Does Arbitraging Matter? Spatial Trade Models and Discriminatory Trade Policies

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When modeling discriminatory trade policies, such as targeted embargoes or targeted subsidies, failure to explicitly include assumptions about the possibility of simultaneous exporting and importing may yield misleading results. Nonlinear programming and "vector sandwich" models implicitly set rules regarding arbitraging which may be at variance with actual policies and/or country behavior. The paper introduces an alternative spatial model which allows the researcher to explicitly incorporate her own assumptions about arbitraging. An analysis of the 1980 U.S. embargo to the USSR shows how the proposed model performs relative to the most frequently used spatial trade models.

Key words: arbitraging, discriminatory trade policies, embargo, spatial models, trade.

The explicit treatment of arbitraging behavior—that is, countries importing and exporting at the same time—in spatial trade models may be necessary for the models to produce valid results. The issue becomes important when analyzing discriminatory national trade policies intended to benefit friends and/or punish enemies. Examples of such policies include the Generalized System of Preferences (GSP), Lome' Convention preferences, targeted export subsidies, PL 480, selective quotas, and targeted embargoes. These policies create multiple prices and generate possibilities to export and import simultaneously to take advantage of price spreads. Discriminatory trade policies include mechanisms to prevent arbitraging.

In this paper we argue that unless trade models explicitly incorporate the possibility of simultaneously exporting and importing, the choice of the trade model implicitly sets the assumptions on arbitraging. For example, spatial trade models using reduced-form trade equations exclude the possibility of a country switching from one side of the market to the other as prices change, or of simultaneously exporting and importing.

In the first part of the paper we discuss the role of arbitraging in the design and management of discriminatory agricultural trade policies. The implications for trade policy analysis of different assumptions about arbitraging are briefly addressed. In the second part, the implicit hypotheses about arbitraging associated with two classes of spatial models, nonlinear programming (NLP) models (which include quadratic programming models as a special case) and vector sandwich (VS) models, are discussed.

An alternative model, presented in the third part of the paper, allows countries to switch from one side of the market to the other as prices change and permits the user to incorporate explicit assumptions about arbitraging. The model is an improvement over other spatial trade models when the policies to be analyzed include, for example, a trade liberalization when preferential trade agreements exist, an embargo, or a targeted export subsidy. A numerical example addresses arbitraging behavior associated with the 1980 U.S. embargo against the USSR and shows how the proposed model compares with frequently used models.

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Discriminatory Agricultural Trade Policies and Arbitraging

World agricultural markets contain many discriminatory trade policies in which arbitraging behavior is a matter of concern. Examples include preferential tariffs, targeted export subsidies, embargoes, customs unions, food aid, and preferential import quotas. For example, countries using targeted export subsidies are concerned with preventing targeted importers from reexporting subsidized imports; countries granting preferential market access are concerned with preventing the reexporting of imports from nonpreferred countries.

In the United States preferential agricultural tariff reduction agreements include the GSP, the Caribbean Basin Initiative, the Israeli free trade agreement and the recently concluded U.S.–Canada Free Trade Agreement. Other discriminatory trade policies include the Agricultural Trade Development and Assistance Act of 1954 (PL 480), the Export Enhancement Program (EEP), and a series of targeted embargoes including those against the USSR in 1974, 1975, and 1980–81. On preferential imports under its GSP scheme, the United States applies a "rule of origin" which requires that at least 35% (50% if two preferred countries involved) of the value of the article is added in the developing country. The same constraints apply to duty free treatment under the Caribbean Basin Initiative (Organization of American States). The United States–Canada free trade agreement contains a substantial set of "rules of origin" to prevent reexport when different third country tariffs apply in the two countries. PL 480 prevents concessional shipments from disrupting commercial markets (lowering prices) and seeks "commitment from participatory countries that will prevent resale or transhipment to other countries, or use (for other than domestic purposes) of surplus agricultural commodities purchased under the act" (sect. 101). In the early years of PL480, the United States did not apply the principle of additionality (sales must be in addition to commercial demand) to barter sales as it did to sales for local currency (Davis). Strong protests from other exporters about arbitraging of barter sales led to a modification of policy in 1957 (Mortensen, Ezekiel, Kristjanson).

Abbott, Paarlberg, and Sharples state that targeted export subsidies "are often criticized because arbitrage or rerouting of exports in transit can frustrate the subsidizing country's policies. International trading firms or importing countries might benefit by reexporting subsidized commodities, as occurred early in the PL 480 program" (p. 724). To enter the EEC under the preferential tariffs granted by the Lome' Convention, exports from the African, Caribbean, and Pacific (ACP) countries must fulfill the conditions stated in Protocol 1 of the convention, concerning the definition of the concept of "originating products."[1] A similar condition is contained in the EEC's GSP scheme (EEC). Borrmann, Borrmann, and Stegger (pp. 117–20) argue that the "rules of origin" may have strongly affected the volume of trade generated through the EEC's GSP scheme. A "country of origin" constraint is also contained in the ASEAN (Association of Southeast Asian Nations) agreement. Koester and Schmitz argue that Kenya imported sugar from the world market and exported it to the EC to capitalize on benefits due under the community preferential import policy. The popular Italian press claimed Israel is exporting frozen orange juice to the EC well in excess of its processing capacity (*La Repubblica,* Dec. 1986).

In all the preferential tariff reduction agreements, the constraints to assure that imports are originating in the beneficiary country are intended to avoid arbitraging and prevent third countries from exploiting the preferential policy. Constraints on the volume of exports receiving preferential treatment are usually included as well.

However, the various "rules of origin" still allow arbitraging. The preferred country can still find it feasible and profitable to import and export at the same time, using low price imports for domestic consumption while exporting domestic production at a higher preferential price. In this case, the quantity arbitraged is implicitly bounded by the "rules of origin" not to exceed domestic consumption.

Finally, a comprehensive study of economic sanctions by Hufbauer and Schott reports that the lack of cooperation of other countries often contributed to sanctions failure. In real-world discriminatory trade policies, arbitraging does matter. Failure to avoid arbitraging may jeopardize the accomplishment of the expected policy goals.

