tr>
<tr>
<td>Father education</td>
<td>4,519</td>
<td>21.7%</td>
<td>6,204</td>
</tr>
<tr>
<td>Mother education</td>
<td>2,632</td>
<td>12.6%</td>
<td>3,944</td>
</tr>
<tr>
<td>Mean University grade</td>
<td>103/110</td>
<td></td>
<td>102/110</td>
</tr>
<tr>
<td>Mean High school grade</td>
<td>48.8</td>
<td></td>
<td>49.4</td>
</tr>
<tr>
<td><strong>Degree subject</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Science</td>
<td>4,037</td>
<td>19.4%</td>
<td>4,904</td>
</tr>
<tr>
<td>Medicine</td>
<td>1,259</td>
<td>6.0%</td>
<td>4,175</td>
</tr>
<tr>
<td>Humanities</td>
<td>4,696</td>
<td>23.8%</td>
<td>4,110</td>
</tr>
<tr>
<td>Econ&amp;Law</td>
<td>7,076</td>
<td>33.9%</td>
<td>7,142</td>
</tr>
<tr>
<td>Engineering</td>
<td>3,509</td>
<td>16.8%</td>
<td>5,036</td>
</tr>
<tr>
<td>Sport Science</td>
<td>-</td>
<td>-</td>
<td>659</td>
</tr>
</tbody>
</table>

Note: Variables defined in Table B1.

<table>
<thead>
<tr>
<th></th>
<th>Frequency</th>
<th>Percentage</th>
<th>Frequency</th>
<th>Percentage</th>
<th>Frequency</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Whole sample</td>
<td>20,844</td>
<td>100%</td>
<td>25,674</td>
<td>100%</td>
<td>26,570</td>
<td>100%</td>
</tr>
<tr>
<td>Obs.</td>
<td>15,334</td>
<td>73.6%</td>
<td>18,165</td>
<td>70.6%</td>
<td>17,928</td>
<td>67.5%</td>
</tr>
<tr>
<td>Employed</td>
<td>1,933</td>
<td>9.3%</td>
<td>1,688</td>
<td>6.6%</td>
<td>1,873</td>
<td>7.0%</td>
</tr>
<tr>
<td>Unemployed</td>
<td>3,577</td>
<td>17.1%</td>
<td>5,040</td>
<td>19.7%</td>
<td>5,981</td>
<td>22.5%</td>
</tr>
<tr>
<td>Not in the labor force</td>
<td>781</td>
<td>3.1%</td>
<td>788</td>
<td>3.0%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Missing</td>
<td>11.2%</td>
<td></td>
<td>8.5%</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment rate</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Employed Individuals</td>
<td>10,636</td>
<td>68.5%</td>
<td>11,302</td>
<td>62.2%</td>
<td>11,242</td>
<td>62.7%</td>
</tr>
<tr>
<td>Dependent workers</td>
<td>2,669</td>
<td>17.3%</td>
<td>3,319</td>
<td>18.3%</td>
<td>2,685</td>
<td>15.0%</td>
</tr>
<tr>
<td>Self-employed</td>
<td>-</td>
<td>-</td>
<td>44</td>
<td>0.2%</td>
<td>1,132</td>
<td>6.3%</td>
</tr>
<tr>
<td>Para-subordinate workers</td>
<td>-</td>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dependent workers</td>
<td>7,981</td>
<td>75.5%</td>
<td>8,199</td>
<td>76.3%</td>
<td>7,412</td>
<td>69.2%</td>
</tr>
<tr>
<td>Permanent</td>
<td>2,586</td>
<td>24.5%</td>
<td>2,542</td>
<td>23.6%</td>
<td>3,292</td>
<td>31.8%</td>
</tr>
<tr>
<td>Temporary</td>
<td>2,965</td>
<td>27.87%</td>
<td>2,848</td>
<td>25.19%</td>
<td>2,778</td>
<td>24.71%</td>
</tr>
<tr>
<td>Mismatched</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wage</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Obs.</td>
<td>11,093</td>
<td>72.3%</td>
<td>13,148</td>
<td>71.8%</td>
<td>15,041</td>
<td>83.9%</td>
</tr>
<tr>
<td>Mean wage</td>
<td>1,026 Euro</td>
<td></td>
<td>1,113 Euro</td>
<td></td>
<td>1,180 Euro</td>
<td></td>
</tr>
</tbody>
</table>

Variables defined in Table B1.