Università della Calabria

Seminario sul tema:

RISK-BASED CAPITAL MODELLING FOR P&C INSURERS

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Agenda

This presentation is based on two papers having as coauthor L. Ballotta (Cass Business School – City University, London)

- Insurance Risk Management and Solvency
- General framework of the model
- The Insurance sub-model
- The Investment sub-model
- The RBC obtained from simulation results (VaR and TVaR)
- The impact of the main model's parameters on the RBC measures
- Further researches and improvements of the Model
- References

Insurance Risk Management and Solvency

- **MAIN PILLARS OF THE INSURANCE MANAGEMENT:**

  the strategic triangle of competing forces:

  – **market share / return for stockholders' capital / financial strength & stability**

- **NEW RISK-BASED CAPITAL REQUIREMENTS:**

  to assess the risk capital of the company according to its own real risk profile (and not according a "fit for all" rule as provided in Solvency I) .

  Simulation models may be used for defining Capital Adequacy – Internal Model approach (e.g. IAA Solvency Working Party, CEIOPS, etc.)

- **INTERNAL RISK MODELS (IRM):**

  to be used:

  – **for solvency purposes (e.g. Pillar 1&2 of Solvency II)**

  – **for management purposes: to define the most appropriate management's strategies**

  IRM could allow for a more comprehensive representation of the business of an individual firm than a standard formula (risk-factor based formula).

- **A NEW APPROACH FOR THE SUPERVISORY AUTHORITIES:**
  – **Stress testing in order to assess the solvency profile of the Insurer**
  – **Validation and approval of the Insurer IRM on the basis of**
    - **Prudential requirements**
    - **Comparability & consistency requirements (with respect to the supervisor's view of key minimum performance criteria)**
  – **Indication of the appropriate course of action to follow in case of an excessive risk of insolvency over the short term**

- **THE AIM OF THIS PRESENTATION:**
  – **to figure out a possible risk model for a P&C insurer incorporating**
    - **the (pure) underwriting risk**
    - **the financial risk**
  – **to analyse the results of different risk measures as capital requirements;**
  – **to show the impact of main parameters on the capital requirements (as e.g. asset allocation, company's dimension, claim size variability, etc.)**

General framework of the model

General framework of the model

- **Company:**
  General Insurer with only **1 line of casualty** insurance
- **Time Horizon: 3** years
- **Aggregate Claim amount: Compound Mixed Poisson** Process
- **Number of Claims:**
  Negative Binomial distribution
- **Claim Size:**
  LogNormal distribution
- **Dynamic Ins. Portfolio:** Volume of premiums increases every year according to **real growth** and **claim inflation**
- **Reinsurance:**
  reinsurance cover is **ignored**
- **Investment Portfolio:**
  1 category of assets for **Equities** and other **5** categories for **Gov.Bonds**, differentiated according to time to maturity (1, 2, 3, 5 and 10 years)
- **Investment Return:**
  - **Geometric Brownian motion** for **equities**
  - **CIR process** for **interest rates**
- **Asset Allocation rule:**
  constant proportion
- **Monte Carlo approach:**
  400,000 simulations
- **Risks not included:**
  - **Claim Reserve risk**
  - **Credit** and **Operational risk**
  - **ALM risk**

Risk Reserve process (U_t)

$$U_t = U_{t-1} + j_t \cdot (U_{t-1} + LR_{t-1} + PR_{t-1} + CF_t) + \left[(PR_{t-1} + \Pi_t - PR_t) - X_t - E_t\right] - TX_t - DV_t$$

- **U_t** = Risk Reserve at time t
- **Π_t = Gross premiums year t**
- **X_t** = Aggregate claims amount year t
- **E_t** = general and acquisition expenses year t
- **CF_t** = Cash Flows year t - at time (t-1)+
  $= (\Pi_t - E_t - C_t) - (TX_{t-1}+DV_{t-1})$

- **j_t** = Investment return rate of year t
- **LR_{t-1}** = Loss Reserve at time t-1
- **PR_{t-1}** = Premium Reserve at time t-1
- **TX_t** = Taxation amount year t
- **DV_t** = Dividends year t

The Insurance sub-model

Gross Premiums, Safety Loading and Loss Reserve

- **Gross Premiums (dynamic rule):**

  $\Pi_t = (1+i)*(1+g)*\Pi_{t-1}$

  i = claim inflation rate (constant) e.g.+5%

  g = real growth rate (constant) – assumed not related to the market level of the premiums – e.g. +5%

- **Loss Reserve and Premium Reserve:**

  $L_t = \delta * \Pi_t$ $PR_t = \xi * \Pi_t$

  with coefficients δ and ξ constant over the time

  (e.g. δ=100% and ξ=35%)

- **Gross Premiums and safety loading:**

  $\Pi_t = (1+\varphi)*E(X_t) + c*\Pi_t$

  where:

  - φ = **safety loading** coefficient (e.g. 5%)
  - **c** = exp. loading coefficient (e.g. 25%)

- The **safety loading coefficient** φ is computed according to the standard deviation principle:

  E(UWP) + E(FP-RFFP) = **b** * STD(UWP+TFP):

  UWP = Undewriting Profit (depending on φ)

  TFP = Total Financial Profit (depending on asset alloc.)

  RFFP = Risk-Free Financial Profit (depending on risk-free rate)

In other words, **the insurer is asking for an expected profit in excess of the risk-free rate from the overall insurance business equal to *b* (e.g. 0.30) for each unit of risk** (measured as standard deviation)

Total Claims Amount year t (X_t)

$$\tilde{X}_t = \sum_{i=1}^{\tilde{k}_t} \tilde{Z}_{i,t}$$

only 1 LoB is here considered

CAT claims not considered

- k_t = **Claim Number** of year *t*

  here assumed to be **Negative Binomial** distributed, i.e.