[1] Products originating in the ACP countries are defined, in simple terms, as products wholly obtained in one or more ACP countries, or products which have undergone sufficient working or processing within the ACP countries (third ACP-EEC Convention, signed in Lome', Togo on 8 Dec. 1984).

Modeling Discriminatory Trade Policies

When discriminatory trade policies are considered, the determination of the net trade positions needs to be based on a spatial trade model, a model capable of reproducing trade flows between each pair of traders. Each region may buy (sell) from (to) different regions at different prices, collecting (paying) different per unit tariffs (subsidies). Any discriminatory trade policy can be equivalently expressed in terms of a tariff or a subsidy. Targeted embargoes are equivalent to country-specific prohibitive export tariffs. Country-specific export (import) quotas may be translated into two export (import) taxes: one, equal to zero, active up to the quota ceiling; the other, prohibitive, active above that ceiling. Food donations represent volume constrained subsidized exports.

Any spatial trade model solution is such that, for each pair of countries, say country i and country j, the domestic prices (p_i and p_j, respectively), must satisfy the following relations (as long as no binding constraint is imposed on the trade flow between the two countries):

$$(1) \qquad (p_j - p_i - t_{ij} + \sigma_{ij} - \pi_{ij}) \leq 0;$$

$$(2) \qquad (p_j - p_i - t_{ij} + \sigma_{ij} - \pi_{ij})x_{ij} = 0;$$

where t_{ij} is the fixed transportation cost to ship one unit of the commodity from region i to region j, σ_{ij} is the export subsidy that country i pays to its producers for each unit exported to country j, π_{ij} is the import tariff that country j imposes on each unit it imports from country i, and x_{ij} is the nonnegative trade flow from country i to country j. If the trade flow from country i to country j is positive, then the per unit transportation cost plus the tariff minus the subsidy must give the wedge between the two domestic prices. If no shipments occur from country i to country j, then the difference between the two domestic prices must be smaller or, at most, equal to the transportation cost plus the tariff minus the subsidy (implying that shipments from country i to country j are not profitable).

Without interventions, the only possible wedges between domestic prices of trading countries are transportation costs. The matrix of the transportation costs is consistent, which means that the minimum cost path to ship from region i to region j is always the one directly connecting the two regions. In this case, there is no rationale for arbitraging. Finding the market equilibrium solution does not depend on differentiating between transportation costs, subsidies, and tariffs. Given domestic demands and supplies, all that matters is the net sum of the transportation cost plus the import tariff minus the export subsidy for each ordered pair [i.e., pairs (i, j) and (j, i) are different] of countries. This quantity represents a generalized transfer cost. The addition to and subtraction from the transportation costs matrix of nondisciminatory tariffs and subsidies does not affect its consistency. However, this property may be disrupted by discriminatory trade policies.[2]

Most agricultural trade models are based on an a priori definition of the sets of the importing and exporting regions. Each country is represented through its excess demand or supply schedule. Thus, the possibility of arbitraging and of a country switching from one side of the market to the other as prices change is assumed away.

When each country's position on the world market is not set a priori, the assumptions about arbitraging are (implicitly) left to the structural characteristics of the specific model.[3] These assumptions may strongly affect the solution obtained. For example, in a model with no transportation costs, and each country left free to export and import at the same time, the imposition of a tariff by an importing country on its imports from all but one of the other countries leaves each country's net trade position unchanged. Only trade flows change because the demand of the tariff-imposing country will be satisfied by the exporting countries bypassing the tariff by costlessly rerouting their exports through the preferred country. However, a very different outcome is obtained if regions cannot import and export at the same time.

Discriminatory Trade Policies and Commonly Used Trade Models

Nonlinear Programming Models

The most commonly used spatial trade models are the Quadratic Programming (QP) models developed by Takayama and Judge (Thompson, p. 28) in which an artificial quadratic net quasi-welfare function is maximized subject to a set

[2] It can vanish as a result of the implementation of preferential tariffs or targeted subsidies. It is lost when country-specific embargoes are imposed. The generalized transfer costs matrix being no longer consistent is a necessary, but not sufficient, condition for arbitraging to be profitable.

[3] It should be noted, as pointed out by a reviewer, that spatial trade models, because of their homogenous good assumption, tend to overemphasize the occurrence of arbitraging. Hence, it is wise to use caution when considering commodities for which this assumption appears to be particularly strong.

of linear constraints. The quasi-welfare function is given by the sum of consumers' and producers' surpluses over all the regions considered. Linear demand and supply functions, large countries, and perfect competition in domestic and world markets are assumed (Takayama and Judge 1964 and 1971, Bawden, Takayama). Rowse expanded the QP formulation of the model to include nonlinear demand and supply functions. Thus, the classical QP model is a special case of the more general nonlinear programming (NLP) formulation.

Takayama and Judge (1971, chap. 10) propose a framework to analyze trading when tariffs and subsidies are present and suggest that it can also be used when discriminatory trade policies are active.[4] They propose two alternative modeling approaches, based on domestic demand and supply functions (Takayama and Judge 1971, chap. 7, 8), and on excess supply/demand functions (Takayama and Judge 1971, chap. 9). They claim that in a large spectrum of cases the two models are equivalent, and that the second one may be more efficient. However, when discriminatory trade policies are considered, the equivalence of the two models may vanish.

The first model, which uses domestic demand and supply functions, leaves each country free to import and export at the same time but implicitly constrains imports to not exceed domestic consumption. The second model leaves the possibility of arbitraging totally free. These results can be easily verified by comparing the Kuhn-Tucker conditions associated with the two models.[5] When the first model is used and the constraint on arbitraging is binding, the arbitraging country's consumption and production prices are not equal. The consumption price is linked to the low price in the region(s) where the imports come from, while the production price is linked to the high price in the region(s) exports are shipped to. This implicit constraint on arbitraging reproduces a condition similar to that imposed by the "rules of origin" observed in real-world preferential trade agreements.

