FORMULE UTILI

A.

1. COSTANTI MATEMATICHE

e	$2.7182818285\ldots$
π	$3.1415926536\ldots$
$\log_{10} 2$	$0.3010299957\ldots$
$\log_{10} e$	$0.4342944819\ldots$
$\log_{10} \pi$	$0.4971498727\ldots$
$\log_e 2$	$0.6931471806\ldots$
$\log_e \pi$	$1.1447298858\ldots$
$\log_e 10$	$2.3025850930\ldots$
$\sqrt{2}$	$1.4142135624\ldots$
$\sqrt{e}$	$1.6487212707\ldots$
$\sqrt{3}$	$1.7320508076\ldots$
$\sqrt{\pi}$	$1.7724538509\ldots$
$\sqrt{5}$	$2.2360679775\ldots$
$\sqrt{10}$	$3.1622776602\ldots$
1°	$0.0174532925\ldots$ radianti
1 radiante	$57^\circ 17' 44''.806\ldots$

2. FUNZIONI TRIGONOMETRICHE

$$\sin x \qquad \cos x \qquad \operatorname{tg} x = \frac{\sin x}{\cos x} \qquad \operatorname{cotg} x = \frac{\cos x}{\sin x}$$

$$(\sin x)^2 + (\cos x)^2 = 1$$

- *Angoli notevoli*

x	$\cos x$	$\sin x$	$\operatorname{tg} x$	$\operatorname{cotg} x$
0	1	0	0	$\pm\infty$
$\frac{\pi}{10} = 18°$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\frac{\sqrt{5}-1}{4}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$	$\sqrt{5+2\sqrt{5}}$
$\frac{\pi}{6} = 30°$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$\frac{\pi}{5} = 36°$	$\frac{\sqrt{5}+1}{4}$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$\sqrt{5-2\sqrt{5}}$	$\sqrt{\frac{5+2\sqrt{5}}{5}}$
$\frac{\pi}{4} = 45°$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$\frac{3\pi}{10} = 54°$	$\frac{1}{4}\sqrt{10-2\sqrt{5}}$	$\frac{\sqrt{5}+1}{4}$	$\sqrt{\frac{5+2\sqrt{5}}{5}}$	$\sqrt{5-2\sqrt{5}}$
$\frac{\pi}{3} = 60°$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$\frac{2\pi}{5} = 72°$	$\frac{\sqrt{5}-1}{4}$	$\frac{1}{4}\sqrt{10+2\sqrt{5}}$	$\sqrt{5+2\sqrt{5}}$	$\sqrt{\frac{5-2\sqrt{5}}{5}}$
$\frac{\pi}{2} = 90°$	0	1	$\pm\infty$	0

- *Simmetrie, archi complementari e supplementari*

$$\sin(-x) = -\sin x \qquad \cos(-x) = \cos x \qquad \operatorname{tg}(-x) = -\operatorname{tg} x$$

$$\sin\left(x \pm \frac{\pi}{2}\right) = \pm\cos x \qquad \sin\left(x \pm \frac{\pi}{2}\right) = \mp\sin x \qquad \operatorname{tg}\left(x \pm \frac{\pi}{2}\right) = -\operatorname{cotg} x$$

$$\sin(x \pm \pi) = \mp\sin x \qquad \cos(x \pm \pi) = \mp\cos x \qquad \operatorname{tg}(x \pm \pi) = \operatorname{tg} x$$

- *Formule di addizione*

$$\sin(x \pm y) = \sin x \cos y \pm \cos x \sin y$$

$$\cos(x \pm y) = \cos x \cos y \mp \sin x \sin y$$

$$\operatorname{tg}(x \pm y) = \frac{\operatorname{tg} x \pm \operatorname{tg} y}{1 \mp \operatorname{tg} x \operatorname{tg} y}$$

- *Formule di duplicazione*

$$\sin 2x = 2\sin x \cos x$$

$$\cos 2x = (\cos x)^2 - (\sin x)^2 = 2(\cos x)^2 - 1 = 1 - 2(\sin x)^2$$

$$\operatorname{tg} 2x = \frac{2\operatorname{tg} x}{1 - (\operatorname{tg} x)^2}$$

- *Formule di bisezione* (scegliere il segno corretto)

$$\sin \frac{x}{2} = \pm\sqrt{\frac{1 - \cos x}{2}}$$

$$\cos \frac{x}{2} = \pm\sqrt{\frac{1 + \cos x}{2}}$$

$$\operatorname{tg} \frac{x}{2} = \frac{1 - \cos x}{\sin x} = \frac{\sin x}{1 + \cos x}$$

- *Formule di prostaferesi*

$$\sin u + \sin v = 2\sin \frac{u+v}{2} \cos \frac{u-v}{2}$$

$$\sin u - \sin v = 2\cos \frac{u+v}{2} \sin \frac{u-v}{2}$$

$$\cos u + \cos v = 2\cos \frac{u+v}{2} \cos \frac{u-v}{2}$$

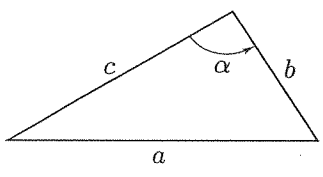
$$\cos u - \cos v = -2\sin \frac{u+v}{2} \sin \frac{u-v}{2}$$

