Time Charters with Purchase Options in the Shipping Business:

Valuation and Risk Management

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Outline

- Motivation
- Model considerations
- A one-factor spot freight rate model
- Valuation of freight rate contingent claims
  - fixed for floating freight rate swaps
  - ships
  - T/C pops in various forms
- Examples and numerical procedures
- Future work and conclusions

Motivation

- Contacted by second largest Danish shipping firm D/S Norden
- D/S Norden is one of 20 largest Danish listed companies, C20

From 2007 annual report:

> • The Company's 71 charter parties with purchase option had a value of USD 1,183 million (USD 533 million) at the beginning of 2007, based on a theoretical valuation. This corresponds to DKK 3,090 per share. Accordingly, at the end of the year, the Company's total theoretical Net Asset Value (NAV) per share was DKK 6,098. The calculation of theoretical value is subject to significant uncertainty, however.

- About 50% percent of the company's reported total net asset value is due to a theoretical calculation of the value of the time charters with purchase options

Motivation

- The intellectual challenge
  - how should T/C pops be valued?
  - how should they be managed optimally?
  - (is the current approach accurate?)
- Economically very significant!
- T/C pops are common in the shipping business – so not only a problem of interest for D/S Norden

T/C pop Example: Golden Ocean Group (NO)

GO sells and leases back ships at fixed daily rates. In addition a Bermudan style purchase option is granted.

Golden Ocean Group Limited: Time Charter and Sale Leaseback Agreement

HAMILTON, Bermuda, Feb. 6, 2007 (PRIME NEWSWIRE) -- Golden Ocean Group Limited ("Golden Ocean" or the "Company") is pleased to advise that the Company has fixed out two out of four Daehan Capesize Vessels ordered in December 2006 (vessel no 3 and no 4).

The vessels are fixed out for a period of 5 years at a net rate of US$36.800 per day to Goldbeam, guaranteed by Jinhui Shipping and Transportation Limited (OSE:JIN).

This is in accordance with the strategy outlined in the press release issued on the 5th of December 2006.

Simultaneously, Golden Ocean has sold the two vessels to Ship Finance International Limited (NYSE:SFL) based on a total delivered price of US$80 million.

Upon delivery from the shipyard, the vessels will commence 15 year bareboat contracts to Golden Ocean, and the charter rate per vessel is agreed as follows:

Year 1-5:	US$27.450 per day
Year 6-10:	US$22.600 per day
Year 11-15:	US$19.750 per day

Including operating expenses, Golden Ocean expects an average break-even rate of around US$31.750 per day during the initial five years period.

Golden Ocean has been granted fixed price purchase options for each of the vessels after 5, 10 and 15 years at US$61 million, US$44 million and US$24 million, respectively.

The deal will boost Golden Ocean's cash position with US$5.5 million per vessel as of delivery. The Charter out deal will further secure a free positive cash

T/C pop example: D/S Norden

Date: 2nd March, 2004

ADDENDUM NO. 1

TO

M/V "TBN" (Hull No. 1454)

Time Charter Party dated 2nd March, 2004

With reference to the captioned Charter Party, it is this day mutually agreed upon and confirmed by and between Shoei Kisen Kaisha, Ltd., or its guaranteed nominee and D/S NORDEN A/S or its guaranteed nominee as Charterers that:

Time Charter hire to be paid at the rate herebelow per day or pro rata including overtime;

1st year	US$15,000.- per day including overtime
2nd year	US$15,000.- per day including overtime
3rd year	US$11,000.- per day including overtime
4th year	US$10,000.- per day including overtime
5th year	US$10,000.- per day including overtime
6th year (Optional Period)	US$13,000.- per day including overtime
7th year (Optional Period)	US$13,500.- per day including overtime
8th year (Optional Period)	US$14,000.- per day including overtime

No address commission to be included on the above Charter hire.

Date: 2nd March, 2004

ADDENDUM NO. 2

TO

M/V "TBN" (Hull No. 1454)

Time Charter Party dated 2nd March, 2004

With reference to the captioned Charter Party, it is this day mutually agreed upon and confirmed by and between Shoei Kisen Kaisha, Ltd., or its guaranteed nominee and D/S NORDEN A/S or its guaranteed nominee as Charterers that:

1. Charterers have the option to purchase the Vessel after the end of the 3rd year and at any time during any optional year, if option to extend charter period is exercised as provided for in Clause 75 thereof.

2. The purchase prices at which Charterers buy the Vessel under this Agreement are as follows:

At the end of 3rd year	JPY2.70 Billion
At the end of 4th year	JPY2.57 Billion
At the end of 5th year	JPY2.44 Billion
At the end of 6th year (Optional Period)	JPY2.31 Billion
At the end of 7th year (Optional Period)	JPY2.18 Billion
At the end of 8th year (Optional Period)	JPY2.05 Billion

The above prices are net receivable to Owners and do not include any address commissions and/or brokerages in them.

3. If Charterers exercise their option to purchase the Vessel at any time other than at the end of respective year, the purchase price shall be obtained in accordance with the following formula:

Purchase price = A −{(A−B) x C/365}

A = The fixed purchase price as above corresponding to the end of the year immediately preceding the year of actual delivery.

Here is the actual ship! "M/S Nord Mercury" (Panamax)

What makes T/C pops non-standard?

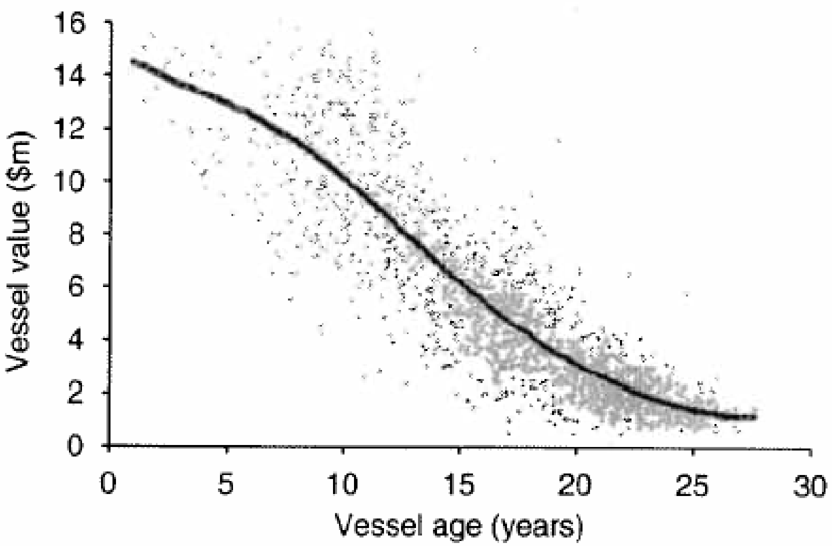
- Underlying asset is physical asset -> option is *real option*
- Hedging real assets/options can be difficult
- Asset (ship) pays dividend and has finite life -> value expected to decrease over time
- Options can be Bermudan/American and have time-varying strike prices
- Option is *add-on* to a lease contract. So when option is exercised, a lease is terminated and option to exercise later is given up
- Strike price can be in different currency (eg. JPY)
- Black-Scholes difficult/impossible to adapt