When discriminatory trade policies are present, the two models can yield different results. This will be the case when the solution obtained by using the model based on excess demand and supply functions is such that (*a*) at least one region exports and imports at the same time, and (*b*) its imports exceed its consumption.

When each country is represented as both a consuming and a producing region, the Kuhn-Tucker conditions of the NLP problem as formulated by Rowse are analogous to those of the QP formulation of the model based on domestic demand and supply functions. Arbitraging is possible, but in each country imports cannot exceed domestic consumption. If each region is a priori defined as an importer or as an exporter, then, a no-arbitraging constraint is implicitly imposed. The assumptions about arbitraging implicit in nonlinear programming models are summarized in table 1.

Whenever the classical QP models or NLP models are used to analyze markets characterized by discriminatory trade policies such that the generalized transfer costs matrix is not consistent, assumptions about arbitraging are implicitly made. Such assumptions may have serious implications for the trade policy analysis.

Vector Sandwich Models

MacKinnon (1975, 1976) proposed a vector sandwich procedure to solve spatial trade equilibrium problems.[6] The procedure allows for nonlinear demand and supply functions as well as transportation costs. Holland developed a microcomputer program based on MacKinnon's procedure to solve relatively small, single commodity, spatial equilibrium models. Holland's program is capable of handling import and export tariffs, both ad valorem and per unit. Constraints may be imposed on specific flows as well as on individual countries' overall imports or

[4] "In this example we use the same demand and supply functions and transportation costs as in chapters 7 and 8, but assume that $\pi_{21} = 1$ and $\pi_{31} = 1$. It is not necessary to assume that $\pi_{1j} = \pi_{2j} = \ldots = \pi_{nj}$ for all j; that is, the tariff may be discriminatory" (Takayama and Judge 1971, p. 201).

[5] For the quantity formulation of the model based on the domestic demand and supply functions the Kuhn-Tucker conditions are given in (7.2.9.d) in Takayama and Judge 1971, p. 133; the analogous conditions for the price formulation of the same model are given in (8.3.7.a) and (8.3.7.b), p. 159 (the equivalence of the price and the quantity formulations is proven in Takayama and Woodland). These conditions imply that, if the domestic price is different from zero, (*a*) domestic consumption must be equal to the portion of the domestic production which is consumed domestically plus the sum of all the imports from the other countries, and (*b*) domestic production must be equal to the portion which is consumed domestically plus the sum of all exports to the other regions. Each country may import and export at the same time, with the constraint that in each country imports cannot exceed domestic consumption. When domestic consumption is entirely satisfied through imports, domestic production is entirely exported. For the second model, the one based on the use of excess supply/demand functions, the Kuhn-Tucker conditions are given in (9.1.27.d), p. 182, and in (9.3.4), p. 194. In this case, if the domestic price is different from zero, domestic production minus domestic consumption plus imports minus exports must be equal to zero. Arbitraging is now left totally unconstrained.

[6] A good introduction to fixed point theory as a tool in finding economic equilibrium solutions is Zangwill and Garcia, chapters 5, 6, and 7.

Table 1. Implicit Assumptions About Arbitraging of the QP, NLP, and VS Models

Model	Each Country is Represented through:	Implicit Assumptions About Arbitraging
Quadratic programming models (Takayama and Judge 1964 and 1971, Bawden, Takayama)	linear domestic demand and supply functions	arbitraging allowed but constrained not to exceed domestic consumption (imports cannot be reexported)
	a continuous linear excess demand and supply function	arbitraging allowed and unconstrained
Nonlinear programming models (Rowse)	nonlinear domestic demand and supply functions	arbitraging allowed but constrained not to exceed domestic consumption (imports cannot be reexported)
	a nonlinear excess demand or supply function	arbitraging not allowed
Vector sandwich models (MacKinnon 1975 and 1976, Holland)	linear or nonlinear domestic demand and supply functions	arbitraging allowed but constrained not to exceed domestic consumption (imports cannot be reexported)
	a linear or nonlinear excess demand or supply function	arbitraging not allowed

exports. However, discriminatory tariffs and subsidies are not explicitly considered, although the user can overcome the problem by providing the generalized transfer costs instead of the transportation costs. Applications of the vector sandwich method in agricultural trade policy analyses include Holland and Sharples, USDA (chap. 11), Haley, and Kahn and Meilke.

If each country is considered as both consuming and producing, arbitraging is allowed but, again, it is constrained not to exceed domestic consumption. Each country may import and export at the same time, but imports may not be reexported (table 1).

The Model

In this section a spatial trade model is presented to analyze settings where disriminatory trade policies make the generalized transfer costs matrix inconsistent. The model has two main features: (*a*) each country can move from one side of the market to the other as prices change, and (*b*) the user is allowed to explicitly specify assumptions on arbitraging through two sets of parameters. The model is given in two forms, one based on domestic supply and demand functions, the other based on excess demand/supply functions. These functions need not be linear. The model based on domestic demand and supply functions can be constrained so that arbitraging (*a*) cannot occur, (*b*) is allowed but is constrained not to exceed domestic consumption, or (*c*) is allowed and left completely unconstrained. When using the model in which each country is represented through its excess demand/supply function, only the first and the third scenarios can be implemented (table 2). The main advantage of the model we propose over the commonly used models is that it provides the researcher with the possibility of easily comparing within a single model structure the effects of discriminatory trade policies under different hypotheses about arbitraging.