  - **k** follows a **Poisson distribution** with a random parameter n*q (as n parameter and q random),
  - **q** is a multiplicative random **structure variable** with mean 1 and distributed as a Gamma(h,h), which captures short-term fluctuations (Note: q is here regarded as time independent variables) ,
  - **n** is the **expected number of claims** (dimensional parameter) increasing according to the real growth rate, i.e. $n_t=n_0*(1+g)^t$

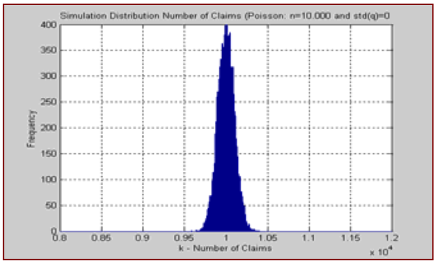
- $Z_{i,t}$ = **Claim Size** for the i-th claim of year *t* (independent of **k**)

  here assumed to be **LogNormal** distributed, with values increasing every year according to the deterministic claim inflation (**i**) only.

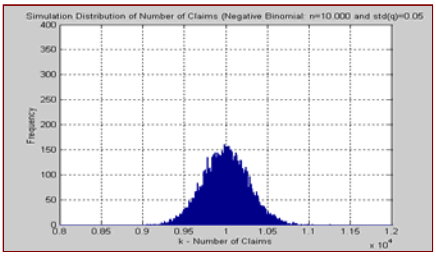
  The claim sizes Z_t **are assumed to be i.i.d.** random variables

- X_t **are time independent variables.** In the real world, though, long-term cycles are present and then significant auto-correlation might be observed (especially for the case of medium/long-term analyses).

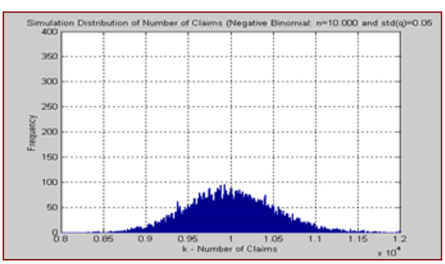
Number of claims distribution
(simulation examples)



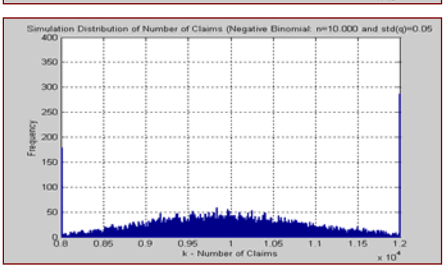
- **Poisson p.d.f.**
  n = 10.000
  σ(q) = **0 %**
  results of 10.000 simulations



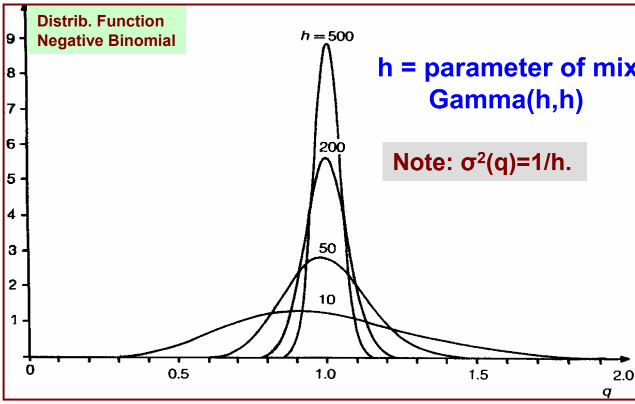
- **Negative Binomial p.d.f.**
  n = 10.000
  σ(q) = **2,5%**
  results of 10.000 simulations



- **Negative Binomial p.d.f.**
  n = 10.000
  σ(q) = **5%**
  results of 10.000 simulations



- **Negative Binomial p.d.f.**
  n = 10.000
  σ(q) = **10%**
  results of 10.000 simulations



h = parameter of mixturing Gamma(h,h)

Note: $\sigma^2(q)=1/h$.

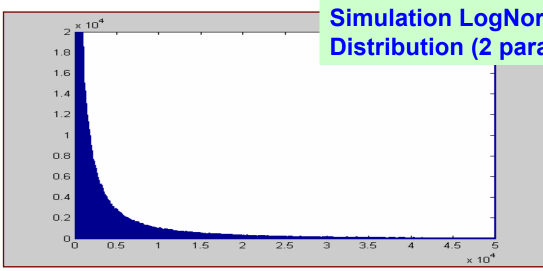
As h is increasing, the mixed Poisson distribution is approximating to the (simple) Poisson distribution.

$c_Z(3+c_Z^2)$

Claim Size distribution
(simulation examples)

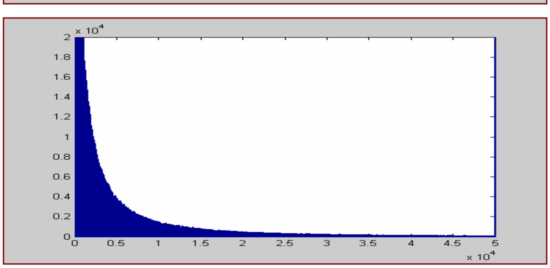
Simulation LogNormal Distribution (2 param.) ⇒

