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Calibrating (Partial Equilibrium) Mathematical Programming Spatial Models

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Partial equilibrium spatial trade models
 (in many cases) *do it better*

partial equilibrium spatial trade models,
 being “naturally” able to generate bilateral
 trade flows (without having to resort to the
Armington assumption), are better suited to
 model discriminatory trade policy instruments
 (such as preferential tariffs, RTAs,
 country/region specific TRQs, country specific
 export subsidies,...)

...but are not perfect.

two often cited *cons* of **partial equilibrium mathematical programming spatial trade models:**

- in the solution of spatial models (which is given by the bilateral trade flows) **trade flows specialization** tends to emerge (i.e. significantly less non-zero bilateral trade flows than those observed)
- **bilateral transportation costs** (or, in more general terms, transaction costs), **are difficult to assess**

one way of addressing the problem is through ***Calibration***

- Bauer and Kasnakoglu (1990)
- Bouamra-Mechemache, Chavaz, Cox and Requillart (2002)
- Jansson and Heckelei (2009)

The paper (it exists!, 70+ pages long...) is structured in two parts:

the *first part* addresses the problem from an analytical point of view

- the classical transportation model
- the T&J trade model, one commodity
- the T&J trade model, k commodities
- the "Equilibrium problem", k commodities

the *second part* does it from an empirical point of view (it presents several numerical examples)

T&J trade model, one commodity (uncalibrated)

$$\max QWF = \sum_{j=1}^R (a_j - D_j x_j^D / 2) x_j^D - \sum_{i=1}^R (b_i + S_i x_i^S / 2) x_i^S - \sum_{i=1}^R \sum_{j=1}^R t c_{ij} x_{ij}$$

$$\text{Subject to: } x_j^D \leq \sum_{i=1}^R x_{ij}$$

$$\sum_{j=1}^R x_{ij} \leq x_i^S$$

$$x_j^D \geq 0, x_i^S \geq 0, x_{ij} \geq 0$$

the solution does not reproduce observed trade flows (...and , as a result, the observed quantities produced and consumed in each country)

T&J trade model, one commodity (calibrated)

$$\max QWF = \sum_{j=1}^R (a_j - D_j x_j^D / 2) x_j^D - \sum_{i=1}^R (b_i + S_i x_i^S / 2) x_i^S - \sum_{i=1}^R \sum_{j=1}^R tc_{ij} x_{ij}$$

$$\text{Subject to: } x_j^D \leq \sum_{i=1}^R x_{ij}$$

$$\sum_{j=1}^R x_{ij} \leq x_i^S \quad x_j^D \geq 0, x_i^S \geq 0, x_{ij} \geq 0$$

let us impose a set of additional constraints:

$$x_{ij} = \bar{x}_{ij}$$

then solve the “augmented” model (which will “forcefully” calibrate trade flows) and generate the dual values λ^*_{ij} associated to the additional constraints

T&J trade model, one commodity (calibrated)

$$\max QWF = \sum_{j=1}^R (a_j - D_j x_j^D / 2) x_j^D - \sum_{i=1}^R (b_i + S_i x_i^S / 2) x_i^S - \sum_{i=1}^R \sum_{j=1}^R (tc_{ij} + \lambda^*_{ij}) x_{ij}$$

$$\text{Subject to: } x_j^D \leq \sum_{i=1}^R x_{ij} \quad \sum_{j=1}^R x_{ij} \leq x_i^S$$

$$\cancel{x_{ij} = \bar{x}_{ij}} \quad x_j^D \geq 0, x_i^S \geq 0, x_{ij} \geq 0$$

this model calibrates observed consumed and produced quantities in each country

at least in general, these quantities are associated to multiple optimal solutions of the problem (*i.e.*, sets of bilateral trade flows), one of them being the set of **observed** trade flows

Why do the λ^*_{ij} show up as a correction of bilateral transaction costs?