Only one commodity is considered in the model proposed. In addition, a partial equilibrium framework, fixed exchange rates, and perfect competition on domestic and world markets

Table 2. Assumptions About Arbitraging Which Can Be Explicitly Incorporated in the Model Proposed

Each Country is Represented through:	Assumptions About Arbitraging Which Can Be Explicitly Incorporated
Linear or nonlinear domestic demand and supply functions	Arbitraging allowed and unconstrained Arbitraging allowed but constrained not to exceed domestic consumption Arbitraging not allowed
A linear or nonlinear continuous excess demand and supply function	Arbitraging allowed and unconstrained Arbitraging not allowed

are assumed. The model maximizes an artificial quasi-welfare function (W) defined as in Samuelson, Takayama and Judge, and Rowse. When domestic demand and supply functions are considered, the model may be stated as follows:

$$(3)\quad \max_{x_{ij}} W = \sum_i \theta_i(y_i) - \sum_i \phi_i(s_i) - \sum_{ij} [(t_{ij} + \pi_{ij} - \sigma_{ij})x_{ij}],$$

subject to

$$(4)\quad \xi_i^2\left\{[1 - (\xi_i - 1)/-2]\left[\sum_j x_{ji} - x_{ii}\right] + [(\xi_i - 1)/-2]\left[\sum_j x_{ij} - x_{ii}\right]\right\} = 0, \quad i = 1, \ldots, n;$$

$$(5)\quad \Psi_i\left(\sum_j x_{ji} - x_{ii}\right) - y_i \leq 0, \quad i = 1, \ldots, n;$$

$$(6)\quad x_i = \sum_j x_{ij} - \sum_j x_{ji}, \quad i = 1, \ldots, n;$$

$$(7)\quad y_i = \max\{x_{ii}, x_{ii} - x_i\}, \quad i = 1, \ldots, n;$$

$$(8)\quad s_i = y_i + x_i, \quad i = 1, \ldots, n;$$

$$(9)\quad x_{ij} \geq 0; \quad i, j = 1, \ldots, n;$$

where i and j denote the regions ($i, j = 1, 2, \ldots, n$); y_i denotes the quantity consumed in country i; s_i denotes the quantity produced in country i; $\theta_i(y_i)$ denotes the integral under the inverse domestic demand of region i, $p_i^d(y_i)$, between 0 and y_i; $\phi_i(s_i)$ denotes the integral under the inverse domestic supply of region i, $p_i^s(s_i)$, between 0 and s_i; x_{ij} denotes the flow of commodity from region i to region j; x_i denotes the total exports (if positive) or the total imports with the sign changed (if negative) of region i; t_{ij} denotes the fixed per unit transportation cost for shipping the commodity from region i to region j;[7] π_{ij} denotes the per unit tariff imposed by region j on its imports from region i; σ_{ij} denotes the subsidy paid by region i for each unit exported to region j; ξ_i denotes a parameter controlling the possibility of the ith region to arbitrage, and, if arbitraging is not allowed, the side of the market on which it appears (this parameter may be set to be equal to -1, 0 or 1: it will be equal to 0 for the nonbeneficiary regions, and for the beneficiary ones which are left free to arbitrage; to -1 for the beneficiary countries which are not allowed to arbitrage and may operate on the market as importers only, to 1 for those which are not allowed to arbitrage and may operate on the market as exporters only); and Ψ_i denotes a parameter constraining arbitraging, when it is allowed, not to exceed domestic consumption (it will be equal to 1 when country i's imports must not exceed its domestic consumption, to 0 otherwise).

Constraints (6)–(9) are self-explanatory. When Ψ_i in (5) is set equal to 1, arbitraging cannot exceed domestic consumption. When Ψ_i is equal to 0 arbitraging is not constrained by (5). Constraint (4) allows the user to impose that regions do not arbitrage. When arbitraging is not allowed, (4) provides the model with the positions to be taken in the market (importer/exporter) by each region.[8]

Once the model is solved, equilibrium prices are computed as

$$(10)\quad p_i^d = p_i^d(y_i^*); \quad i = 1, \ldots, n;$$

$$(11)\quad p_i^s = p_i^s(s_i^*); \quad i = 1, \ldots, n.$$

Each country's producers and consumers welfare is defined (fig. 1) as the area between the inverse demand function and the price (the consumers' price if consumers and producers prices are not equal) line, plus the area between the price (the producers' price if consumers and producers prices are not equal) line and the horizontal axis or the inverse supply function, plus the tariff revenue (which is assumed to be redistributed to consumers and producers as a lump sum transfer):

$$(12)\quad W_i = [\theta_i(y_i^*) - y_i^* p_i^d(y_i^*)] + \left[p_i^s(s_i^*)s_i^* - \int_{\max[0,S^{ri}]}^{s_i^*} p_i^s(s_i)\, ds_i\right] + \sum_j \pi_{ji} x_{ji}, \quad i = 1, \ldots, n,$$

[7] Nonlinear transportation costs may be easily included. They are assumed to be linear to keep the discussion as close as possible to the standard QP setting.

[8] In the case of a preferential tariff, for example, each beneficiary country has to impose a prohibitive tariff either on its imports or on its exports. The decision on which kind of tariff to implement implies an explicit policy choice. Imports are taxed when the country wants to make use of the preference. Exports are taxed, to make arbitraging unprofitable, when it finds itself better off by importing. The decision is based on the maximization of the beneficiary country's welfare. In many cases, this choice may be easy, as it is the case when only one country is granted a preferential treatment and it is already exporting prior to the implementation of the preferential tariff. In other cases the choice may not be so obvious (Anania). An importing beneficiary country may reach, for example, a higher welfare by remaining on the importers' side of the market even if the preferential treatment granted would make it possible for it to switch to being an exporter.

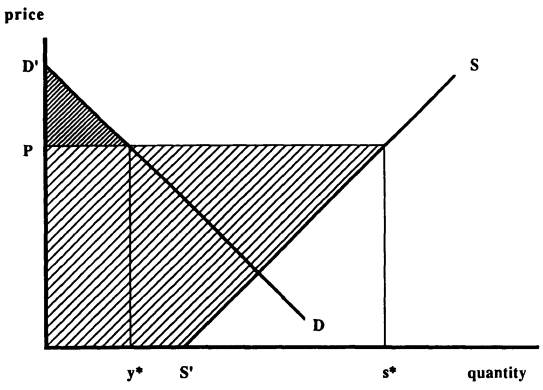

Figure 1. Welfare components for an exporting country

where S_i' is the intercept of the inverse supply function on the horizontal axis. In figure 1 $S'S$ is the inverse supply function, $D'D$ is the inverse demand function, p is the equilibrium price, y^* and s^* are the quantities consumed and produced, respectively, and the crosshatched areas sum to the country's producers' and consumers' welfare.