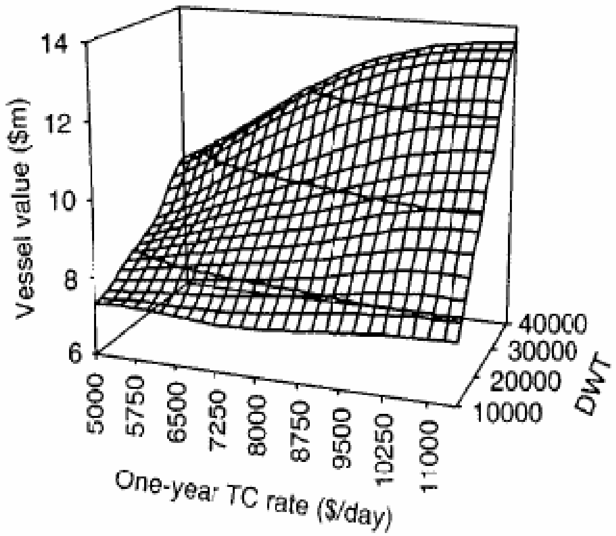
Model considerations

- First question: What determines the value of a ship?
- Adland & Koekebakker (2007): ”Three most important factors are size, age, and freight rates”
- Only freight rates are uncertain/stochastic. This should be first choice of factor process
- Second factor could be exchange rate – if it matters
- Oil price, interest rates are of second order importance

Adland & Koekebakker (2007)

Handysize bulk carriers (10,000-40,000 DWT). 1960 transactions in 1993-2003 (Clarksons)





A one-factor spot freight rate model

- Basic model first proposed in:
- Bjerksund & Ekern (1995), ”Contingent Claims Evaluation of Mean-reverting Cash Flows in Shipping”.
- The *Spot freight-rate* is modeled as a *mean-reverting* stochastic process
- Authors price forward contracts and European options on T/C contracts

The Bjerksund-Ekern-Vasicek model

- Basic assumption: Instantaneous (daily) cash flow from operating ship follows OU-process:

$$D(t)dt = (aX(t) - b)dt$$

time measured in years

cargo size, 1 if X quoted for entire ship

Instant. CF~daily profit

freight rate

cost flow rate

$$dX(t) = \kappa(\theta - X(t))dt + \sigma dW(t)$$

N()-noise

rate of mean reversion

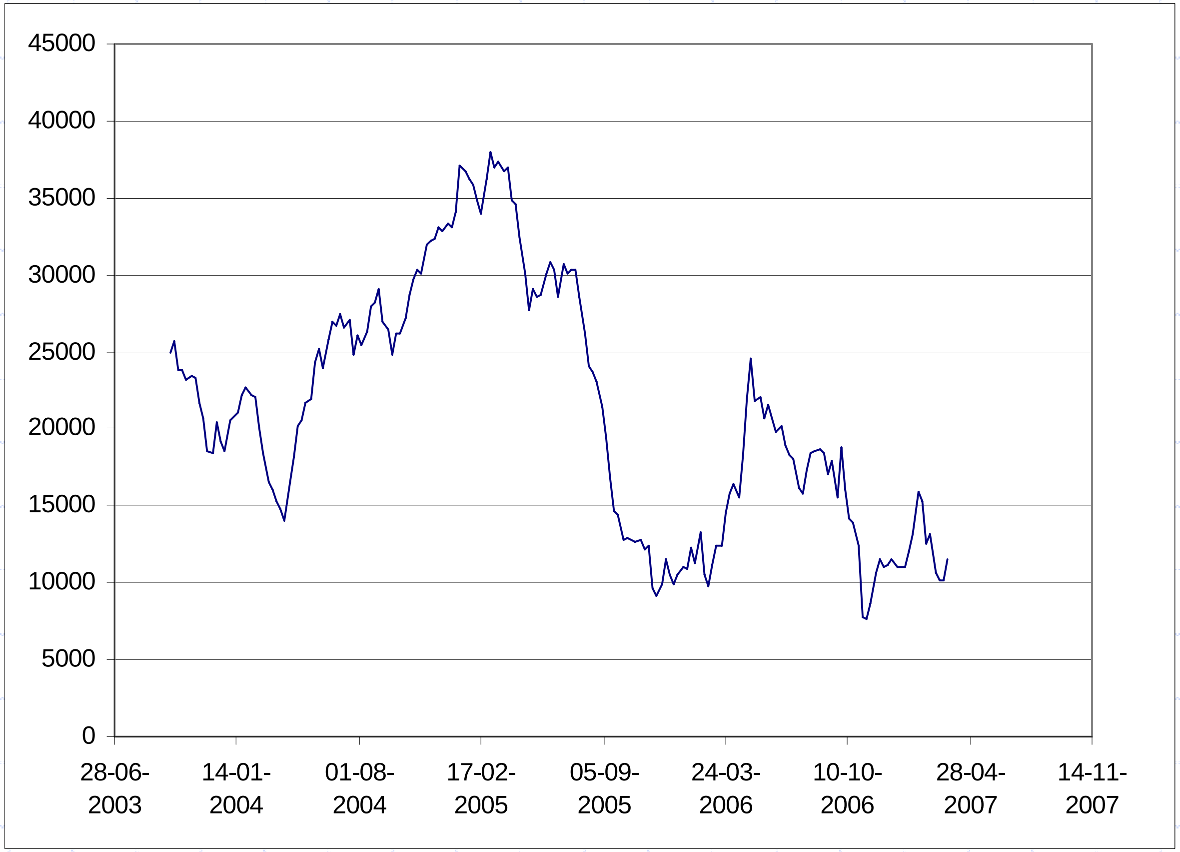
long term mean

volatility

Simulated freight rate-process

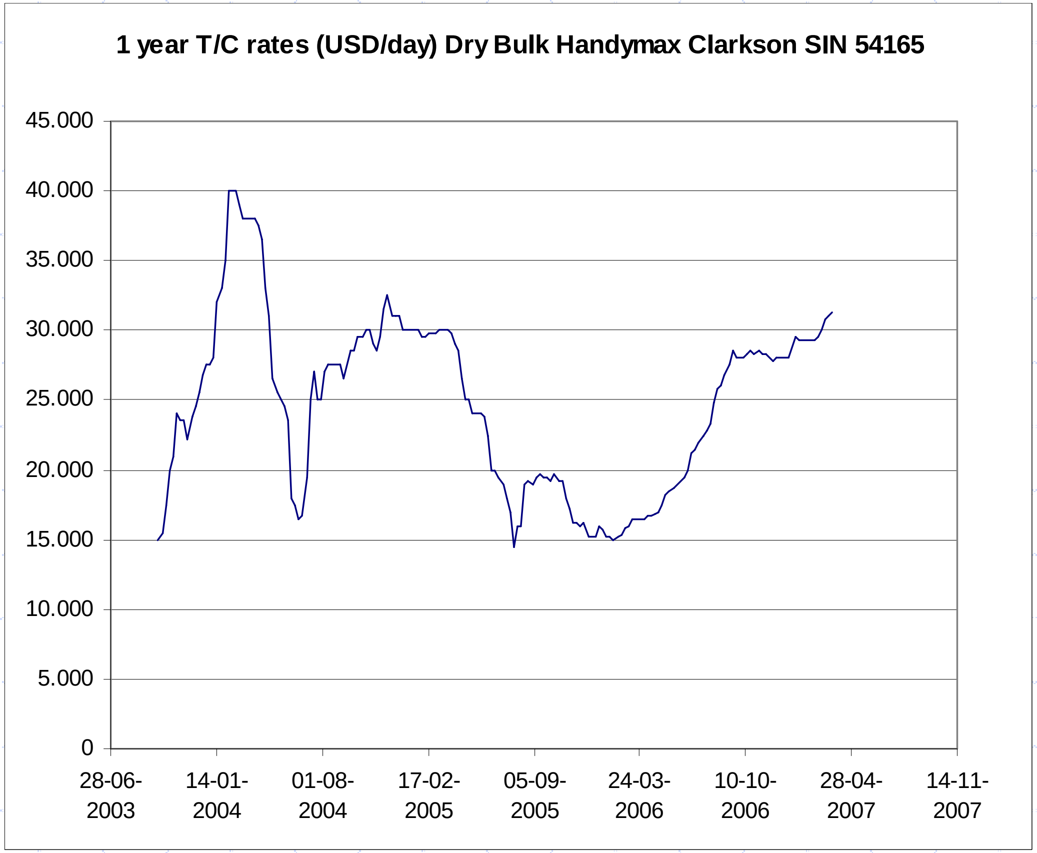
weekly steps

$$X(0) = 25000, \theta = 25000, \sigma = 10000 \text{ all USD/day}, \kappa = 0{,}25$$