let us consider the Lagrangean function of the augmented problem:

$$L = \sum_{j=1}^R (a_j - D_j x_j^D / 2) x_j^D - \sum_{i=1}^R (b_i + S_i x_i^S / 2) x_i^S - \sum_{i=1}^R \sum_{j=1}^R tc_{ij} x_{ij} \\ + \sum_{j=1}^R p_j^D (\sum_{i=1}^R x_{ij} - x_j^D) + \sum_{i=1}^R p_i^S (x_i^S - \sum_{j=1}^R x_{ij}) + \sum_{i=1}^R \sum_{j=1}^R \lambda_{ij} (\bar{x}_{ij} - x_{ij})$$

and the Karush-Kuhn-Tucker equilibrium conditions:

$$\frac{\partial L}{\partial x_{ij}} = p_j^D - p_i^S - tc_{ij} - \lambda_{ij} \leq 0$$

$$\frac{\partial L}{\partial x_{ij}} x_{ij} = 0$$

T&J trade model, k commodities

$$\max QWF = \sum_{j=1}^R (\mathbf{a}_j - \mathbf{D}_j \mathbf{x}_j^D / 2)' \mathbf{x}_j^D - \sum_{i=1}^R (\mathbf{b}_i + \mathbf{S}_i \mathbf{x}_i^S / 2)' \mathbf{x}_i^S - \sum_{i=1}^R \sum_{j=1}^R \mathbf{t}c'_{ij} \mathbf{x}_{ij}$$

$$\text{Subject to: } \mathbf{x}_j^D \leq \sum_{i=1}^R \mathbf{x}_{ij} \quad \sum_{j=1}^R \mathbf{x}_{ij} \leq \mathbf{x}_i^S \quad x_j^D \geq 0, x_i^S \geq 0, x_{ij} \geq 0$$

the statement of the problem in the form of maximizing a QW objective function that assumes a quadratic structure imposes the requirement that matrices D_j and S_i be **symmetric and positive semidefinite**; ...this is quite a strong assumption, since there is no reason why D_j and S_i should satisfy these conditions

Solution: solving a (calibrated) “Equilibrium problem”

primal relations:

$$\begin{aligned} \mathbf{p}_j^D \geq \mathbf{0} \quad \mathbf{x}_j^D \leq \sum_{i=1}^R \mathbf{x}_{ij} \quad & \left(\sum_{i=1}^R \mathbf{x}_{ij} - \mathbf{x}_j^D \right)' \mathbf{p}_j^D = \mathbf{0} \\ \mathbf{p}_i^S \geq \mathbf{0} \quad \sum_{j=1}^R \mathbf{x}_{ij} \leq \mathbf{x}_i^S \quad & \left(\mathbf{x}_i^S - \sum_{j=1}^R \mathbf{x}_{ij} \right)' \mathbf{p}_i^S = \mathbf{0} \\ \lambda_{ij} \text{ free,} \quad \mathbf{x}_{ij} = \bar{\mathbf{x}}_{ij} \quad & \left(\bar{\mathbf{x}}_{ij} - \mathbf{x}_{ij} \right)' \lambda_{ij} = \mathbf{0} \end{aligned}$$

dual relations:

$$\begin{aligned} \mathbf{x}_j^D \geq \mathbf{0} \quad \mathbf{a}_j - \mathbf{D}_j \mathbf{x}_j^D \leq \mathbf{p}_j^D \quad & \left(\mathbf{p}_j^D - \mathbf{a}_j + \mathbf{D}_j \mathbf{x}_j^D \right)' \mathbf{x}_j^D = \mathbf{0} \\ \mathbf{x}_i^S \geq \mathbf{0} \quad \mathbf{p}_i^S \leq \mathbf{b}_i + \mathbf{S}_i \mathbf{x}_i^S \quad & \left(\mathbf{b}_i + \mathbf{S}_i \mathbf{x}_i^S - \mathbf{p}_i^S \right)' \mathbf{x}_i^S = \mathbf{0} \\ \mathbf{x}_{ij} \geq \mathbf{0} \quad \mathbf{p}_j^D \leq \mathbf{p}_i^S + (\mathbf{t} \mathbf{c}_{ij} + \lambda_{ij}) \quad & \left[\mathbf{p}_i^S + (\mathbf{t} \mathbf{c}_{ij} + \lambda_{ij}) - \mathbf{p}_j^D \right]' \mathbf{x}_{ij} = \mathbf{0} \end{aligned}$$

$$\min \left\{ \sum_{ij} [\mathbf{z}'_{jp1} \mathbf{p}_j^D + \mathbf{z}'_{ip2} \mathbf{p}_i^S + \mathbf{z}'_{jd1} \mathbf{x}_j^D + \mathbf{z}'_{id2} \mathbf{x}_i^S + \mathbf{z}'_{ijd3} \mathbf{x}_{ij}] \right.$$

