On
Traffic Light Options
for Solvency Protection in the
Life Insurance and Pension Business

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Seminar at the University of Calabria

October 29, 2008
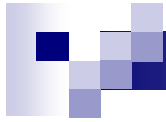
Agenda

- Background

- A new structured derivative

- The Model

- Valuation

- Illustrations

Balance sheet of an L&P company

	Assets	Liabilities
Strongly preferred by many pension fund managers →	Stocks	Defined/guaranteed benefits (bond-like with long duration)
Long duration hard to get →	Bonds (short duration)	
	Real estate	Equity/buffer

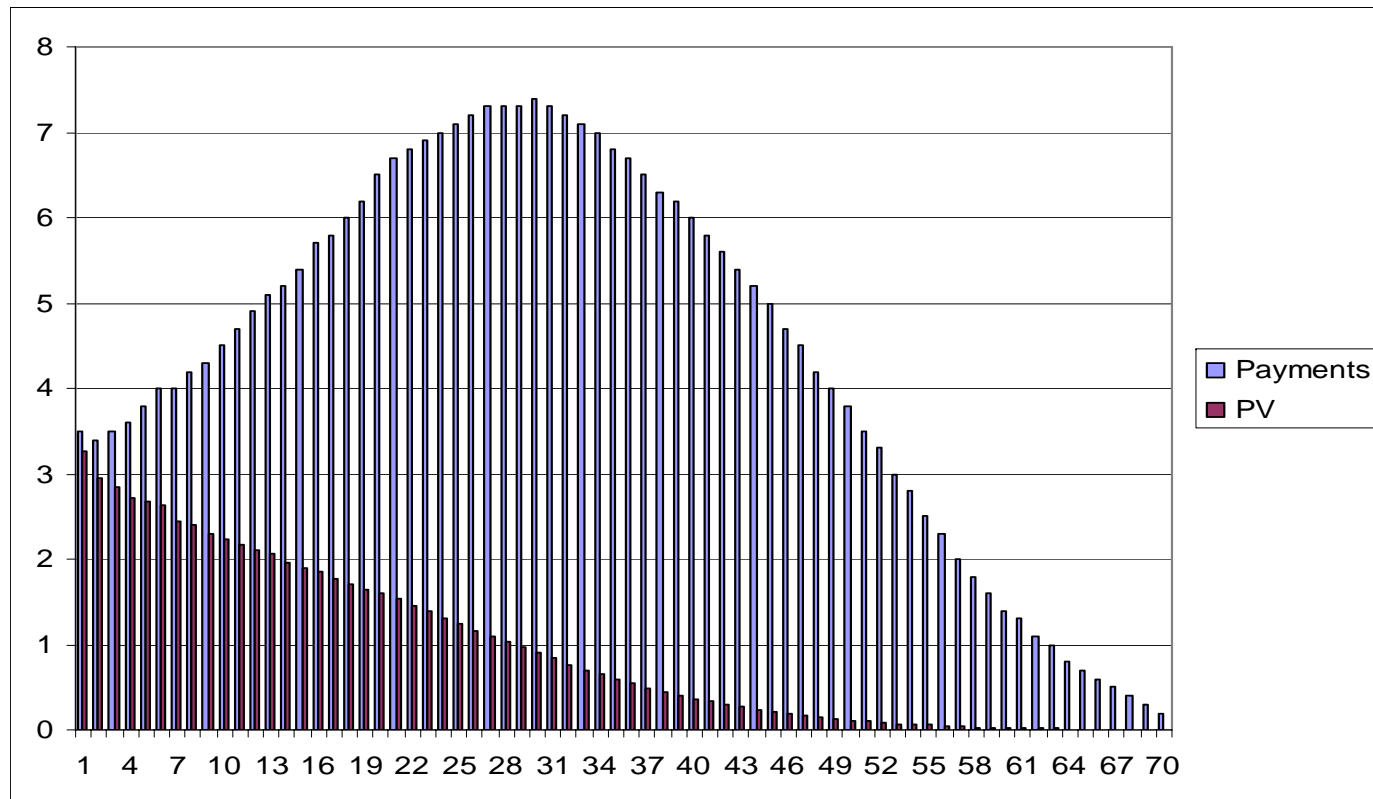**Potentially a serious and significant asset-liability mismatch!**