$E(Z)=m$
$Std(Z)=m*c_Z$
$Skew(Z)=c_Z*(3+c^2_Z)$



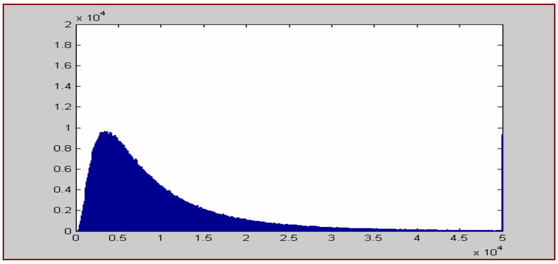
E(Z) = m = € 10.000

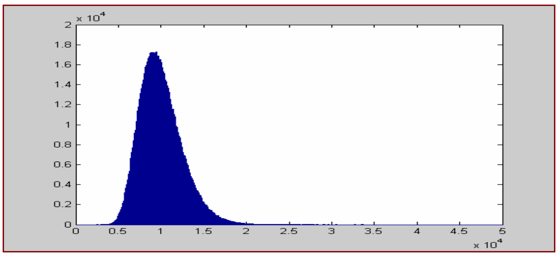
c_Z = 10



m = € 10.000

c_Z = 5



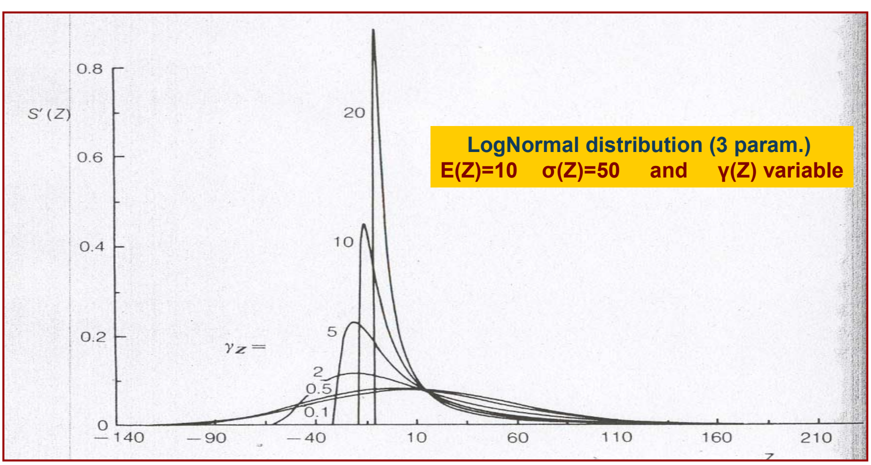


m = € 10.000

c_Z = 1,00

m = € 10.000

c_Z = 0,25



The (exact) Moments of the Total Claim amount X

(NO structure variable)

$$E(\tilde{X}) = n \cdot a_{1Z} = nm$$

$$\sigma^2(\tilde{X}) = n \cdot a_{2Z}$$

$$\gamma(\tilde{X}) = \frac{1}{\sqrt{n}} \cdot \frac{a_{3Z}}{(a_{2Z})^{3/2}}$$

a_{jZ} = j-th moment about origin claim size Z

Note:

- **As n (dimension parameter) is increasing, Variance is increasing to ∞ and skewness is decreasing to 0.**
- **Skewness in this case is always > 0.**

(YES structure variable q)

$$E(\tilde{X}) = n \cdot a_{1Z} = n \cdot m$$

$$\sigma^2(\tilde{X}) = n \cdot a_{2Z} + n^2 \cdot m^2 \cdot \sigma^2(\tilde{q})$$

$$\gamma(\tilde{X}) = \frac{na_{3Z} + 3n^2 ma_{2Z}\sigma^2(\tilde{q}) + 2n^3 m^3 \cdot \sigma^3(\tilde{q}) \cdot \gamma(q)}{\sigma^3(\tilde{X})}$$

- **Variance** is obviously larger
- **Skewness** may also be negative for extremely high negative values of the structure variable's skewness (q).

Variability of Loss Ratio

- **Loss Ratio = X / E(X) = Claims / Risk Premiums**

$$\lim_{n\to\infty}\frac{\sigma(\tilde{X})}{E(\tilde{X})}=\lim_{n\to\infty}\sigma\left(\frac{\tilde{X}}{E(\tilde{X})}\right)=\lim_{n\to\infty}\sqrt{\frac{1+c_Z^2}{n}+\sigma^2(\tilde{q})}=\sigma(\tilde{q})$$

- As we can see, **the growth of dimensional parameter n** (i.e. a larger insurance portfolio) **is not deleting the variability of the loss ratio**, because of the structure variable **q**, which represents a **systematic risk** (diversifiable only by reinsurance covers as e.g. Quota Share and Stop-Loss).

Standard parameters of the Insurance Model

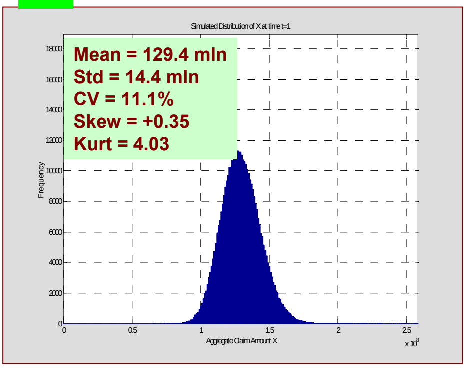
Initial expected number of claims (n_0)	20.000
St. Deviation structure variable σ(q)	0.10
Initial expected claim size ($) E(Z)	6.000
Variability coefficient of claim size (c_Z)	7

Claim Inflation (i)	5%
Real Growth rate (g)	5%
Expenses Loading coefficient (c)	25%
Safety Loading coefficient (φ)	Depending on asset allocation
Loss Reserve ratio (δ)	100%
Premium Reserve ratio (ξ)	35%
Taxation rate (tx)	35%
Dividends rate (dv)	20%