Actual freight rate time series

weekly observations



Implications of OU-specification

- Solution of SDE

$$X(T) = X(t)e^{-\kappa(T-t)} + \theta\left(1 - e^{-\kappa(T-t)}\right) + \sigma \int_t^T e^{-\kappa(T-u)}\, d\mathcal{W}(u),$$

- Mean and variance of factor process

$$E_t\left\{X(T)\right\} = X(t)e^{-\kappa(T-t)} + \theta\left(1 - e^{-\kappa(T-t)}\right),$$

$$Var_t\left\{X(T)\right\} = \sigma^2 \int_t^T e^{-2\kappa(T-u)} du = \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa(T-t)}\right)$$

Valuation of freight rate contingent claims

- Claims, *V(X,t)*, that depend on freight rate and time must satisfy pde:

$$\kappa\big(\theta^* - X(t)\big)V_x + \frac{1}{2}\sigma^2 V_{xx} + V_t - rV + \Gamma(X,t) = 0.$$

$$\theta^* \equiv \theta - \frac{\sigma\lambda}{\kappa}$$

- Probabilistic representation of solution

Both are used in valuation of contracts considered in the paper

$$V(X,t) = E_t^Q\left\{\int_t^T \Gamma(X_u,u)e^{-r(u-t)}\,du + e^{-r(T-t)}V(X_T,T)\right\}$$

$$dX(t) = \kappa\big(\theta^* - X(t)\big)\,dt + \sigma\,d\mathcal{W}^Q(t),$$

Risk neutralized freight rate process

Valuing freight rate contingent claims

- Claim to receive spot freight rate flow from time *t* to time *T:*

$$
\begin{aligned}
V^{Flow}(X(t),t;T) &= E_t^Q\left\{\int_t^T e^{-r(u-t)}X(u)\,du\right\} \\
&= \Big(X(t)-\theta^*\Big)A(T-t,r+\kappa)+\theta^*A(T-t,r),
\end{aligned}
$$

$A(\tau,\delta) \equiv \frac{1-e^{-\delta\tau}}{\delta}$ annuity factor

Valuing freight rate contingent claims

- Fixed for floating freight rate swap

$$\bar{X}_{t,T} = \theta^* + \frac{A(T-t, r+\kappa)}{A(T-t,r)}\Big(X(t) - \theta^*\Big)$$

r=5%
θ=20000*
T=5yrs

		$X(0)$							
		5000	10000	15000	20000	25000	30000	35000	40000
	0.10	8073	12049	16024	20000	23976	27951	31927	35902
	0.25	11220	14147	17073	20000	22927	25853	28780	31707
	0.50	14229	16153	18076	20000	21924	23847	25771	27694
κ	1.00	16788	17859	18929	20000	21071	22141	23212	24283
	2.00	18346	18897	19449	20000	20551	21103	21654	22205
	5.00	19329	19552	19776	20000	20224	20448	20671	20895
	10.00	19663	19775	19888	20000	20112	20225	20337	20450

Valuing freight rate contingent claims

- Value of fixed for floating swap

$$V^{Swap}(t;s,T) = \left[\bar{X}_{t,T} - \bar{X}_{s,T}\right] A(T-t,r)$$

r=5%
θ=20000*
T=5yrs
κ=0.25

The value of a 5-year time charter contract (in millions)
Dependence on current and previously contracted daily T/C rates
$r = 5\%$, $\theta^* = 20000$ per day, $\kappa = 0.25$

Spot and current T/C rate		$\bar{X}_{s,T}$, previously contracted rate			
$X(t)$	$\bar{X}_{t,T}$	5000	10000	20000	30000
5000	11220	9.906	1.943	-13.984	-29.910
10000	14147	14.567	6.604	-9.322	-25.249
20000	20000	23.890	15.926	0.000	-15.926
30000	25853	33.212	25.249	9.322	-6.604

Valuing freight rate contingent claims

- A ship is also a freight rate contingent claim
- We define a ship as the right to receive the spot freight rate flow during the ships useful service life PLUS scrap value

$$V^{ship}(X(t),t;\bar{T}) = \Big(X(t) - \theta^*\Big)A(\bar{T}-t, r+\kappa) + \theta^* A(\bar{T}-t, r) + e^{-r(\bar{T}-t)}\bar{V}.$$

- Value depends on freight rate, age, (and ship size via *X*)

Example: Ship valuation

The dependence of ship values (in millions)
on remaining ship life $(\bar{T} - t)$ and the spot freight rate $(X(t))$
$\kappa = 0.25$, $\theta^* = 20000$ per day, $r = 5\%$, $\bar{V} = 5\text{m}$

		Remaining ship life $(\bar{T} - t)$				
		5	10	15	20	25
$X(t)$	5000	21.763	42.588	60.541	74.909	86.186
	10000	26.424	48.290	66.474	80.895	92.182
	15000	31.085	53.991	72.408	86.880	98.179
	20000	35.747	59.692	78.341	92.865	104.176
	25000	40.408	65.394	84.274	98.850	110.173
	30000	45.069	71.095	90.208	104.835	116.169
	35000	49.730	76.796	96.141	110.820	122.166
	40000	54.392	82.497	102.074	116.805	128.163