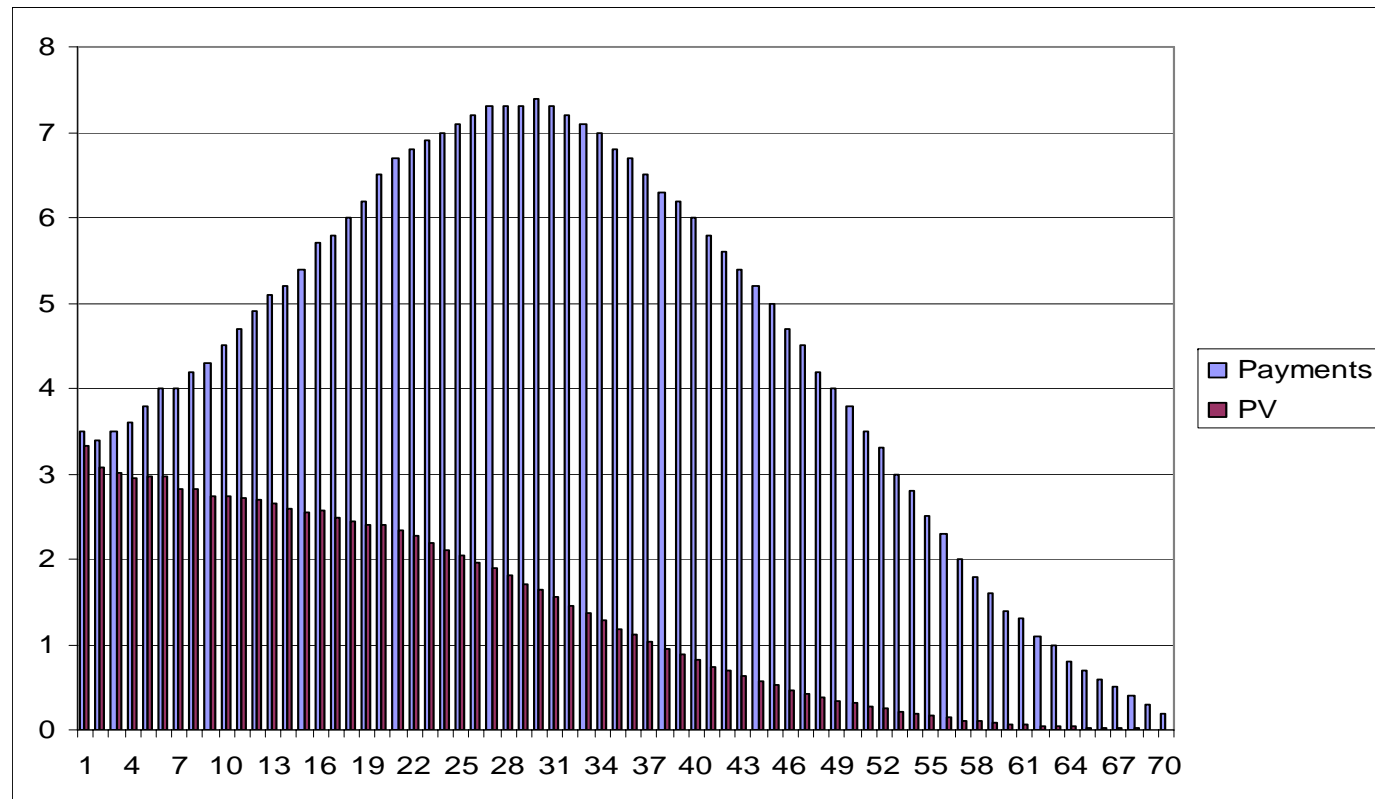Fig 1: A typical pension fund's liability cash flow profile
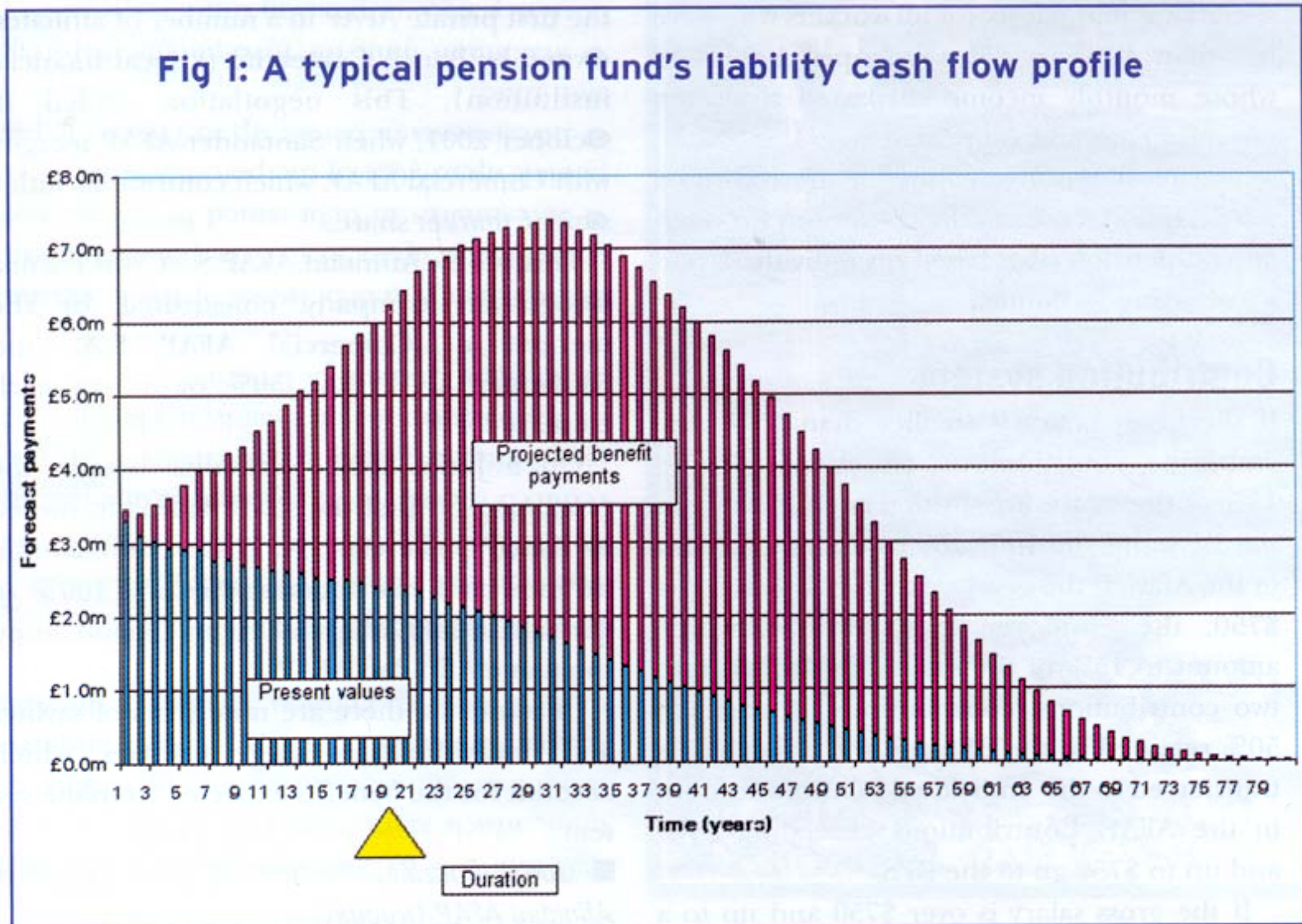
7% discount rate -> PV=66