Often estimates of the domestic supply and demand functions for each region are not available, while estimates of the excess demand/supply functions are. In addition, the excess functions can be more easily estimated. Thus, a model formulation based on excess functions is presented as well. The structure of this model is essentially the same as the one based on the domestic demand and supply schedules. In figure 2 a simple two-country world market case is presented. S_aS_a and S_bS_b are the inverse excess demand/supply functions of countries A and B, respectively. The market equilibrium is such that region A imports $-X_a$ from region B ($-X_a$ is equal to X_b). The equilibrium prices in the two regions are P_a and P_b, respectively. The difference between the two prices is equal to the transportation cost of shipping one unit from B to A, plus the per unit import tariff imposed by country A, minus the per unit export subsidy paid by country B. The crosshatched areas in figure 2 represent the gains from trade. When excess supply/demand functions are used, W, the artificial quasi-welfare function, may be defined as

(13)
$$W = \sum_i [-\chi_i(x_i)] - \sum_{ij} [(t_{ij} + \pi_{ij} - \sigma_{ij})x_{ij}],$$

where $\chi_i(x_i)$ denotes the integral under the inverse excess supply/demand function of region i, $p_i(x_i)$, between 0 and x_i.

In figure 2 the gains from trade of regions A and B are given by the sum of the areas CDP_a and P_bGF. These gains may be obtained by subtracting the areas FGX_b0 and CP_aP_bE from the area $CDOX_a$. This is given by expression (13) for the n-countries case. $\Sigma_i[-\chi_i(x_i)]$, in fact, gives the algebraic sum of the areas under each region's excess supply/demand function, as a positive term if the region is importing ($x_i < 0$), negative if it is exporting ($x_i > 0$). The net gains from trade are obtained by subtracting the transportation costs plus the tariff revenues minus the

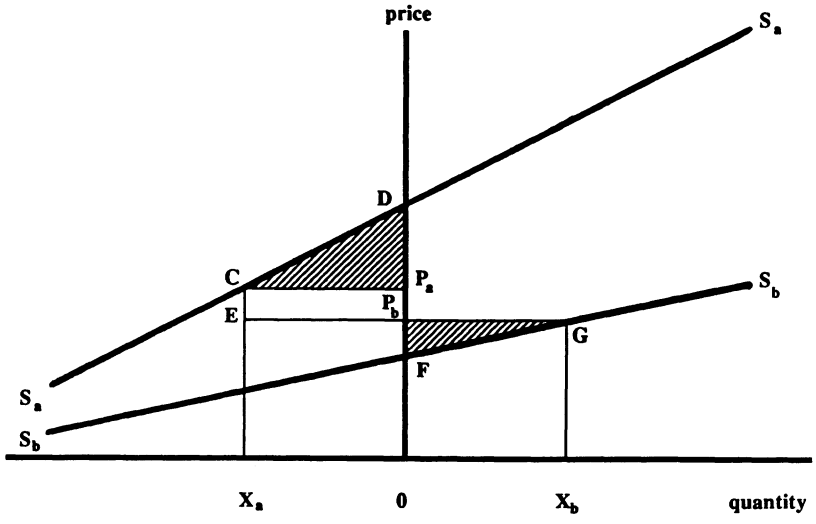

Figure 2. Two-country world trade equilibrium

subsidy expenditure (the algebraic sum of tariffs, subsidies and transportation costs is given in fig. 2 by area CP_aP_bE).

When excess demand/supply functions are used, arbitraging can no longer be constrained to not exceed domestic consumption.[9]

The problem now is stated as

$$\max_{x_{ij}} W, \tag{14}$$

subject to

$$\xi_i^2\left\{[1-(\xi_i-1)/-2]\sum_j x_{ji} + [(\xi_i-1)/-2]\sum_j x_{ij}\right\} = 0, \quad i=1,\ldots,n; \tag{15}$$

$$x_i = \sum_j x_{ij} - \sum_j x_{ji}, \quad i=1,\ldots,n; \tag{16}$$

$$x_{ij} \geq 0; \quad i,j=1,\ldots,n. \tag{17}$$

Equilibrium prices and individual countries gains from trade are computed as

$$p_i = p_i(x_i^*), \quad i=1,\ldots,n; \tag{18}$$

$$W_i = [p_i(x_i^*)x_i^*] - \chi_i(x_i^*) + \sum_j (\pi_{ji}x_{ji}^*), \quad i=1,\ldots,n. \tag{19}$$

A Numerical Example

In this section a numerical example illustrates how the results from the proposed model compare with those which may be obtained using other spatial trade models. While the exercise provides some interesting insights about embargoes, its objective does not go beyond showing the effectiveness of the proposed model in analyzing the effects of discriminatory trade policies by explicitly investigating alternative hypotheses regarding arbitraging and cooperation by the other actors involved.

The focus is on the 1980 U.S. embargo of the USSR. It lasted from 4 January 1980 to 24 April 1981 and covered several agricultural products, including wheat, feed grains, soybeans, meat, and dairy products. The embargo was only partial, because the United States fulfilled its commitment to the 1975 U.S.–USSR trade agreement allowing the export to the USSR of 8 million tons of grains in 1979/80 and 1980/81.

The embargo was motivated solely on the basis of a foreign policy concern, as a retaliation to protest the "USSR invasion of Afghanistan." The decision was based on a Central Intelligence Agency (CIA) estimate that the embargo would strongly affect meat consumption in the USSR (USDA). The CIA estimate assumed full cooperation of all other exporters and ignored the possibility of countries arbitraging. The actual short-run impact of the embargo was substantially smaller than expected. It had a very small impact, if any, on meat consumption in the USSR. The USSR strategy was essentially based on (*a*) replacing imports from the United States by increased imports from other sources, (*b*) increasing imports of substitute goods, and (*c*) slightly reducing its stocks.