Subject to:

$$\begin{aligned} \mathbf{x}_j^D + \mathbf{z}_{jp1} &= \sum_{i=1}^R \mathbf{x}_{ij} & \sum_{i=1}^R \mathbf{x}_{ij} + \mathbf{z}_{ip2} &= \mathbf{x}_i^S \\ \mathbf{x}_{ij} &= \bar{\mathbf{x}}_{ij} & \mathbf{a}_j - \mathbf{D}_j \mathbf{x}_j^D + \mathbf{z}_{jd1} &= \mathbf{p}_j^D \\ \mathbf{p}_i^S + \mathbf{z}_{id2} &= \mathbf{b}_i + \mathbf{S}_i \mathbf{x}_i^S & \mathbf{p}_j^D + \mathbf{z}_{ijd3} &= \mathbf{p}_i^S + (\mathbf{t} \mathbf{c}_{ij} + \lambda_{ij}) \\ \mathbf{p}_j^D \geq \mathbf{0} \quad \mathbf{p}_i^S \geq \mathbf{0} \quad \lambda_{ij} \text{ free} \quad \mathbf{x}_j^D \geq \mathbf{0} \quad \mathbf{x}_i^S \geq \mathbf{0} \quad \mathbf{x}_{ij} \geq \mathbf{0} \end{aligned}$$

Phase I: solve this model (which will calibrate observed production and consumption in each country) and generate the values of λ_{ij}^*

$$\min \left\{ \sum_{ij} [z'_{jP1} \mathbf{p}_j^D + z'_{iP2} \mathbf{p}_i^S + z'_{jD1} \mathbf{x}_j^D + z'_{iD2} \mathbf{x}_i^S + z'_{ijD3} \mathbf{x}_{ij}^D] \right.$$

Subject to:

$$\mathbf{x}_j^D + \mathbf{z}_{jP1} = \sum_{i=1}^R \mathbf{x}_{ij} \quad \sum_{i=1}^R \mathbf{x}_{ij} + \mathbf{z}_{iP2} = \mathbf{x}_i^S$$

$$\mathbf{x}_{ij} \leq \bar{\mathbf{x}}_{ij} \quad \mathbf{a}_j - \mathbf{D}_j \mathbf{x}_j^D + \mathbf{z}_{jD1} = \mathbf{p}_j^D$$

$$\mathbf{p}_i^S + \mathbf{z}_{iD2} = \mathbf{b}_i + \mathbf{S}_i \mathbf{x}_i^S \quad \mathbf{p}_j^D + \mathbf{z}_{ijD3} = \mathbf{p}_i^S + (\mathbf{tc}_{ij} + \lambda_{ij}^*)$$

$$\mathbf{p}_j^D \geq \mathbf{0} \quad \mathbf{p}_i^S \geq \mathbf{0} \quad \mathbf{x}_j^D \geq \mathbf{0} \quad \mathbf{x}_i^S \geq \mathbf{0} \quad \mathbf{x}_{ij} \geq \mathbf{0}$$

Phase II: the λ_{ij}^* are inserted in the model and the calibration constraints eliminated.

this model will calibrate observed production and consumption in each country

One of the seven numerical examples developed in the draft paper: 4 countries, 3 goods, no symmetry required for the matrices of demand and supply function slopes

the matrix of the demand intercepts:

$$\mathbf{A} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} A \\ B \\ U \\ E \end{matrix} & \begin{bmatrix} 30.0 & 25.0 & 20.0 \\ 22.0 & 18.0 & 15.0 \\ 25.0 & 10.0 & 18.0 \\ 28.0 & 20.0 & 19.0 \end{bmatrix} \end{matrix}$$

↑
countries

the matrix of the demand slopes:

$$\mathbf{D} = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} A.1 \\ A.2 \\ A.3 \\ B.1 \\ B.2 \\ B.3 \\ U.1 \\ U.2 \\ U.3 \\ E.1 \\ E.2 \\ E.3 \end{matrix} & \begin{bmatrix} 1.2 & 0.2 & -0.2 \\ 0.3 & 2.1 & 0.2 \\ -0.1 & 0.1 & 0.7 \\ 0.8 & -0.1 & 0.2 \\ -0.2 & 1.6 & 0.4 \\ 0.3 & 0.3 & 2.6 \\ 0.8 & 0.2 & 0.5 \\ 0.3 & 0.9 & -0.1 \\ 0.4 & 0.0 & 1.7 \\ 1.1 & 0.1 & 0.3 \\ 0.0 & 0.8 & 0.2 \\ 0.4 & 0.3 & 0.9 \end{bmatrix} \end{matrix}$$