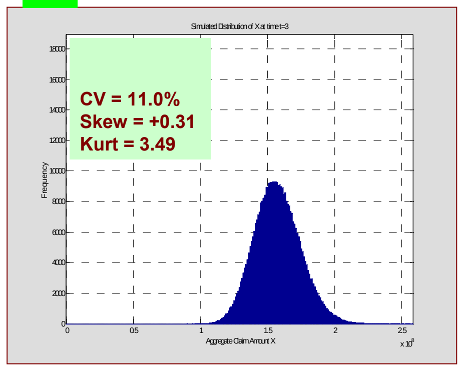
Initial Risk Premium (mill $)	120.0
Initial Gross Premiums (mill $) π_0	Depending on asset allocation

The simulated claim distribution X
(Standard Insurer)

t=1



t=3



The Investment sub-model

The Investment Model

- The insurer invests
  - **α%** of the available resources in an **equity index, *S***, and
  - **(1- α)%** in a portfolio of **zero coupon bonds, *P***, with different redemption dates
    - **$\beta^{(\tau)}$ %** is **invested in the bond with time to maturity τ**
  - the **asset allocation** and the **asset mix** are **constant** over time

Geom.Brownian Motion (equities)

CIR process (bonds)

$$dS_t = \mu S_t dt + \sigma S_t dW_t; \quad dr_t = \kappa(\theta - r_t)dt + v\sqrt{r_t}dZ_t$$

$$dP_t(t+i) = a(t;t+i)P_t(t+i)dt + b(t;t+i)P_t(t+i)dZ_t$$

$$i \in N := \{1,2,3,5,10\}$$

$$dW_t dZ_t = \rho dt \Rightarrow Z_t = \rho W_t + \sqrt{1-\rho^2}Y_t$$

$$\frac{dA_t}{A_t} = [\alpha\mu + (1-\alpha)a(t;t+i)]dt + [\alpha\sigma + (1-\alpha)\rho\Sigma(t;t+i)]dW_t + (1-\alpha)\sqrt{1-\rho^2}\Sigma(t;t+i)dY_t$$

$$a(t;t+i) = r_t + \lambda(t,r); \qquad \Sigma(t;t+i) = \sum_{i\in N}\beta^{(i)}b(t;t+i)$$

Cash Flows and Total Assets

- We also assume that at the beginning of every year the insurer invests the cashflows (***CF***), originated by the "pure" insurance business in the financial portfolio ***A***
  - The cashflows arise from consideration of
    - **Premium income**, (**Π_t**) depending on the assumed overall annual rate of growth (**g** and **i**)
    - General and acquisition **expenses** of the year (**c*B**)
    - the amount of **claims** deferred from the previous year and paid in the current year, **C_t^d**
    - the amount of **claims** occurred in the current year and settled during the same period, **C_t^c**
    - the payment of **taxation** regarding the previous financial year **TX_{t-1}**
    - The payment of **dividends** to stockholders regarding the previous financial year **DV_{t-1}**

  CF_t = Cash Flows year t at time (t-1)+ **= $(\Pi_t - E_t - C_t) - (TX_{t-1}+DV_{t-1})$**

The value of Total Assets at time t

$$A_t = \alpha\cdot(A_{t-1}+CF_t)\frac{S_t}{S_{t-1}} + (1-\alpha)\cdot(A_{t-1}+CF_t)\sum_{i\in N}\beta^{(i)}\frac{P(t,t-1+i)}{P(t-1,t-1+i)}$$

Asset Allocation and Asset Mix

- **Asset allocation**

		Insurer A	St. Insurer	Insurer B	Insurer C
Equity	α	0%	15%	30%	100%
Bond port.	$1-\alpha$	100%	85%	70%	0%

- **Asset mix (bond portfolio)**

Maturity (years)	*i*	*1*	*2*	*3*	*5*	*10*
Weight	$\beta(i)$	40%	25%	15%	10%	10%

Parameters of the Investment Model

Equity Index		
Expected rate of return	μ	10%
Volatility	σ	20%

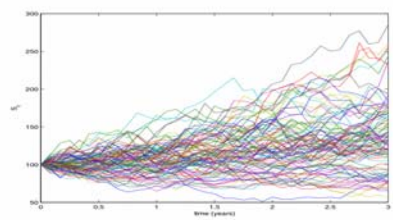
Interest rate (CIR)		
Long run mean	θ	4.78%
Speed	κ	0.10
Diffusion	υ	4.70%
Market price of interest rate risk	λ	-0.005
Correlation	ρ	-0.2
Current short rate	r_0	4.38%

Risk-free rate

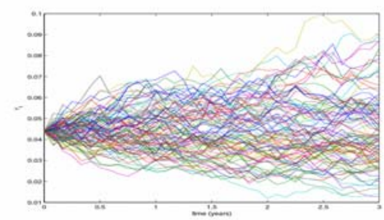
Zero Yield Curve		
1 year	r(0,1)	4.47%
2 years	r(0,2)	4.38%
3 years	r(0,3)	4.39%
5 years	r(0,5)	4.43%
10 years	r(0,10)	4.49%

Source: Bank of England (31/12/2004)

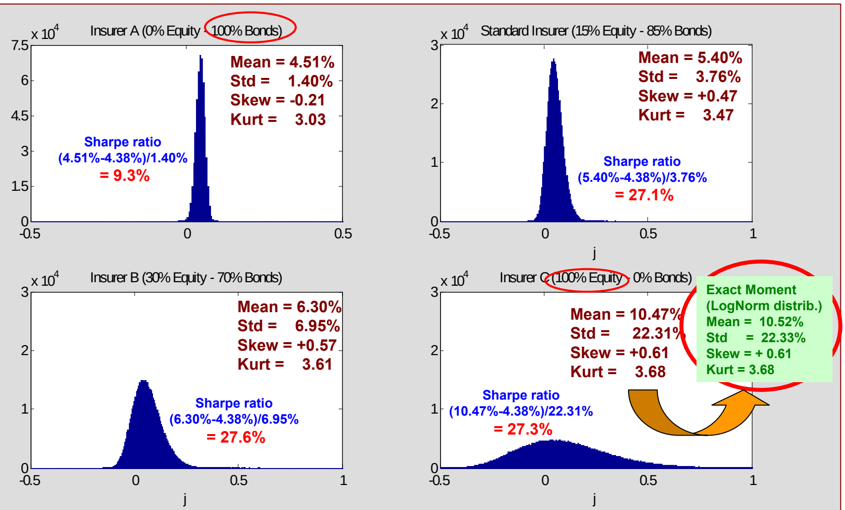
The Equity Index S_t over 3 years
(100 simulations by GBM model)