Scrap value = 5m

Example: Ship valuation

- Ship value is linear in *X(t)* and thus normal distributed with

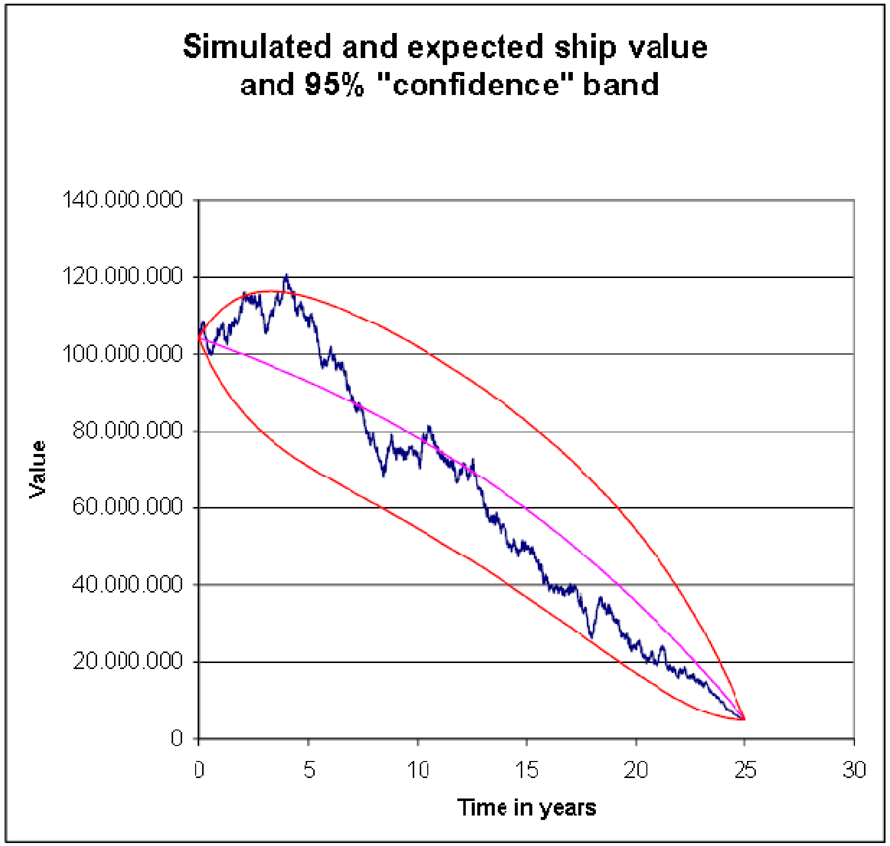
$$E_t^Q\left\{V^{ship}(X(T),T;\bar{T})\right\} = e^{-\kappa(T-t)}\Big(X(t)-\theta^*\Big)A(\bar{T}-T,r+\kappa) \\ +\theta^* A(\bar{T}-T,r)+e^{-r(\bar{T}-T)}\bar{V},$$

$$E_t\left\{V^{ship}(X(T),T;\bar{T})\right\} = \left(e^{-\kappa(T-t)}\left(X(t)-\theta\right)+\left(\theta-\theta^*\right)\right)A(\bar{T}-T,r+\kappa) \\ +\theta^* A(\bar{T}-T,r)+e^{-r(\bar{T}-T)}\bar{V},$$

$$Var_t^{(Q)}\left\{V^{ship}(X(T),T;\bar{T})\right\} = A^2(\bar{T}-T,r+\kappa)\frac{\sigma^2}{2\kappa}\left(1-e^{-2\kappa(T-t)}\right).$$

Simulated ship value dynamics & 95% confidence bands

κ=0.25
θ^*=20000/day
σ=5000/day
$\underline{r}$=5%
$\underline{T}$=25yrs
$\overline{V}$=5m



Valuing freight rate contingent claims

- European option to purchase ship. Payoff:

$$c(X(T),T) = \max\left(V^{ship}(X(T),T;\bar{V}) - K;0\right)$$

- Pricing formula (slight generalization of result in Bjerksund & Ekern)

$$c(X(t),t) = e^{-r(T-t)}A(\bar{T}-T,r+\kappa)v_{t,T}\Big(\xi N(\xi)+n(\xi)\Big),$$

$$\begin{aligned}
\xi &= \frac{m_{t,T}-\bar{\bar{K}}}{v_{t,T}}, \\
m_{t,T} &= E_t^Q\left\{X(T)\right\} = X(t)e^{-\kappa(T-t)} + \theta^*\Big(1-e^{-\kappa(T-t)}\Big), \\
\bar{\bar{K}} &= \frac{K+\theta^*\Big(A(\bar{T}-T,r+\kappa)-A(\bar{T}-T,r)\Big)-e^{-r(\bar{T}-T)}\bar{V}}{A(\bar{T}-T,r+\kappa)}, \\
v_{t,T}^2 &= Var_t^{(Q)}\left\{X(T)\right\} = \frac{\sigma^2}{2\kappa}\Big(1-e^{-2\kappa(T-t)}\Big),
\end{aligned}$$

European ship option values

Value of European option to buy ship (in millions)

Dependence on spot freight rate and freight rate volatility

$r = 5\%,\ \kappa = 0.25,\ \theta^* = 20000$ per day, $\bar{V} = 5$m

$t = 0,\ T = 5,\ \bar{T} = 25,\ K = 93$m

		Freight rate volatility, σ (daily)				
		1000	3000	5000	7000	9000
Spot freight rate $X(0)$	5000	0.000	0.268	0.980	1.846	2.773
	10000	0.006	0.512	1.371	2.312	3.282
	15000	0.080	0.899	1.864	2.854	3.851
	20000	0.453	1.460	2.467	3.475	4.483
	25000	1.341	2.206	3.182	4.177	5.177
	30000	2.575	3.128	4.007	4.958	5.933

T/C contract with Purchase option

- Value is simple sum of swap value and European option value

The value of a time charter contract with European purchase option (in millions)

Dependence on contracted charter rate and current spot freight rate

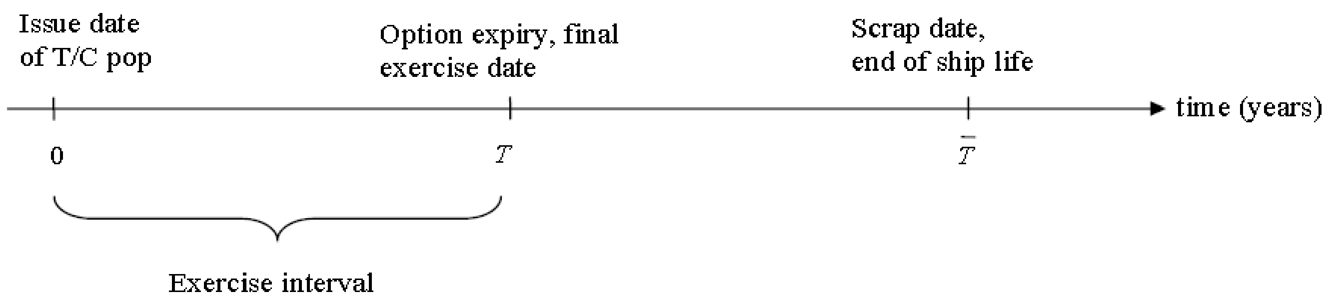
$r = 5\%,\ \kappa = 0.25,\ \theta^* = 20000$ per day, $\sigma = 5,000$ per day

$\bar{V} = 5m,\ t = 0,\ T = 5, \bar{T} = 25,\ K = 93$m

		Fixed T/C rate (daily)					
		5000	10000	15000	20000	25000	30000
Spot freight rate $X(0)$	5000	10.885	2.922	-5.041	-13.004	-20.967	-28.930
	10000	15.938	7.975	0.012	-7.951	-15.915	-23.878
	15000	21.093	13.130	5.166	-2.797	-10.760	-18.723
	20000	26.357	18.394	10.430	2.467	-5.496	-13.459
	25000	31.733	23.770	15.807	7.843	-0.120	-8.083
	30000	37.219	29.256	21.293	13.330	5.367	-2.596

But in practice the options are not always European...