One of the seven numerical examples developed in the draft paper: 4 countries, 3 goods, no symmetry required for the matrices of demand and supply function slopes

the matrix of the demand intercepts:

$$A = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} A \\ B \\ U \\ E \end{matrix} & \begin{bmatrix} 30.0 & 25.0 & 20.0 \\ 22.0 & 18.0 & 15.0 \\ 25.0 & 10.0 & 18.0 \\ 28.0 & 20.0 & 19.0 \end{bmatrix} \end{matrix}$$

↑
countries

the matrix of the demand slopes is not symmetric

$$D = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} A.1 \\ A.2 \\ A.3 \\ B.1 \\ B.2 \\ B.3 \\ U.1 \\ U.2 \\ U.3 \\ E.1 \\ E.2 \\ E.3 \end{matrix} & \begin{bmatrix} 1.2 & 0.2 & -0.2 \\ 0.3 & 2.1 & 0.2 \\ -0.1 & 0.1 & 0.7 \\ 0.8 & -0.1 & 0.2 \\ -0.2 & 1.6 & 0.4 \\ 0.3 & 0.3 & 2.6 \\ 0.8 & 0.2 & 0.5 \\ 0.3 & 0.9 & -0.1 \\ 0.4 & 0.0 & 1.7 \\ 1.1 & 0.1 & 0.3 \\ 0.0 & 0.8 & 0.2 \\ 0.4 & 0.3 & 0.9 \end{bmatrix} \end{matrix}$$

the matrix of the supply intercepts:

$$B = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} A \\ B \\ U \\ E \end{matrix} & \begin{bmatrix} 0.4 & 0.1 & 0.7 \\ 0.2 & -0.4 & 0.3 \\ -0.6 & 0.2 & -0.4 \\ -0.5 & -1.6 & -1.2 \end{bmatrix} \end{matrix}$$

the matrix of the supply slopes:

$$S = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} A.1 \\ A.2 \\ A.3 \\ B.1 \\ B.2 \\ B.3 \\ U.1 \\ U.2 \\ U.3 \\ E.1 \\ E.2 \\ E.3 \end{matrix} & \begin{bmatrix} 1.4 & -0.4 & 0.3 \\ -0.2 & 2.1 & 0.2 \\ 0.2 & 0.3 & 1.7 \\ 2.4 & 0.5 & 0.2 \\ 0.7 & 1.6 & 0.3 \\ 0.1 & 0.5 & 1.8 \\ 1.9 & -0.1 & 0.5 \\ -0.1 & 2.8 & 0.4 \\ 0.6 & 0.5 & 2.1 \\ 0.6 & -0.1 & 0.2 \\ -0.1 & 1.1 & 0.5 \\ 0.3 & 0.3 & 0.5 \end{bmatrix} \end{matrix}$$

the matrix of the transaction costs:

$$TC = \begin{matrix} & \begin{matrix} 1 & 2 & 3 \end{matrix} \\ \begin{matrix} A.A \\ A.B \\ A.U \\ A.E \\ B.A \\ B.B \\ B.U \\ B.E \\ U.A \\ U.B \\ U.U \\ U.E \\ E.A \\ E.B \\ E.U \\ E.E \end{matrix} & \begin{bmatrix} 0.5 & 0.5 & 0.5 \\ 1.5 & 1.5 & 1.5 \\ 1.0 & 1.0 & 1.0 \\ 3.0 & 3.0 & 3.0 \\ 1.5 & 1.5 & 1.5 \\ 0.5 & 0.5 & 0.5 \\ 2.2 & 2.2 & 2.2 \\ 4.0 & 4.0 & 4.0 \\ 1.0 & 1.0 & 1.0 \\ 2.2 & 2.2 & 2.2 \\ 0.5 & 0.5 & 0.5 \\ 3.7 & 3.7 & 3.7 \\ 3.0 & 3.0 & 3.0 \\ 4.0 & 4.0 & 4.0 \\ 3.7 & 3.7 & 3.7 \\ 0.5 & 0.5 & 0.5 \end{bmatrix} \end{matrix}$$

	1	2	3
A.A	3.910	1.740	4.637
A.B	2.834	2.887	
A.U	3.684		
B.A			4.835
uncalibrated solution: $X^* = B.B$	5.356	3.037	
U.A		2.704	2.037
U.U	7.618		0.837
E.A	9.809		0.450
E.E	12.909	12.124	0.158

	1	2	3
A.A	3.000	2.000	3.000
A.B	2.500	2.500	
A.U	2.000		
B.A	0.500		4.000
"observed" trade flows: $\bar{X} = B.B$	5.000	2.000	
U.A	1.000	1.000	1.000
U.U	6.000		
E.A	10.000		
E.E	12.000	10.000	