The short rate of interest over 3 years
(100 simulations by CIR model)

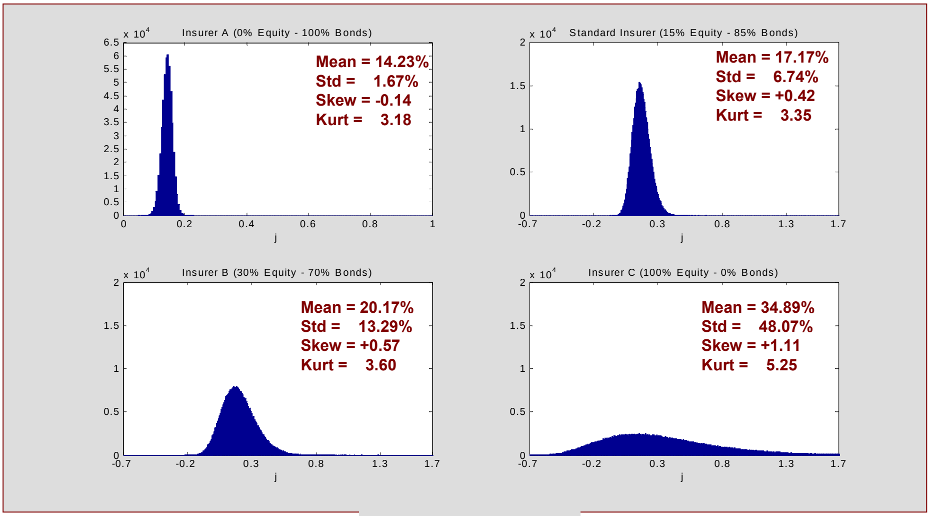


Simulated rate of total return (over 1 year)



500.000 simulations

Simulated rate of total return (over 3 years)



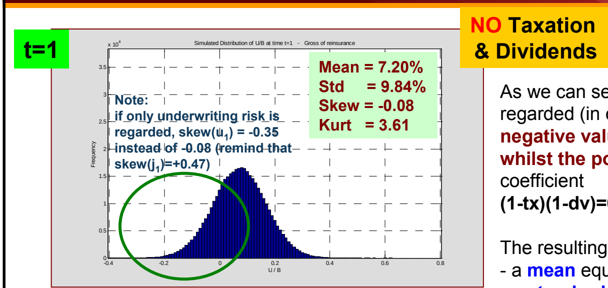
500.000 simulations

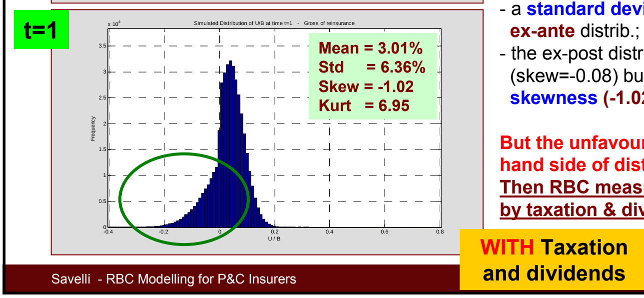
The RBC obtained from simulation results

(VaR and TVaR for TH=1-2-3 years)

Prob. Distrib. of the capital ratio u_t

(Standard Insurer – α=15%)

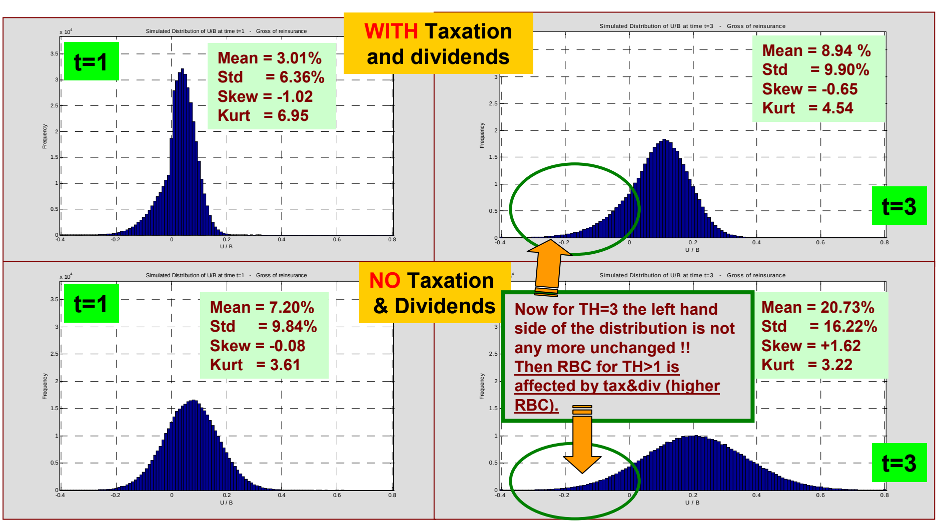




As we can see when also taxation and dividends are regarded (in our case tx=35% and dv=20%) the **negative values distribution remain unchanged whilst the postive values are rescaled** by the coefficient
(1-tx)(1-dv)=0.65*0.80=**0.52**.