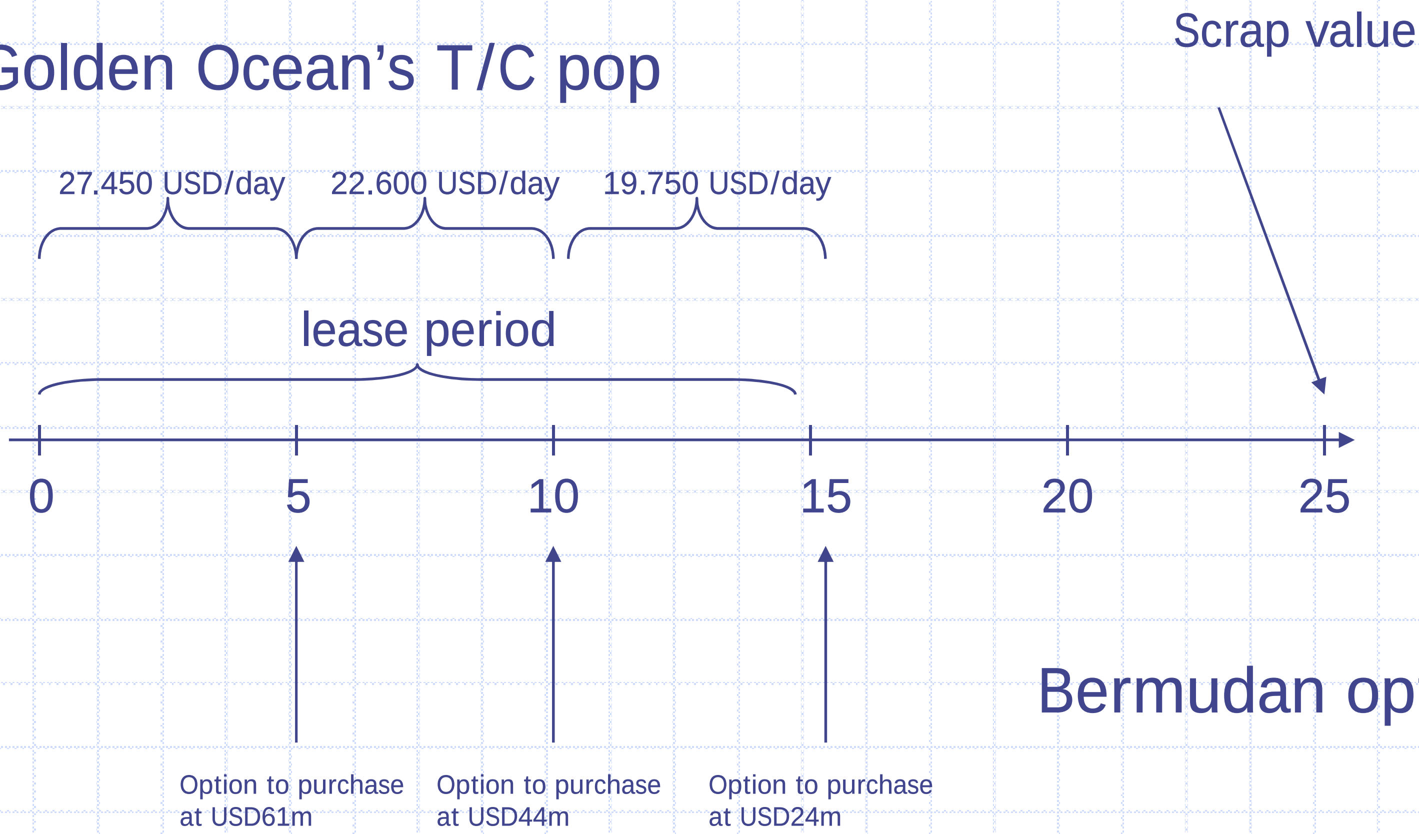
- The pure American-style TCPOP:



$$
\begin{aligned}
C(X_0,0) = \sup_{\tau\in\mathcal{S}_{[0,T]}} \; & E_0^Q \Bigg\{ \int_0^\tau e^{-ru}\Big(X(u)-\bar{X}(u)\Big)\,du \\
& + e^{-r\tau}\Big(V^{ship}(X(\tau),\tau)-K(\tau)\Big)\cdot 1_{\{\tau<T\}} \\
& + e^{-rT}\Big(V^{ship}(X(T),T)-K(T)\Big)^{+}\cdot 1_{\{\tau=T\}} \Bigg\},
\end{aligned}
$$