5% discount rate -> PV=93

3% discount rate -> PV=142
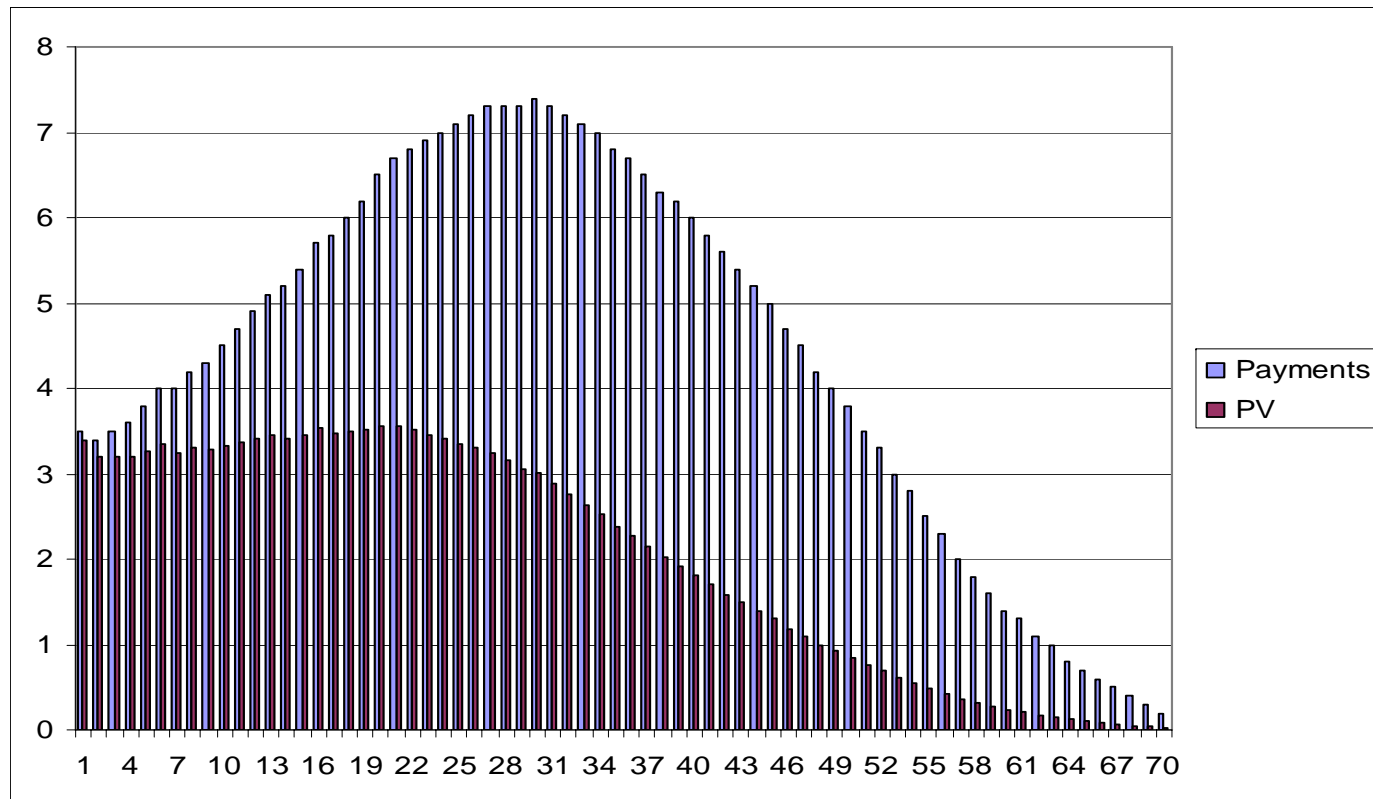
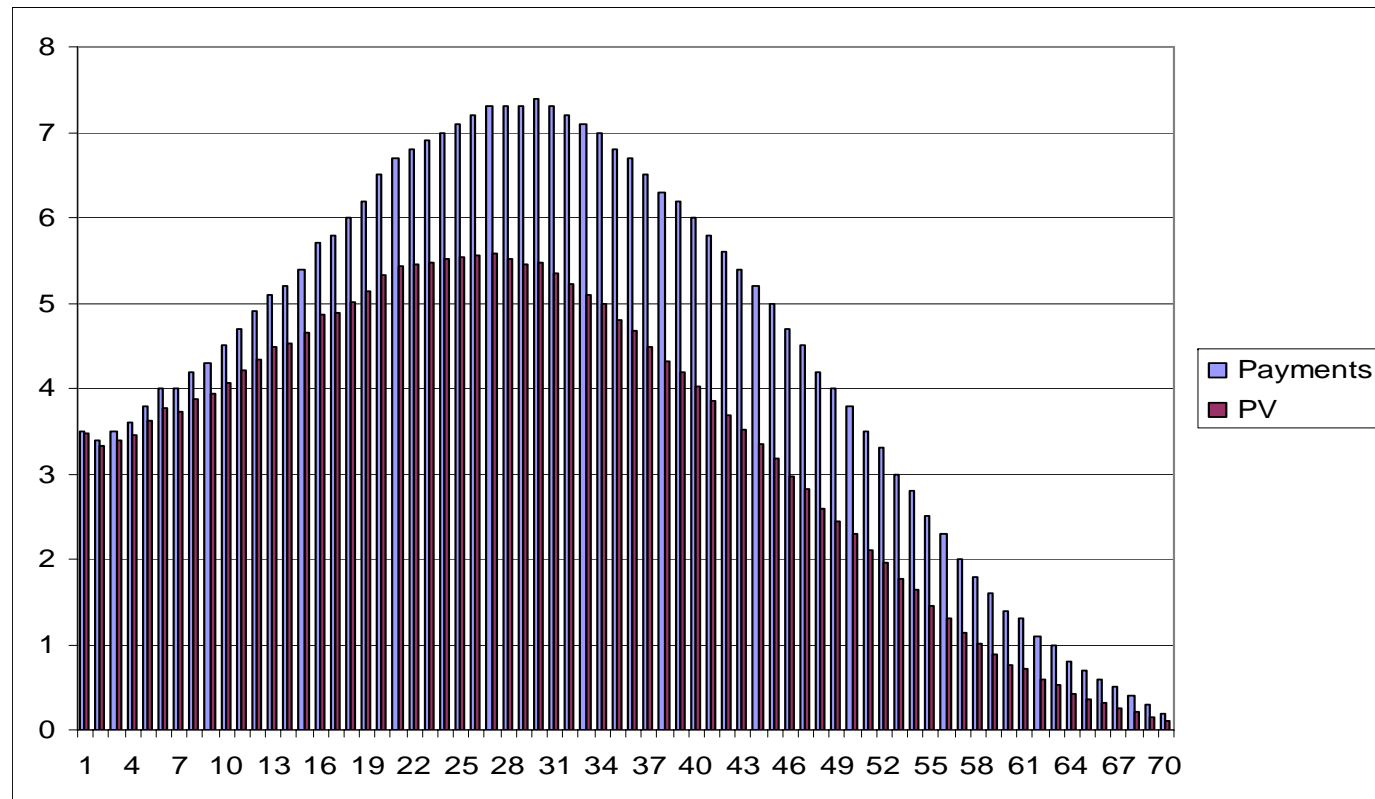# 1% discount rate -> PV=234

Risk management

- Companies have been hurt by falling interest rates

- Have learned a lesson and started buying interest rate protection (swaps, swaptions etc)

- But appetite for stocks is intact and holdings are typically significant (20-40% of assets)

- Companies must comply with DFSAs traffic light regulation/stress tests so equity positions must also be covered…

The Traffic Light System

- **Main features are:**
  - ☐ Semi-annual reporting of key solvency variables following certain (hypothetical) scenarios:
  - ☐ Red light: If – following a 12% decline in stock prices, a 0.7% change (fall) in the interest rate level, and an 8% decline in real estate values – base capital will drop below a certain minimum requirement/critical level. ⇒ monthly reports
  - ☐ Yellow light: Same with 30% for stocks, 1% i.r. change, and 12% for real estate. ⇒ quarterly reports
  - ☐ Green light: If solvency/base capital can withstand the above scenario.