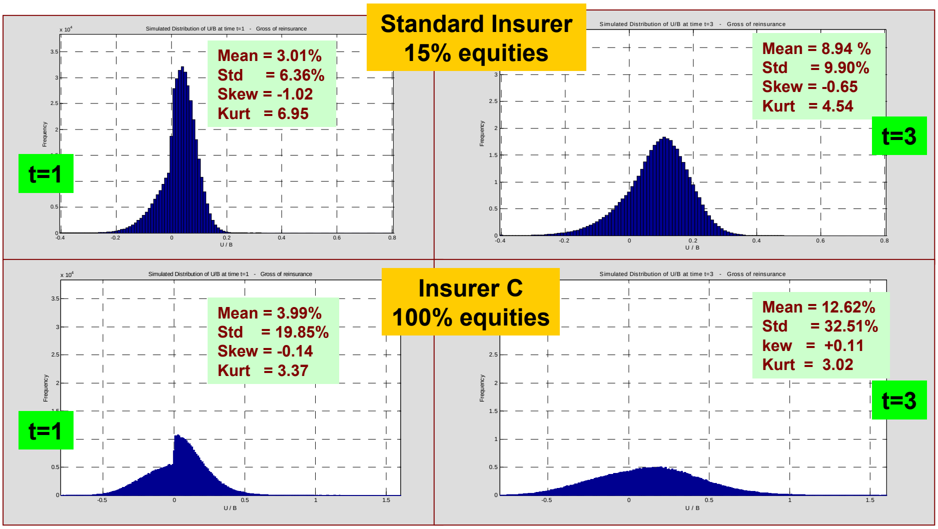
The resulting ex-post distribution has then:
- a **mean** equal to **40% of the ex-ante** distrib.;
- a **standard deviation** equal to roughly **65% of the ex-ante** distrib.;
- the ex-post distribution is not any more symmetric (skew=-0.08) but has a significant **negative skewness** (**-1.02**).

But the unfavourable quantiles (see at the left hand side of distribution) remain unchanged !!
Then RBC measures with TH=1 are not affected by taxation & dividends



Prob. Distrib. of the capital ratio U/Π

(net of taxation and dividends)



Moments of the capital ratio

NOTE: Risk Reserve U is affected by both financial and underwriting result

	Insurer A (0% Equities - 100% Bonds) Saf.loading φ = +3.19%			Standard Insurer (15% Equities - 85% Bonds) Saf.loading φ = +2.07%		
	T=1	T=2	T=3	T=1	T=2	T=3
Mean (%)	2.94	5.89	8.64	3.01	6.07	8.94
Std (%)	5.54	7.38	8.55	6.36	8.52	9.90
Skew	-1.31	-1.03	-0.84	-1.02	-0.80	-0.65
Kurt	8.93	6.98	5.37	6.95	5.54	4.54

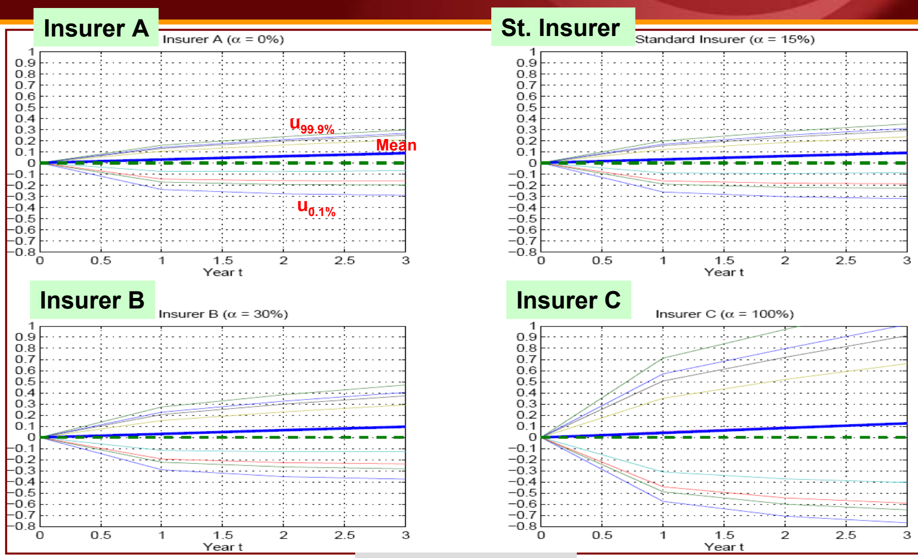
	Insurer B (30% Equities - 70% Bonds) Saf.loading φ = +1.57%			Insurer C (100% Equities - 0% Bonds) Saf.loading φ = +1.54%		
Mean (%)	3.17	6.46	9.55	3.99	8.42	12.62
Std (%)	8.05	10.91	12.78	19.85	27.17	32.51
Skew	-0.61	-0.47	-0.36	-0.14	-0.02	+0.11
Kurt	4.90	4.12	3.69	3.37	3.02	3.02

From Ins. A to Ins. C the distribution of U/Π becomes more and more Normal distributed (skewness close to 0 and kurtosis to 3)

400,000 simulations

Quantiles of the capital ratio

(0.1% 0.5% 1% 5% 50% 95% 99.0% 99.5% 99.9%)