- *Formule parametriche*

Posto $t = \operatorname{tg}(x/2)$:

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2} \qquad \operatorname{tg} x = \frac{2t}{1-t^2}$$

- *Teorema di Carnot*



$$a^2 = b^2 + c^2 - 2bc \cdot \cos\alpha$$

4. DERIVATE ELEMENTARI

$f(x)$	$f'(x)$
x^α	$\alpha x^{\alpha-1}$
$\lvert x\rvert$	$\operatorname{sgn} x$
$\log\lvert x\rvert$	$1/x$
$\log_a\lvert x\rvert$	$1/(x\log a)$
e^x	e^x
a^x	$a^x\log a$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\operatorname{tg} x$	$1+(\operatorname{tg} x)^2 = 1/(\cos x)^2$
$\operatorname{ctg} x$	$-1-(\operatorname{ctg} x)^2 = -1/(\sin x)^2$
$\operatorname{Sh} x$	$\operatorname{Ch} x$
$\operatorname{Ch} x$	$\operatorname{Sh} x$
$\operatorname{Th} x$	$1-(\operatorname{Th} x)^2 = 1/(\operatorname{Ch} x)^2$
$\operatorname{Cth} x$	$1-(\operatorname{Cth} x)^2 = -1/(\operatorname{Sh} x)^2$
$\log\lvert\sin x\rvert$	$-\operatorname{ctg} x$
$\log\lvert\cos x\rvert$	$\operatorname{tg} x$
$\log\lvert\operatorname{Sh} x\rvert$	$\operatorname{Cth} x$
$\log\operatorname{Ch} x$	$\operatorname{Th} x$
$\arcsin x$	$1/\sqrt{1-x^2}$
$\arccos x$	$-1/\sqrt{1-x^2}$
$\operatorname{arctg} x$	$1/(1+x^2)$
$\operatorname{arccotg} x$	$-1/(1+x^2)$

5. REGOLE DI DERIVAZIONE

$$\begin{aligned}
\mathrm{D}(\lambda f(x)+\mu g(x)) &= \lambda f'(x)+\mu g'(x)\\
\mathrm{D}(f(x)g(x)) &= f'(x)g(x)+f(x)g'(x)\\
\mathrm{D}\,\frac{f(x)}{g(x)} &= \frac{f'(x)g(x)-f(x)g'(x)}{g(x)^2}\\
\mathrm{D}\,f(g(x)) &= f'(g(x))g'(x)\\
\mathrm{D}\,f(g(h(x))) &= f'(g(h(x)))g'(h(x))h'(x)\\
\mathrm{D}\,e^{f(x)} &= e^{f(x)}f'(x)
\end{aligned}$$

$$\mathrm{D}\log|f(x)| = \frac{f'(x)}{f(x)}$$

$$\mathrm{D}[f(x)]^{g(x)} = [f(x)]^{g(x)}\left\{g'(x)\log f(x) + \frac{g(x)f'(x)}{f(x)}\right\}$$

6. SVILUPPI DI MAC LAURIN DELLE PRINCIPALI FUNZIONI

$f(x)$	Sviluppo	
e^x	$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!} + \cdots$	
$\mathrm{Sh}\,x$	$x + \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots + \frac{x^{2n+1}}{(2n+1)!} + \cdots$	
$\mathrm{Ch}\,x$	$1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots + \frac{x^{2n}}{(2n)!} + \cdots$	
$\sin x$	$x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n\frac{x^{2n+1}}{(2n+1)!} + \cdots$	
$\cos x$	$1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \cdots + (-1)^n\frac{x^{2n}}{(2n)!} + \cdots$	
$\log(1+x)$	$x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots + (-1)^{n-1}\frac{x^n}{n} + \cdots$	
$\mathrm{arctg}\,x$	$x - \frac{x^3}{3} + \cdots + (-1)^n\frac{x^{2n+1}}{2n+1} + \cdots$	
$(1+x)^\alpha$	$1 + \alpha x + \binom{\alpha}{2}x^2 + \cdots + \binom{\alpha}{n}x^n + \cdots$	$\alpha \in \mathbb{R}$

In particolare:

$\frac{1}{1+x}$	$1 - x + x^2 - x^3 + \cdots + (-1)^n x^n + \cdots$	$(\alpha = -1)$
$\sqrt{1+x}$	$1 + \frac{x}{2} - \frac{x^2}{8} + \frac{x^3}{16} - \cdots + \frac{(-1)^{n+1}(2n-3)!!x^n}{(2n)!!}$	$(\alpha = \frac{1}{2})$
$\frac{1}{\sqrt{1+x}}$	$1 - \frac{x}{2} + \frac{3x^2}{8} - \frac{5x^3}{16} + \cdots + \frac{(-1)^n(2n-1)!!x^n}{(2n)!!}$	$(\alpha = -\frac{1}{2})$

ove

$$\binom{\alpha}{n} = \frac{\alpha(\alpha-1)(\alpha-2)\cdots(\alpha-n+1)}{n!} \qquad \text{(coefficiente binomiale generalizzato)}$$

$$k!! = k(k-2)(k-4)\cdots 2 \qquad (k \text{ semifattoriale})$$