The main data source is the comprehensive study mandated by the U.S. Congress (USDA). Consistent with the approach followed so far, only one commodity (wheat) is considered in a partial equilibrium framework. Excess supply/demand functions for twenty-seven regions are derived from base net trade positions, prices, and trade elasticities used in USDA. The transportation costs matrix expands on that used in Holland and Sharples.[10] The model is short run in nature; that is, production is fixed and only consumption, stocks, and trade flows change in response to changes in prices. The model time framework is 1980, the only full year the embargo was in place. Domestic as well as trade policies have been incorporated by including price transmission elasticities in the computation of the trade price elasticities (USDA).

The objective is an a priori evaluation of the effects of a zero constraint on wheat exports from the United States to the USSR. Hence, the base solution has no constraints on the trade flows. This solution is then compared with five different scenarios in which the embargo is active and different hypotheses regarding arbitraging and the cooperation of the other countries are assumed. In the first one it is assumed that no country cooperates and arbitraging can occur; that is, the only constraint imposed is the zero constraint on the U.S.–USSR trade flow. The second scenario assumes that all the other ex-

[9] However, in this formulation of the problem, as well as in the one based on domestic demand and supply functions, each country's arbitraging may be easily constrained not to exceed a specific amount by inserting an ad hoc constraint.

[10] Detailed information regarding the excess supply/demand functions and the transportation costs matrix used are in Anania and McCalla, which can be obtained from the authors.

porters in the base scenario (Canada, the European Community, Oceania, and Argentina) agree not to increase their exports to the USSR above the pre-embargo levels. However, importers are left free to arbitrage. In the third scenario all countries are cooperating; that is, exporters agree not to increase their exports to the USSR, and importers agree not to arbitrage. The fourth and fifth scenarios differ from the second and the third, respectively, only because Argentina is now assumed not to cooperate. (In 1980 Canada, the European Community, and Oceania agreed not to increase their exports to the USSR, even if their actual level of cooperation remains questionable, while Argentina announced that it was not going to cooperate.)

Trade flows and net trade positions in the base solution are given in table 3. In the pre-embargo scenario the United States exports 33.7 million tons of wheat, and exports to the USSR equal 5.3 million tons. The other net exporters are Canada (17.5 million tons), the European Community (9.1), Oceania (12.2), and Argentina (4.9). Major importers are Japan (5.7 million tons), East Europe (5.4), USSR (14.4), China (12.2), Egypt (5.3), and Middle East (5.3).

Under the first scenario, in which (*a*) the United States stops its exports to the USSR, and (*b*) other countries do not cooperate (table 4), the embargo has negligible effects on the twenty-seven regions' net trade positions. USSR wheat imports from the United States are replaced by increased imports from Canada and, as a result, USSR total wheat imports decline by only 59,000 tons. The United States, in turn, made up for the embargo on its exports to USSR by (*a*) increasing its exports toward regions they were already exporting to, and (*b*) exporting 2 million tons to East Europe and half a million tons to Egypt. Essentially, if the United States imposes the embargo with no cooperation from the other market participants, the policy results in a complete failure. Net trade positions remain (almost) unchanged, and only marginal welfare losses arise from increased transportation costs due to the changes in the trade flows.

In the second scenario the U.S. embargo receives full cooperation from all the regions exporting in the base solution (Canada, EC, Argentina, and Oceania). They agree not to increase their exports to the USSR above the base solution levels. However, importers are left free to arbitrage. This scenario actually is very close to the one the United States was trying to reach in 1980. The results of this simulation (table 5) show that obtaining exporters' cooperation does

Table 3. The 1980 U.S. Embargo of the USSR: Base Solution; Trade Flows and Net Trade Positions (million tons)

	Source					
Destination	U.S.	Canada	EC	Oceania	Argentina	Net Trade Positions
Other Western Europe	1.755					−1.755
Japan	5.698					−5.698
South Africa				.011		−.011
East Europe		5.351				−5.351
USSR	5.282	9.090				−14.373
China	9.395			2.792		−12.187
Mexico	.793					−.793
Central America	2.129					−2.129
Brazil	4.786					−4.786
Venezuela	.744					−.744
South America	3.119					−3.119
Sub-Saharan Africa		2.007			.297	−2.305
Nigeria		.495			.510	−1.006
Egypt		.554	4.729			−5.283
North Africa			4.381			−4.381
India				.067		−.067
South Asia					2.748	−2.748
Indonesia				1.505		−1.505
Thailand				.177		−.177
Southeast Asia					1.345	−1.345
East Asia				2.421		−2.421
Middle East				5.275		−5.275
Net trade positions	33.702	17.498	9.110	12.248	4.901	65.211

Table 4. The 1980 U.S. Embargo of the USSR, Scenario No. 1: Embargo Active, No Country Cooperating; Trade Flows and Net Trade Positions (million tons)

	Source					
Destination	U.S.	Canada	EC	Oceania	Argentina	Net Trade Positions
Other Western Europe	1.758					−1.758
Japan	5.698					−5.698
South Africa				.011		−.011
East Europe	2.073	3.255				−5.328
USSR		14.314				−14.314
China	11.941			.264		−12.205
Mexico	.796					−.796
Central America	2.130					−2.130
Brazil	4.787					−4.787
Venezuela	.744					−.744
South America	3.122					−3.122
Sub-Saharan Africa					2.306	−2.306
Nigeria					1.006	−1.006
Egypt	.532		4.746			−5.278
North Africa			4.373			−4.373
India				.068		−.068
South Asia				2.024	.727	−2.752
Indonesia				1.506		−1.506
Thailand				.177		−.177
Southeast Asia				.485	.862	−1.347
East Asia				2.423		−2.423
Middle East				5.284		−5.284
Net trade positions	33.582	17.569	9.119	12.243	4.900	65.167

Table 5. The 1980 U.S. Embargo of the USSR, Scenario No. 2: Embargo Active, All Exporters Cooperating; Trade Flows and Net Trade Positions (million tons)