400.000 simulations

Risk-Based Capital ratio according VaR and TVaR approach

- The maximum loss for an insurer over a target **time horizon (0,t)** within a given **confidence level 1-ε** (e.g. 99%):

$$rbc_{1-\varepsilon}^{VaR}(0,t) = \frac{-u_{\varepsilon}^{VaR}(t)\cdot\Pi_t\cdot\left[1+(1-tx)E(\tilde{j})\right]^{-t}}{\Pi_0}$$

$$rbc_{1-\varepsilon}^{TVaR}(0,t) = \frac{-u_{\varepsilon}^{TVaR}(t)\cdot\Pi_t\cdot\left[1+(1-tx)E(\tilde{j})\right]^{-t}}{\Pi_0}$$

whereas $u^{VaR}_{\varepsilon}(t)$ is the ε-th quantile of the capital ratio U/Π distribution at time t

The Risk-Based Capital ratio with 99.5% confidence (Var and TVaR approach)

RBC ratio (per unit of initial Gross Premiums π_0)	Insurer A (0% Equities - 100% Bonds) Saf.loading $\varphi = +3.19\%$			Standard Insurer (15% Equities - 85% Bonds) Saf.loading $\varphi = +2.07\%$		
	T=1	T=2	T=3	T=1	T=2	T=3
VaR 99.5%	18.5	22.4	24.3	20.3	25.0	27.5
TVaR 99.5 %	23.8	29.1	32.1	25.5	31.8	35.1

	Insurer B (30% Equities - 70% Bonds) Saf.loading $\varphi = +1.57\%$			Insurer C (100% Equities - 0% Bonds) Saf.loading $\varphi = +1.54\%$		
VaR 99.5%	23.6	29.8	33.5	50.4	63.9	71.6
TVaR 99.5 %	28.7	36.5	40.9	56.0	71.0	79.5

400,000 simulations

Some comments on the RBC measures

Regarding the Standard Insurer only (α=15%):

- For whatever risk measure (VaR or TVaR) and confidence level (99.0%/99.5%/99,9%):

  - if **TH** is increasing **from 1 to 2 years**, capital requirements are increasing by roughly **20-25%**;

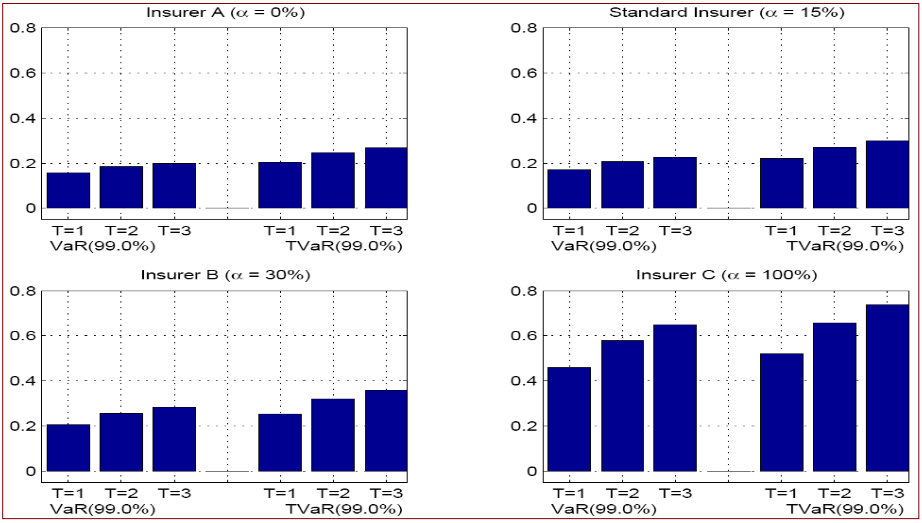
  - if TH is increasing **from 2 to 3 years**, capital requirements are further increasing by **10-13%** approx.;

- If a **TVaR(99.0%)** is compared with a **VaR(99.5%)** the capital requirement is larger by **10%**

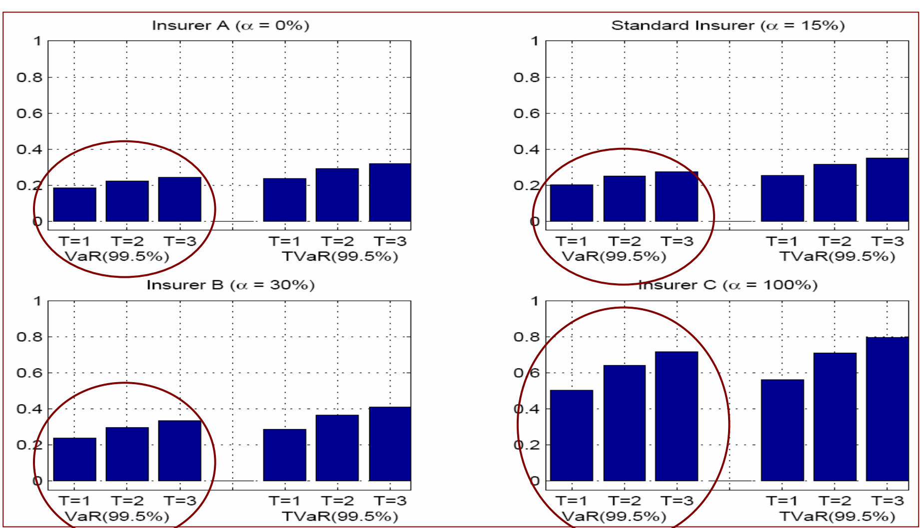
  (that seems be confirmed by the results obtained by FOPI in the SST Field Test 2005, +13% for non-life companies and +9% for life companies);

- In case **a TVaR instead of a VaR approach** (with the same confidence level 1-ε) is used, the capital requirement is increased by **25-30%**