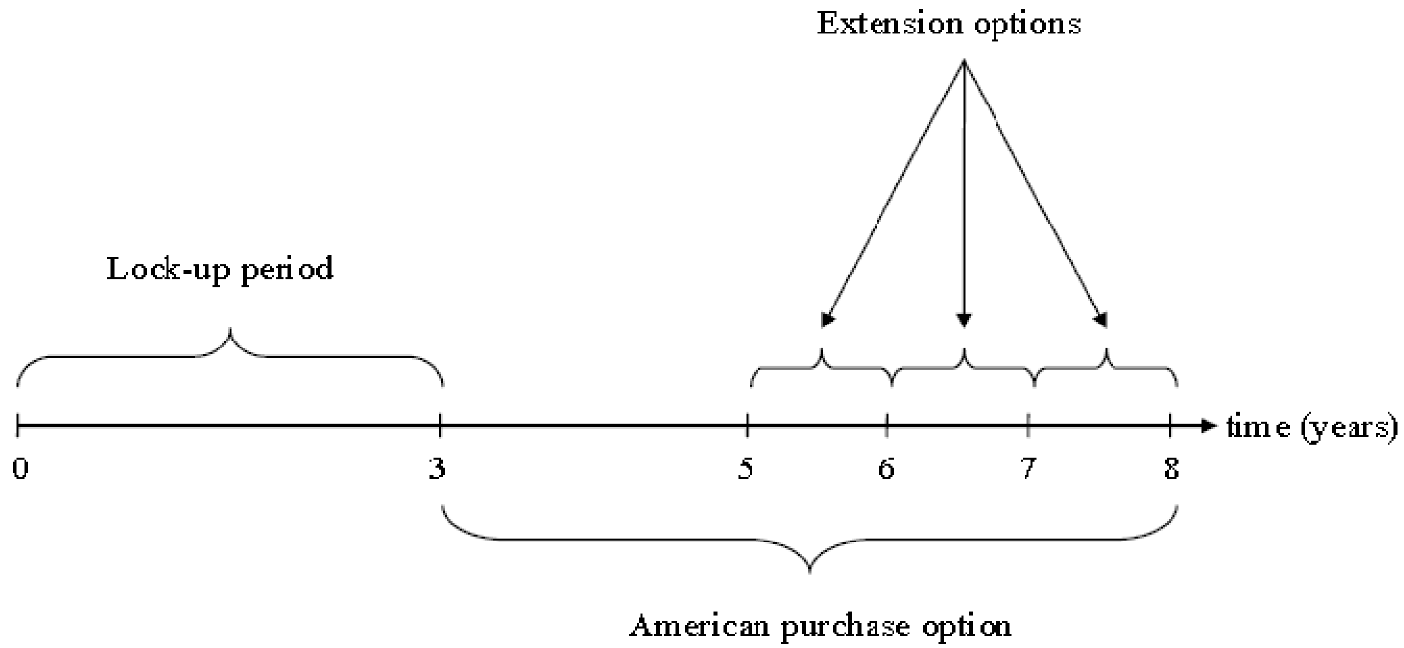
The Golden Ocean structure

- Golden Ocean's T/C pop

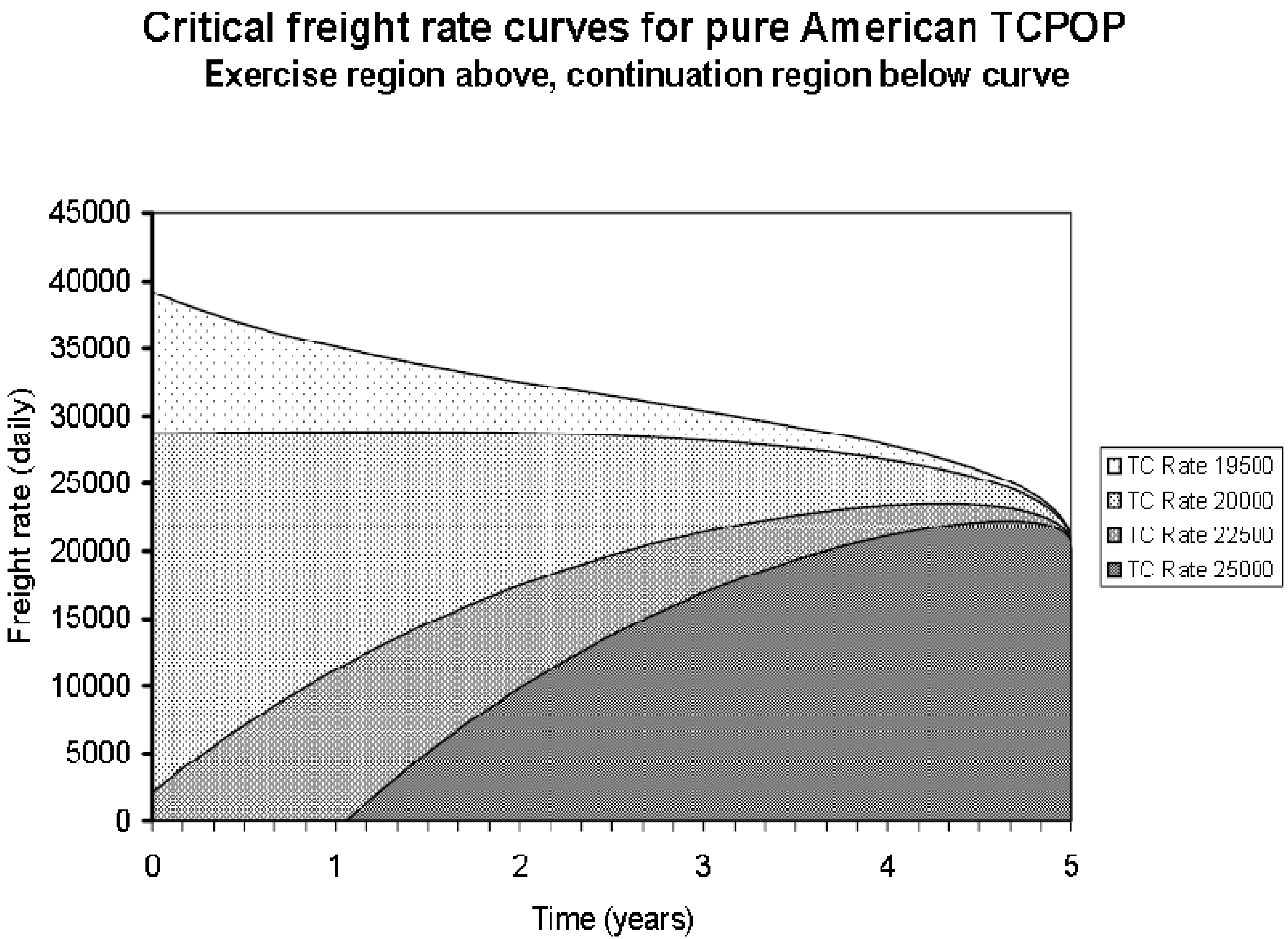


Bermudan option!

D/S Norden's TC pop



The pure American contract



Value of time charter contract with American-style purchase option (in millions)

(Value of similar European-style contract in parentheses below)

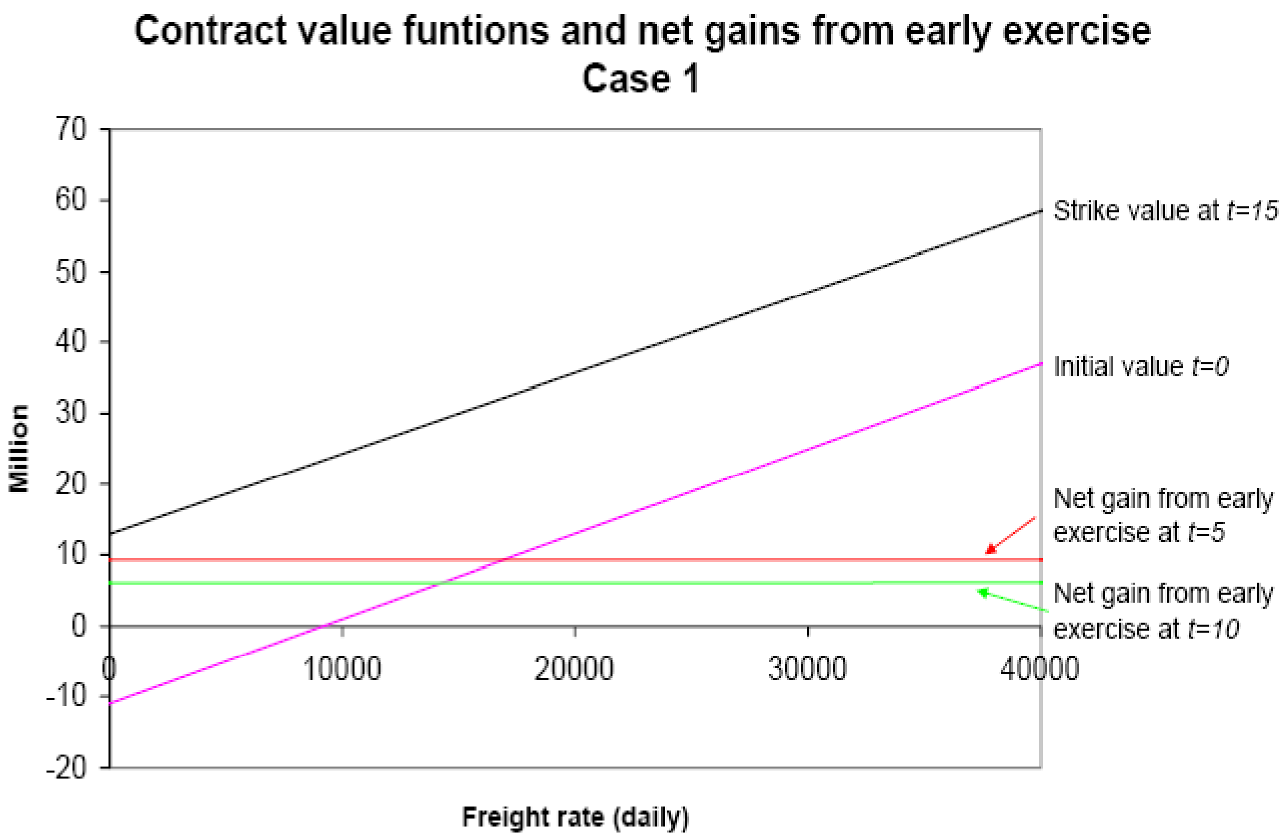
$r = 5\%$, $\kappa = 0.25$, $\theta^* = 20000$ per day, $\sigma = 5,000$ per day