(Sweden has now implemented similar system)

Correlation products

- So solvency is hurt when interest rates fall…
- …and when stock prices fall…
- Protection can be bought separately…
- … but you may then be <u>over</u>insured.
- Hence *correlation products* have been introduced.

The Traffic Light Option

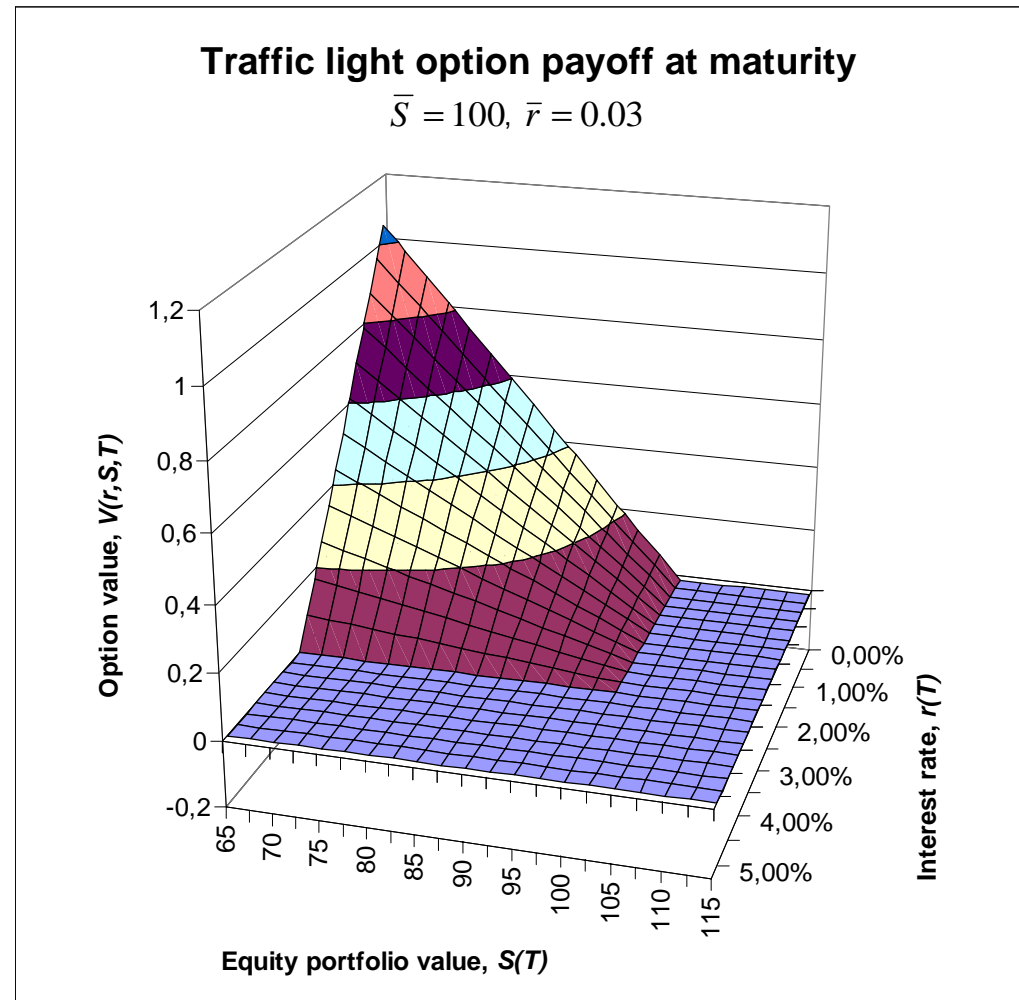A European option with payoff if and only if <u>both</u> stock pf. value and the interest rate drop below given strikes,

$$V(r, S, T) = [\bar{r} - r(T)]^{+} \cdot [\bar{S} - S(T)]^{+}$$

To my knowledge not previously analyzed in the literature.

Challenge is to establish time *t* pricing formula.