	1	2	3
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A.A	3.000	2.000	3.000
A.B	2.500	2.500	
A.U	2.000		
B.A	0.500		4.000
"observed" trade flows: $\bar{X} = B.B$	5.000	2.000	
U.A	1.000	1.000	1.000
U.U	6.000		
E.A	10.000		
E.E	12.000	10.000	

the λ_{ij}^* obtained when the calibration constraints $x_{ij} = \bar{x}_{ij}$ are imposed (phase I)

	1	2	3
A.A	3.100	3.600	6.400
A.B	4.950	2.150	1.250
A.U	7.600	-2.050	5.150
A.E	0.800	0.350	-0.450
B.A	-3.100	3.400	5.000
B.B	0.750	3.950	1.850
B.U	1.200	-2.450	3.550
B.E	-5.400	0.150	-1.850
U.A	-0.500	9.050	8.150
U.B	1.150	7.400	2.800
U.U	5.000	4.400	7.900
U.E	-3.000	5.600	1.100
E.A	-1.100	2.550	12.550
E.B	0.750	1.100	7.400
E.U	3.200	-3.300	11.100
E.E	1.600	4.300	10.700

Phase II: the λ_{ij}^* are inserted in the model and the calibration constraints eliminated.

the solution calibrates observed production and consumption in each country:

	1	2	3	
$\bar{x}^S =$	A	7.500	4.500	3.000
	B	5.500	2.000	4.000
	U	7.000	1.000	1.000
	E	22.000	10.000	
		1	2	3
	A	14.500	3.000	8.000
	B	7.500	4.500	
	U	8.000		
	E	12.000	10.000	

	1	2	3	
$x^{S*} =$	A	7.500	4.500	3.000
	B	5.500	2.000	4.000
	U	7.000	1.000	1.000
	E	22.000	10.000	
		1	2	3
	A	14.500	3.000	8.000
	B	7.500	4.500	
	U	8.000		
	E	12.000	10.000	

the problem has multiple optimal solutions, all associated to the same quantities produced and consumed in each country

three examples of obtained optimal sets of trade flows associated to the same optimal solution:

	1	2	3		1	2	3		1	2	3	
X_1^*	AA	3.000	2.000	3.000	AA	7.500	2.000	3.000	AA	0.500		3.000
	AB	2.500	2.500		AB		2.500		AB	0.500	2.500	
	AU	2.000			BA	0.500		4.000	AE	6.500	2.000	
	BA	0.500		4.000	BB		2.000		BA			4.000
	BB	5.000	2.000		BE	5.000			BB		2.000	
	UA	1.000	1.000	1.000	UE	7.000	1.000	1.000	BE	5.500		
	UU	6.000			EA	6.500	1.000		UA			1.000
	EA	10.000			EB	7.500			UB	7.000		
	EE	12.000	10.000		EU	8.000			UE		1.000	
					EE		9.000		EA	14.000	3.000	
									EU	8.000		
									EE		7.000	

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three examples of obtained optimal sets of trade flows associated to the same optimal solution:

	1	2	3		1	2	3		1	2	3	
X_1^*	AA	3.000	2.000	3.000	AA	7.500	2.000	3.000	AA	0.500		3.000
	AB	2.500	2.500		AB		2.500		AB	0.500	2.500	
	AU	2.000			BA	0.500		4.000	AE	6.500	2.000	
	BA	0.500		4.000	BB		2.000		BA			4.000
	BB	5.000	2.000		BE	5.000			BB		2.000	
	UA	1.000	1.000	1.000	UE	7.000	1.000	1.000	BE	5.500		
	UU	6.000			EA	6.500	1.000		UA			1.000
	EA	10.000			EB	7.500			UB	7.000		
	EE	12.000	10.000		EU	8.000			UE		1.000	
					EE		9.000		EA	14.000	3.000	
									EU	8.000		
									EE		7.000	



“observed” trade flows

Conclusions

- a calibration procedure has been developed to make the base model reproduce observed produced and consumed quantities in each country (*not trade flows*)
- the calibrated model is potentially effective in correcting commodity-specific bilateral transaction costs, which are difficult to assess

Conclusions

- however, because corrected transaction costs will now “include” all factors effecting trade which are not explicitly (and effectively...) represented in the model, (including policies...), if these factors are not modeled, or are incorrectly modeled, policy simulations will generate distorted results
- the calibration procedure proposed does not solve the issue of the over-specialization of trade flows when the calibrated model is used to perform policy simulations