$t = 0$, $T = 5$,$\bar{T} = 25$, $\bar{V} = 5$m

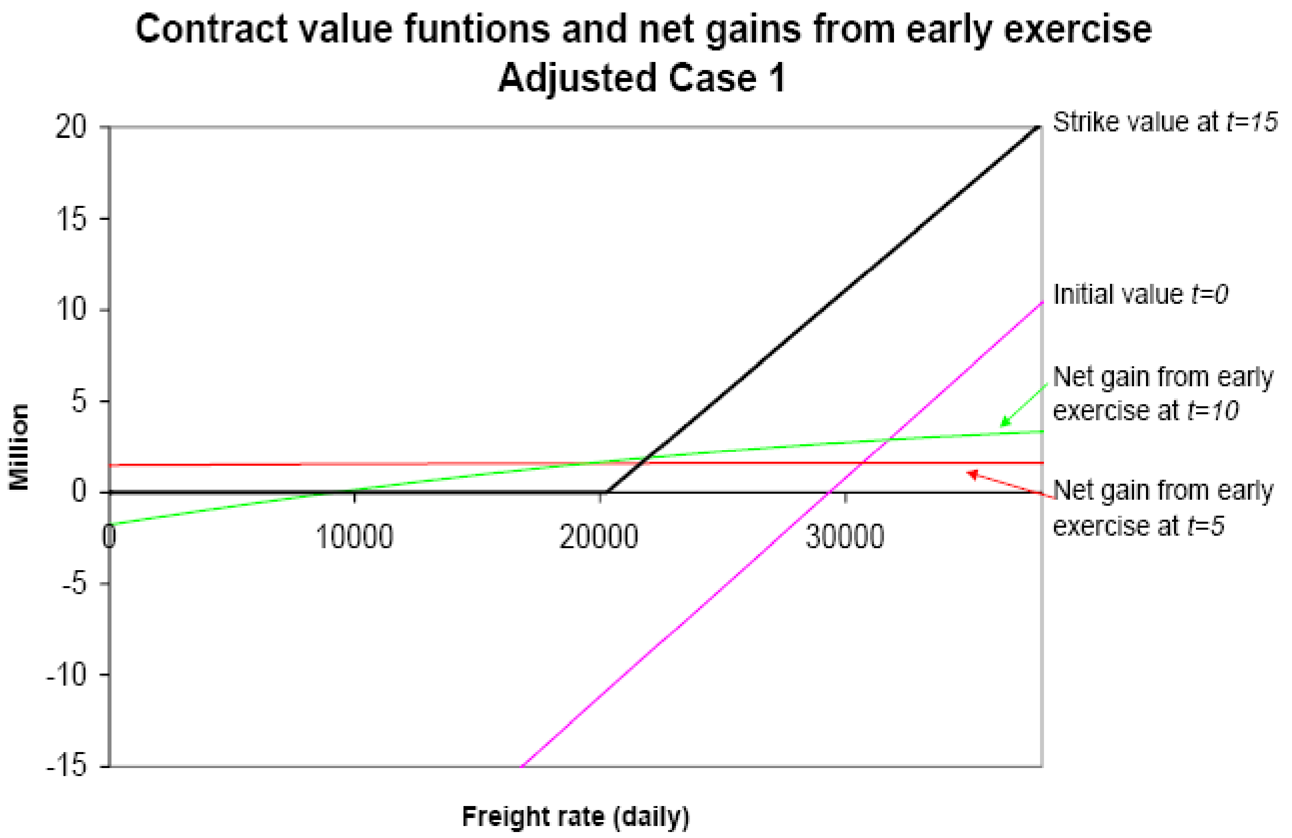
Strike price, $K(t)$, linear between $K(0) = 102.000$m and $K(5) = 92.865$m

		Contracted daily T/C rate, $\bar{X}$				
		19500	20000	22500	25000	30000
$X(0)$	5000	−12.151 (−12.180)	−12.927 (−12.977)	−15.814 (−16.958)	−15.814 (−20.940)	−15.814 (−28.903)
	10000	−7.073 (−7.120)	−7.835 (−7.916)	−9.818 (−11.898)	−9.818 (−15.879)	−9.818 (−23.843)
	15000	−1.883 (−1.957)	−2.619 (−2.753)	−3.821 (−6.735)	−3.821 (−10.716)	−3.821 (−18.680)
	20000	3.431 (3.316)	2.747 (2.520)	2.176 (−1.462)	2.176 (−5.444)	2.176 (−13.407)
	25000	8.880 (8.701)	8.306 (7.905)	8.173 (3.923)	8.173 (−0.059)	8.173 (−8.022)
	30000	14.474 (14.196)	14.169 (13.399)	14.169 (9.418)	14.169 (5.436)	14.169 (−2.527)

Case 1: Golden Ocean



Case 1: Golden Ocean adjusted



Value of Bermudan T/C–POP (in millions)

(Value of similar European-style contract in parentheses below)

$r = 5\%$, $\theta^* = 20000$ per day, $\sigma = 5,000$ per day

$t = 0$, $\bar{T} = 25$, $\bar{V} = 5$m

Strike prices; $K(5) = 61$m, $K(10) = 44$m, $K(15) = 24$m

		Mean reversion rate, κ			
		0.25	0.50	0.75	1.00
$X(0)$	5000	−5.029 (−15.970)	3.142 (−7.795)	6.186 (−4.709)	7.545 (−3.002)
	10000	0.967 (−9.975)	6.415 (−4.528)	8.459 (−2.484)	9.528 (−1.413)
	15000	6.964 (−3.979)	9.688 (−1.256)	10.710 (−0.234)	11.245 (0.301)
	20000	12.960 (2.017)	12.960 (2.016)	12.960 (2.016)	12.960 (2.015)
	25000	18.957 (8.014)	16.233 (5.289)	15.209 (4.265)	14.673 (3.729)
	30000	24.953 (14.009)	19.503 (8.560)	17.456 (6.513)	16.380 (5.440)
	35000	30.947 (20.003)	22.759 (11.822)	19.603 (8.709)	17.386 (6.844)