Traffic Light Option Payoff

Traffic light option payoff at maturity
$\overline{S} = 100, \ \overline{r} = 0.03$

Dynamic model

- The basic *P*-measure model is Black-Scholes-Vasicek:

$$dS(t) \;=\; \mu S(t)\, dt + \sigma_S S(t)\, d\mathcal{W}_S(t)$$

$$dr(t) \;=\; \kappa\big(\theta - r(t)\big)\, dt + \sigma_r\, d\mathcal{W}_r(t),$$

Obs!

$$d\mathcal{W}_S(t) \cdot d\mathcal{W}_r(t) = \rho\, dt.$$

Exploit change of numeraire technique

■ Forward neutral/Q^T-dynamics

$$\frac{dS(t)}{S(t)} = \left(r(t) - \rho\sigma_S\sigma_r\Psi(T-t)\right)dt + \sigma_S\,d\mathcal{W}_S^{Q^T}(t)$$

$$dr(t) = \left(\kappa(\bar{\theta} - r(t)) - \sigma_r^2\Psi(T-t)\right)dt + \sigma_r\,d\mathcal{W}_r^{Q^T}(t)$$

$$d\mathcal{W}_S^{Q^T}(t) \cdot d\mathcal{W}_r^{Q^T}(t) = \rho\,dt$$

The mathematical challenge
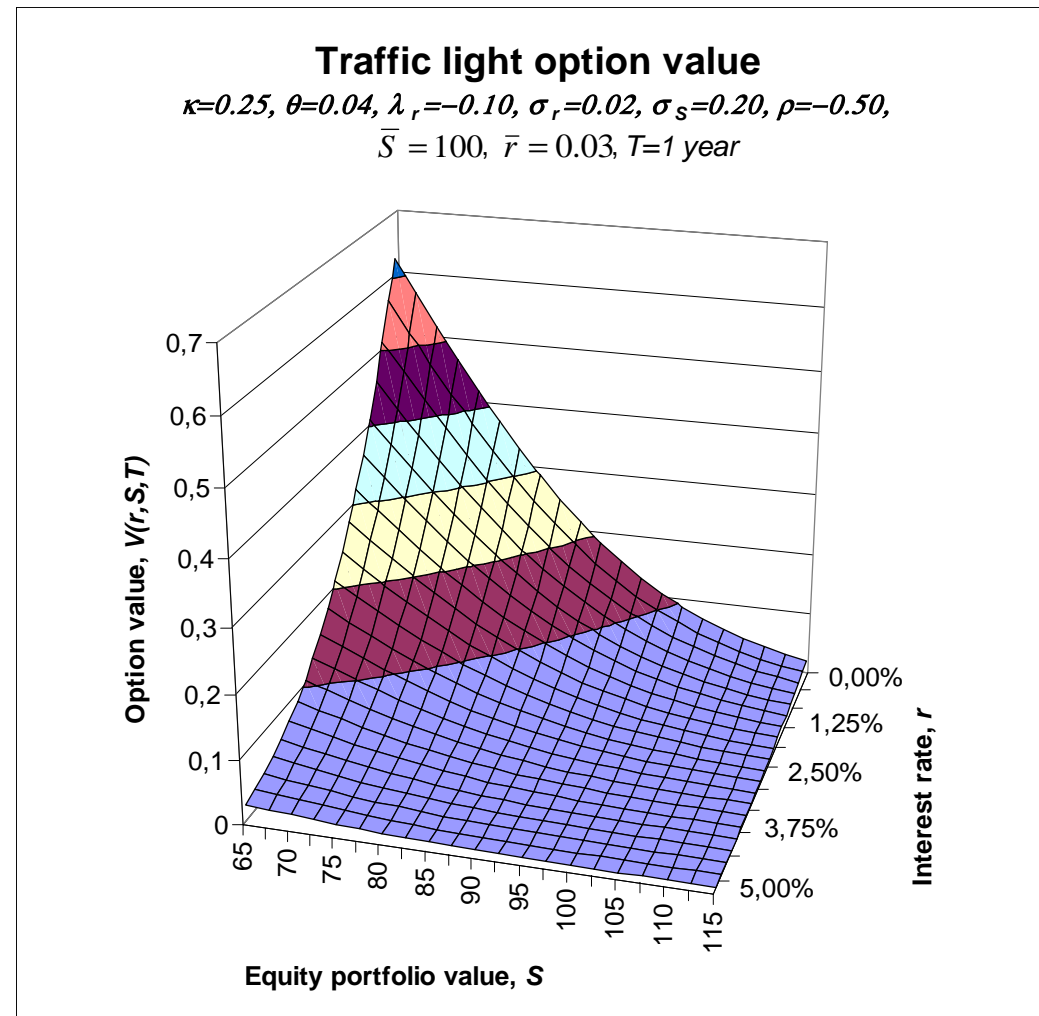
Traffic light option price is:

$$V(r, S, t) = P(r, t; T) \cdot E^{Q^T} \left\{ [\bar{r} - r(T)]^+ \cdot [\bar{S} - S(T)]^+ | \mathcal{F}_t \right\}$$

Vasicek (1977) zero-coupon bond price

Explicit formula derived. See Prop 1 in the paper.