	Source							Net Trade
Destination	U.S.	Canada	EC	Oceania	Argentina	East Eur.	O.W. Eur.	Positions
Other Western Europe	3.594							−1.762
Japan	5.699							−5.699
South Africa				.011				−.011
East Europe		8.421						−5.347
USSR		9.090				3.074	1.832	−13.996
China	12.000			.228				−12.228
Mexico	.799							−.799
Central America	2.131							−2.131
Brazil	4.789							−4.789
Venezuela	.745							−.745
South America	3.126							−3.126
Sub-Saharan Africa					2.308			−2.308
Nigeria					1.006			−1.006
Egypt	.548		4.734					−5.282
North Africa			4.379					−4.379
India				.069				−.069
South Asia				1.174	1.584			−2.758
Indonesia				1.508				−1.508
Thailand				.177				−.177
Southeast Asia				1.348				−1.348
East Asia				2.426				−2.426
Middle East				5.295				−5.295
Net trade positions	33.430	17.511	9.113	12.236	4.898	−5.347	−1.762	64.940

not guarantee effective results. USSR wheat imports are now predicted to decrease only by 400,000 tons. USSR import price, on the other hand, raises by $10.48 because of increased transportation costs (table 9). The USSR substitutes for its imports from the United States thanks to arbitrage. It imports 3 and 1.8 million tons from East Europe and from the non-EC western European countries, respectively. Both regions are net importers and arbitrage increased exports from the United States (East Europe) and from Canada (other West Europe). U.S. exports decrease by only 272,000 tons, while price falls by $1.32. World wheat trade falls only by 271,000 tons. Hence, these simulations suggest that, even if the United States had obtained cooperation from the other exporters, this would not have assured a significant impact of the embargo because of arbitraging. Arbitraging, however, was not considered a relevant issue during the policy design and implementation.

In the third scenario (table 6) all countries cooperate—exporting countries restrict their exports to the USSR not to exceed pre-embargo levels, and importing countries do not arbitrage. The embargo impact is now significant. USSR wheat imports equal only 9.090 million tons, 5.3 million tons below the pre-embargo level. If Canada, which is the only country exporting to the USSR, does not exploit market power, then USSR import price is $13.49 lower than the pre-embargo one (table 9). U.S. exports decrease by 2.8 million tons and the export price by $13.69. World wheat trade decreases by 3.7 million tons. It should be noted that the United States is not the only region paying a price for the U.S. embargo. Canada's exports fall by almost 600,000 tons and its export price by U.S. $13.49. EC, Argentina, and Oceania all experience lower exports and prices because of the increased competition from U.S. exports in their traditional markets. (The same argument is in Paddock.)

The fourth and fifth scenarios (table 7, 8) show that Argentina not cooperating was a sufficient condition to make the U.S. effort hopeless. In fact, when all exporters but Argentina are constrained to export to the USSR volumes not exceeding the pre-embargo levels, the impact of the embargo is very small. USSR imports decline only by 371,000 tons when arbitraging is allowed (and some arbitrage occurs), and by 382,000 tons when importing countries are assumed to cooperate fully.

The third scenario, in which all exporters cooperate and no arbitraging occurs, likely reflects the one assumed by the CIA when anticipating

Table 6. The 1980 U.S. Embargo of the USSR, Scenario No. 3: Embargo Active, All Countries Cooperating; Trade Flows and Net Trade Positions (million tons)

	Source					
Destination	U.S.	Canada	EC	Oceania	Argentina	Net Trade Positions
Other Western Europe	1.827					−1.827
Japan	5.713					−5.713
South Africa				.013		−.013
East Europe		5.547				−5.547
USSR		9.090				−9.090
China	11.575			1.043		−12.618
Mexico	.850					−.850
Central America	2.146					−2.146
Brazil	4.818					−4.814
Venezuela	.762					−.762
South America	3.184					−3.184
Sub-Saharan Africa		1.412			.928	−2.340
Nigeria		.015			.993	−1.008
Egypt			5.345			−5.345
North Africa		.843	3.651			−4.494
India				.083		−.083
South Asia				1.310	1.553	−2.863
Indonesia				1.531		−1.531
Thailand				.180		−.180
Southeast Asia					1.381	−1.381
East Asia				2.480		−2.480
Middle East				5.480		−5.480
Net trade positions	30.870	16.907	8.996	12.120	4.856	61.501

Table 7. The 1980 U.S. Embargo of the USSR, Scenario No. 4: Embargo Active, All Exporters but Argentina Cooperating; Trade Flows and Net Trade Positions (million tons)

	Source						Net Trade
Destination	U.S.	Canada	EC	Oceania	Argentina	O.W. Eur.	Positions
Other Western Europe	2.008						−1.763
Japan	5.699						−5.699
South Africa				.011			−.011
East Europe		5.347					−5.347
USSR		9.090			4.668	.244	−14.002
China	12.234						−12.234
Mexico	.800						−.800
Central America	2.131						−2.131
Brazil	4.789						−4.789
Venezuela	.746						−.746
South America	3.126						−3.126
Sub-Saharan Africa		2.146			.158		−2.304
Nigeria		.929			.077		−1.005
Egypt			5.282				−5.282
North Africa	.552		3.829				−4.380
India				.067			−.067
South Asia				2.748			−2.748
Indonesia				1.506			−1.506
Thailand				.177			−.177
Southeast Asia				1.345			−1.345
East Asia				2.422			−2.422
Middle East		1.308		3.970			−5.278
Net trade positions	33.392	17.511	9.111	12.247	4.903	−1.763	64.912

Table 8. The 1980 U.S. Embargo of the USSR, Scenario No. 5: Embargo Active, All Countries but Argentina Cooperating; Trade Flows and Net Trade Positions (million tons)

	Source					
Destination	U.S.	Canada	EC	Oceania	Argentina	Net Trade Positions
Other Western Europe	1.764					−1.764
Japan	5.699					−5.699
South Africa				.011		−.011
East Europe	.214	5.128				−5.342
USSR		9.090			4.901	−13.991
China	12.237					−12.237
Mexico	.800					−.800
Central America	2.131					−2.131
Brazil	4.789					−4.789
Venezuela	.746					−.746
South America	3.127					−3.127
Sub-Saharan Africa		2.303				−2.303
Nigeria		1.003			.002	−1.005
Egypt			5.283			−5.283
North Africa	.554		3.827			−4.381
India				.067		−.067
South Asia				2.749		−2.749
Indonesia				1.506		−1.506
Thailand				.177		.177
Southeast Asia				1.346		−1.346
East Asia	1.312			1.110		−2.422
Middle East				5.279		−5.279
Net trade positions	33.373	17.525	9.110	12.246	4.904	64.908