Case 2: Norden's TC pop

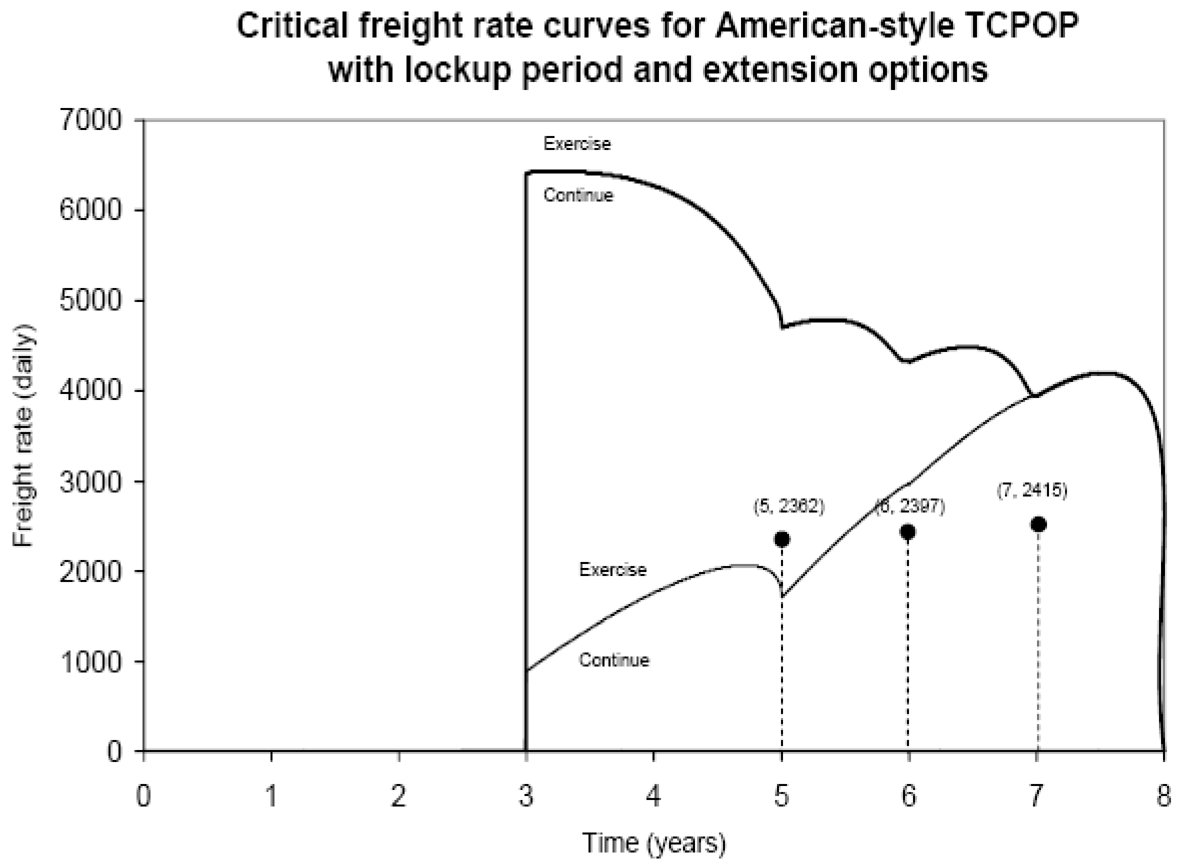


Figure 9: $\kappa = 0.25$, $r = 0.05$, $\bar{T} = 25$, $\bar{V} = 1\text{m}$, $\theta^* = 9000$, $\sigma = 3000$, strike price function linear between $K(3) = 24.55\text{m}$ and $K(8) = 18.64\text{m}$.

Case 2: Norden's TC pop

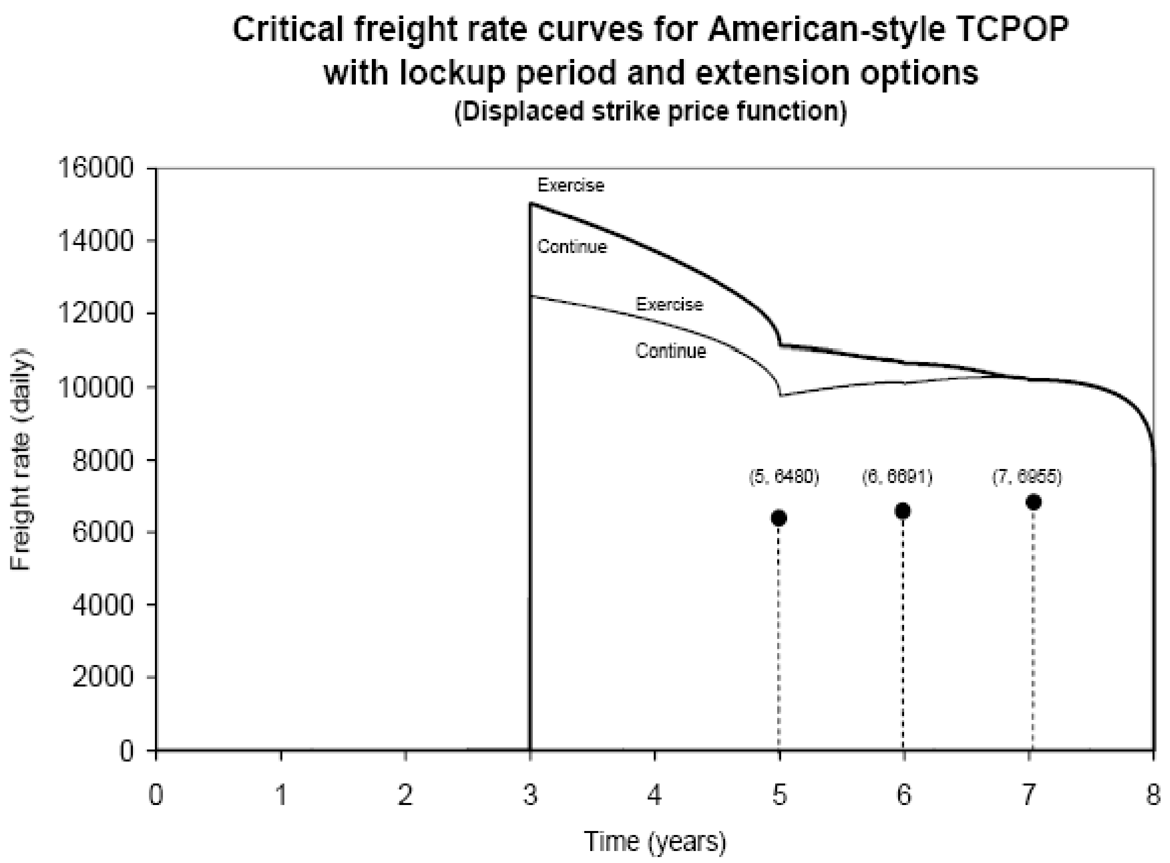


Figure 10: $\kappa = 0.05$, $r = 0.05$, $\bar{T} = 25$, $\bar{V} = 1\text{m}$, $\theta^* = 9000$, $\sigma = 3000$, strike price function linear between $K(3) = 40.00\text{m}$ and $K(8) = 34.09\text{m}$.

Value of Complex American-style T/C–POP (in millions)

(Value without extension option in parentheses below)

$r = 5\%,\ \kappa = 0.05,\ \theta^* = 9000,\ \sigma = 3000,\ t = 0,\ \bar{T} = 25,\ \bar{V} = 1\text{m}$

		Actual strike function			Displaced strike function		
		USD/JPY					
		100	110	120	100	110	120
Freight rate $X(0)$	2500	−7.843 (−11.186)	−6.666 (−9.418)	−5.570 (7.841)	−12.773 (−19.005)	−12.120 (−17.950)	−11.392 (−16.766)
	5000	−1.307 (−3.344)	0.161 (−1.413)	1.489 (0.262)	−8.017 (−12.798)	−7.040 (−11.388)	−5.985 (−9.849)
	7500	5.819 (4.702)	7.534 (6.732)	9.045 (8.458)	−2.741 (−6.233)	−1.375 (−4.432)	0.051 (−2.534)
	10000	13.412 (12.867)	15.308 (14.948)	16.942 (16.696)	3.115 (0.701)	4.906 (2.900)	6.713 (5.124)
	12500	21.325 (21.092)	23.335 (23.194)	25.040 (24.951)	9.555 (7.986)	11.772 (10.557)	13.926 (13.043)
	15000	29.428 (29.341)	31.498 (31.450)	33.328 (33.210)	16.532 (15.579)	19.135 (18.465)	21.572 (21.132)
	17500	37.627 (37.599)	39.724 (39.710)	41.478 (41.470)	23.957 (23.421)	26.874 (26.542)	29.511 (29.318)
	20000	45.867 (45.859)	47.975 (47.971)	49.733 (49.731)	31.718 (31.441)	34.865 (34.718)	37.625 (37.550)

Extensions/future work

- Consider alternative models of freight rate dynamics
- For example, Geometric Mean Reverting process

$$dX(t) = \kappa(\theta - \ln X(t))X(t)dt + \sigma X(t)dW(t)$$

- Guaranteed to stay positive
- See Jostein Tvedt, "Valuation of VLCCs under income uncertainty", Maritime Policy and Management, 1997.
- Empirical task of estimating parameters in the freight rate process
- Inclusion of one or more additional factors, eg. JPY/USD exchange rate

Conclusions / future work

- Would be great to calibrate model and check calculations done by D/S Norden
- Hopefully their estimates are ”water-tight” ☺
- Evaluate sensitivities and do risk (VaR) analyses
- Lots more…

- Thank you for your attention!