Option value *T=1* year

Traffic light option value

κ=0.25, θ=0.04, λ_r=−0.10, σ_r=0.02, σ_s=0.20, ρ=−0.50, $\overline{S}=100$, $\overline{r}=0.03$, *T=1 year*

Option value, *V(r,S,T)*

Interest rate, *r*

Equity portfolio value, *S*

Correlation is essential

Dependence on correlation
ATM, T=1
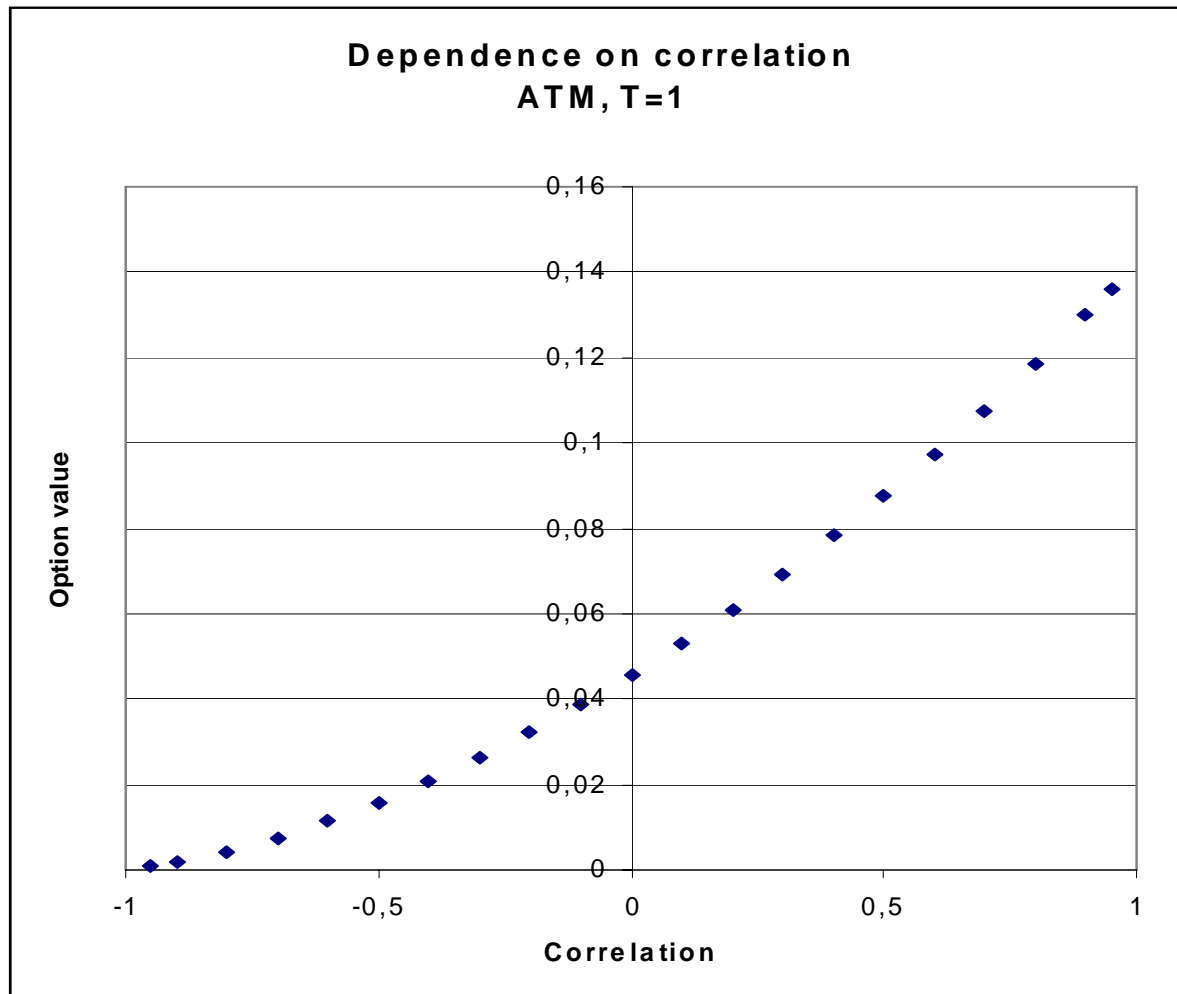
Example: No hedge, before