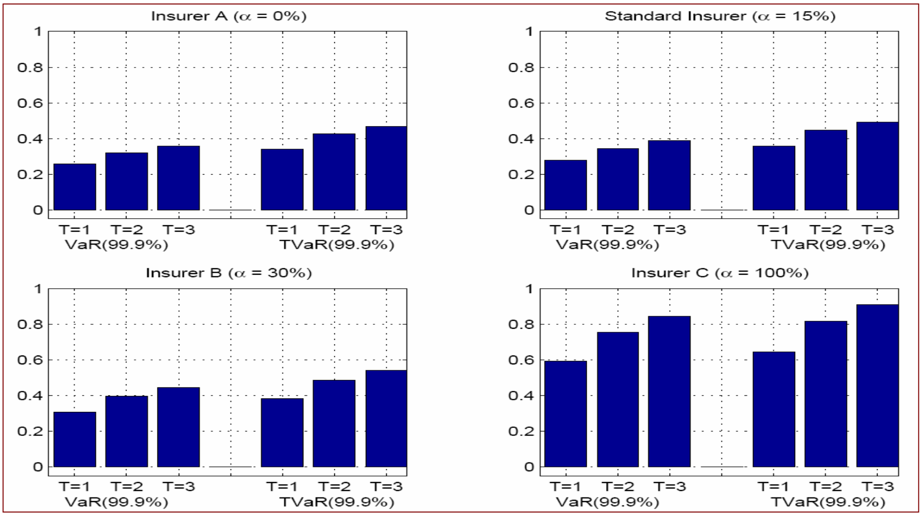
RBC/Premiums according VaR and TVaR at 99.0%



RBC/Premiums according VaR and TVaR at 99.5%



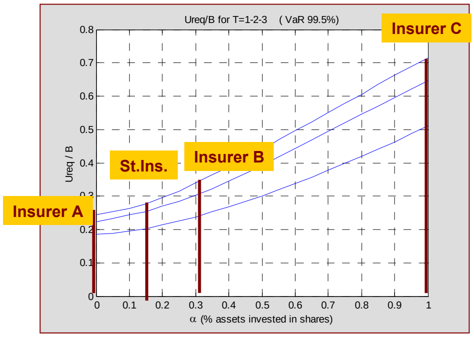
RBC/Premiums according VaR and TVaR at 99.9%



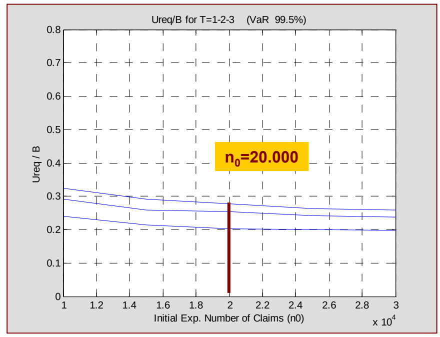
The impact of the main model's parameters on the RBC

Sensitivity of RBC (VaR 99.5%) according to the main parameters

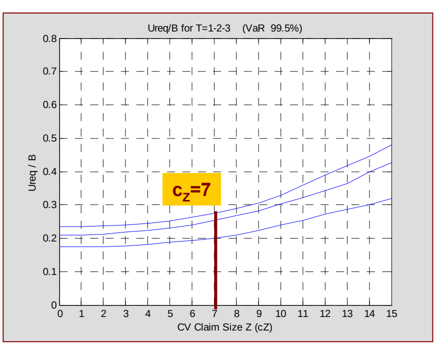
% Equities (α)



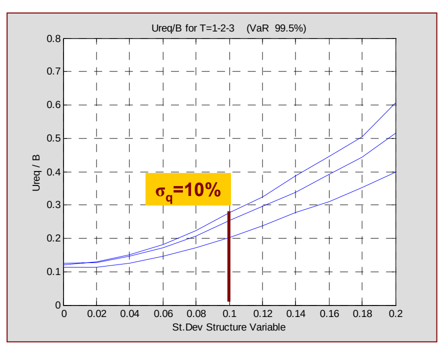
Initial Exp. Numb. Claims (n_0)



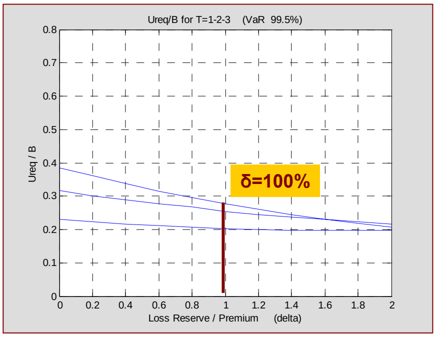
CV claim size (c_Z)



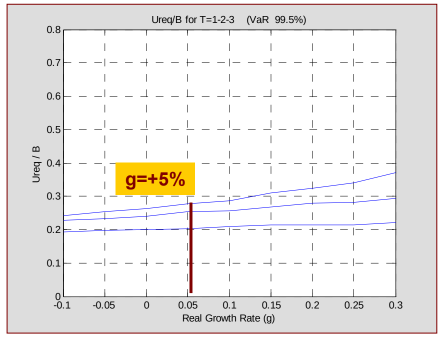
St. Dev. Struct. Variable (σ_q)



Loss Reserve ratio (δ)



Real Growth rate (g)



Further Researches and improvements of the Model

- Run-Off dynamics of **Claims Reserving** (in some countries Claim Reserving Risk has been playing a prominent role in case of insolvencies)
- To regard also **Credit Risk**, **Market Risk** and **Operational Risk**
- Premium Rating and **Premium Cycles**
- **Correlation** among different insurance lines (other than by catastrophe events - Copula analyses)
- **Reinsurance** and **ART**
- Claims amount of a line simulated separately for **small** and **large claims**
- **Dynamic Asset allocation** strategies and non-life ALM
- **Dynamic dividends policy** and taxation
- Analyse the impact of **IAS**
- **Modeling a multiplayer market** with high policyholders' sensitivity to either premium measure and insurer's financial strength, with special reference to TPML (Game Theory)

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