Table 9. The 1980 U.S. Embargo of the USSR: Changes in Import and Export Prices as a Consequence of the Imposition of the Embargo (US$ per ton)

	Base Solution	Scenarios				
		1	2	3	4	5
U.S.	163.28	−.58	−1.32	−13.69	−1.50	−1.60
Canada	165.28	+1.62	+.28	−13.49[a]	+.30	+.60
EC	177.18	+1.02	+.28	−13.49	−.10	.00
Other Western Europe	179.88	−.58	−1.32	−13.69	−1.50	−1.60
Japan	179.88	−.58	−1.32	−13.69	−1.50	−1.60
Oceania	166.98	−.58	−1.32	−13.69	−.20	−.30
South Africa	192.78	−.58	−1.32	−13.69	−.20	−.30
East Europe	181.28	+1.62	+.28	−13.49	+.30	+.60
USSR	183.08	+1.62	+10.48	−13.49[a]	+10.30	+10.60
China	192.18	−.58	−1.32	−13.69	−1.50	−1.60
Mexico	177.08	−.58	−1.32	−13.69	−1.50	−1.60
Central America	177.08	−.58	−1.32	−13.69	−1.50	−1.60
Brazil	178.28	−.58	−1.32	−13.69	−1.50	−1.60
Argentina	167.98	−.38	−1.12	−13.49	−.30	+.60
Venezuela	179.78	−.58	−1.32	−13.69	−1.50	−1.60
South America	179.78	−.58	−1.32	−13.69	−1.50	−1.60
Sub-Saharan Africa	196.48	−.38	−1.12	−13.49	+.30	+.60
Nigeria	196.48	−.38	−1.12	−13.49	+.30	+.60
Egypt	191.78	+1.02	+.28	−13.49	−.10	.00
North Africa	191.78	+1.02	+.28	−13.49	−.10	.00
India	196.98	−.58	−1.32	−13.69	−.20	−.30
South Asia	194.78	−.38	−1.12	−13.49	.00	−.10
Indonesia	190.18	−.58	−1.32	−13.69	−.20	−.30
Thailand	188.18	−.58	−1.32	−13.69	−.20	−.30
Southeast Asia	194.78	−.58	−1.22	−13.49	.00	−.10
East Asia	190.18	−.58	−1.32	−13.69	−.20	−.30
Middle East	189.98	−.58	−1.32	−13.69	−.20	−.30

[a] Assuming Canada does not exercise market power.

a strong embargo impact. The fourth scenario—in which arbitraging is allowed, Canada, Oceania, and the EC keep their exports at the pre-embargo levels and Argentina does not cooperate—represents the one closest to the actual outcome.

The results of this analysis suggest that embargoes are not effective policy instruments. Rather, embargo effectiveness rests on one of the two following conditions: (*a*) all countries cooperate (exporters by freezing their exports to the target country, importers by not arbitraging), or (*b*) all exporters agree to freeze at the pre-embargo levels their exports not only to the target country but to all importing regions. Both conditions are difficult to achieve.

Finally, if all exporters agree to cooperate, they share part of the cost of the embargo because the embargo imposing country's exports will now displace part of their pre-embargo exports. This implies that asking for cooperation in an embargo scheme should be supported by either a reimbursement for the costs or by the guarantee that the embargo-imposing country's exports would not exceed the pre-embargo level minus the volume exported to the country the embargo is imposed on.[11]

[11] A second numerical example providing additional evidence on how the model proposed performs compared with NLP and VS models is in Anania and McCalla. This example addresses the hypothetical *ex ante* analysis of the granting of a preferential tariff reduction. Four different solutions are discussed. In the first solution—obtained using the QP and the VS models based on domestic demand and supply functions and the model proposed in this paper, all assuming, implicitly or explicitly, that arbitraging is allowed but constrained not to exceed domestic consumption—one of the beneficiary countries switches from being an importer to being an exporter due to the tariff preference it is granted. Arbitraging occurs and the constraint on its volume is binding. The second solution has been obtained using the QP model based on excess demand and supply functions, and the model proposed leaving arbitraging unconstrained. The nonpreferred exporter increases its exports, completely bypassing the discriminatory tariff. Its exports to the preference granting country are rerouted through one of the countries benefiting of the preferential treatment. The welfare of the preferred countries is lower than that occurring under a nondiscriminatory tariff. The third and the fourth solutions are obtained using the model proposed imposing no arbitraging and assuming two alternative hypotheses regarding the beneficiary countries' behavior: that they collude, and that each of them makes its choice between being an exporter and being an importer on the basis of its own welfare only. In these two scenarios the preferred countries reach the highest welfare, and the most efficient policy outcomes (in terms of the welfare the preference-granting country has to give up in order to induce a one-unit increase in the welfare of the beneficiary countries) are obtained.

Conclusions

Arbitraging is an important issue in discriminatory trade policy design and implementation. It may turn discriminatory trade policies into ineffective and very costly policy options. Preventing arbitraging is a very difficult task.

In this paper a simple model has been proposed to analyze discriminatory trade policies. It improves upon commonly used spatial trade models by allowing countries to move from one side of the market to the other as equilibrium prices change, and it allows researchers to properly specify their own assumptions about arbitraging and/or to obtain different solutions as a function of different policy constraints or different levels of effectiveness in enforcing such constraints.

The example presented, focusing on the 1980 U.S. embargo of the USSR, provides evidence on the importance of carefully considering arbitraging in the design and implementation of discriminatory trade policies. Different hypotheses regarding arbitraging (or, more often, the apparently "neutral" choice among alternative models thought to be equivalent) may yield contrasting forecasts of the trade policy impact.

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