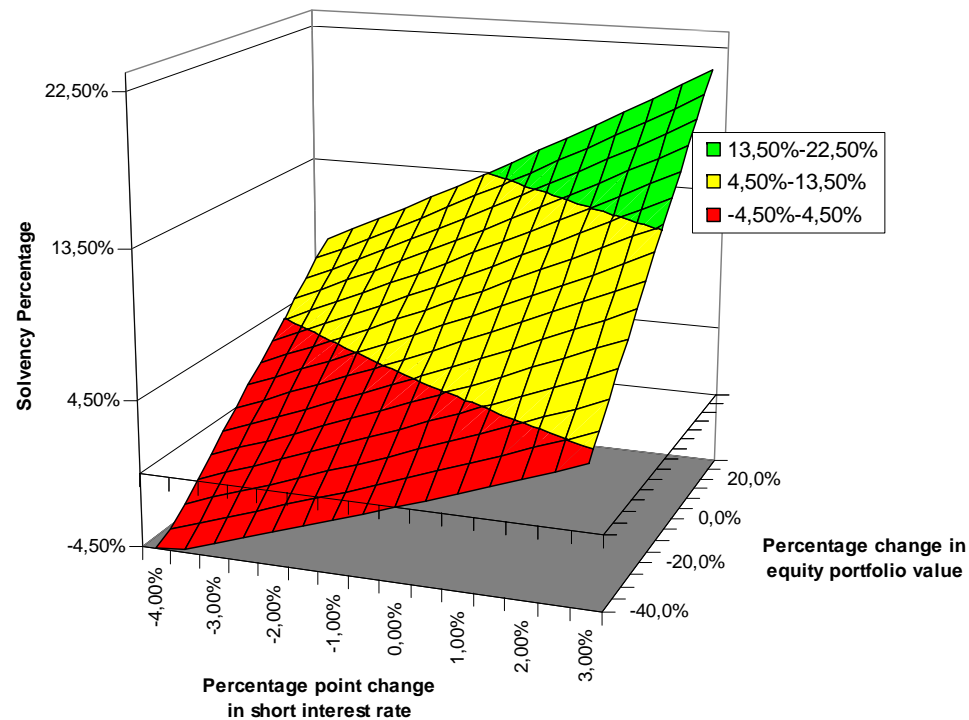| Assets				Liabilities
Stocks	20.00	92.00	Pension obligations ($D = 20$ years)	
Bonds ($D = 6$ years)	80.00	8.00	Equity (Solvency ratio: 8.70%)	
Total	100.00	100.00	Total	

Example: No hedge, after yellow scenario

Assets			Liabilities
Stocks	14.00	95.73	Pension obligations ($D = 20$ years)
Bonds ($D = 6$ years)	82.53	0.80	Equity (Solvency ratio: 0.83%)
Total	96.53	96.53	Total

The unhedged balance sheet

The Unhedged Balance Sheet
Solvency Percentage when interest rates
and equity values change

Example: TLO hedging, before

Assets			Liabilities
Stocks	20.00	92.00	Pension obligations ($D = 20$ years)
Bonds ($D = 6$ years)	78.93		
TL options	1.07	8.00	Equity (Solvency ratio: 8.70%)
Total	100.00	100.00	Total

$$150 \text{ TLOs with } \bar{r} = 0.04, \bar{S} = 21, T = 6$$

Example: TLO hedging, after yellow scenario

Assets				Liabilities
Stocks	14.00	95.73		Pension obligations ($D = 20$ years)
Bonds ($D = 6$ years)	81.42			
TL options	4.18	3.87		Equity (Solvency ratio: 4.05%)
Total	99.60	99.60		Total

The TLO hedged balance sheet

The Hedged Balance Sheet
Solvency Percentage when interest rates
and equity values change
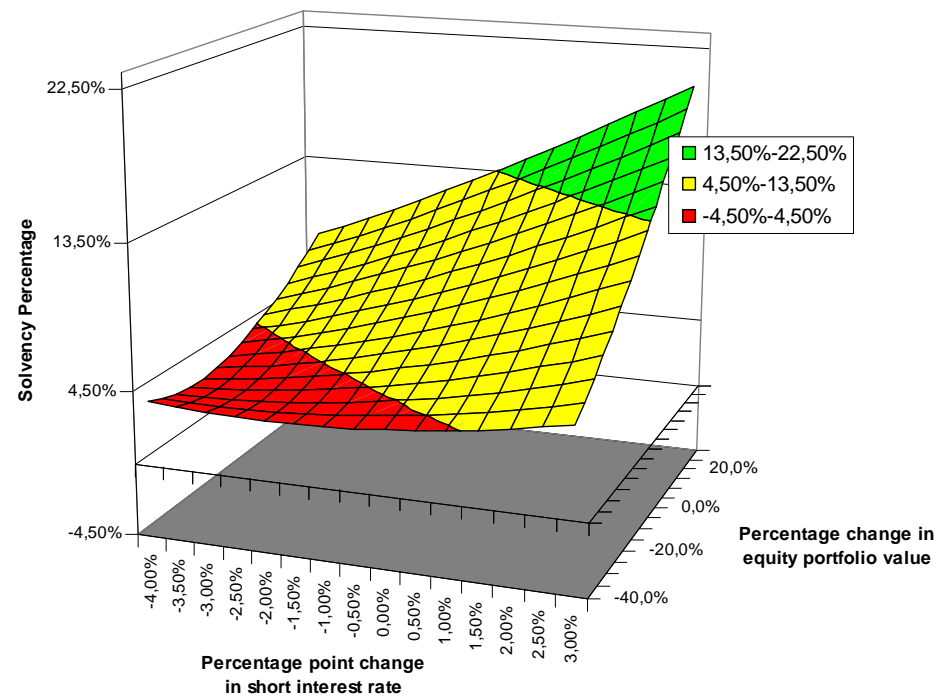
Remaining issues

- More realistic term structure model (Hull-White type)
- Optimal design of TLO (maturity, strikes)
- Other payoff structures

Time is up!! Thank you for your attention.